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Abstract

Section 1 presents analytical results on the consistency of the equalized rate of surplus value in the ‘New Interpretation’ (NI) that are briefly referred to in the paper. Section 2 contains results of robustness tests using alternative assumptions about the frequency of the equalization of the rate of surplus value for the empirical calculations in the paper.

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1 Brief Note on the Equalized Rate of Surplus Value

This section explains the equivalence of the value and money rate of surplus value in the New Interpretation (Duménil 1980, 1984; Foley 1982, 1986). This explanation is juxtaposed with what Foley (2000) calls the ‘dual-system’ approach to Marx.

Let \( n \) be the number of commodities with a single production process and commodity for each industry \( i = 1, \ldots, n \). Let \( p \) denote the \( 1 \times n \) vector of prices, let \( l \) be the \( 1 \times n \) vector of labor requirements for production, let \( A \) denote the \( n \times n \) matrix of capital requirements, and let \( I \) denote the \( n \)-dimensional identity matrix. The \( n \times 1 \) vector \( x \) represents the gross product and the \( n \times 1 \) vector \( y = (I - A)x \) represents net product. With the money wage given by \( w \), the money rate of surplus value \( e \) can be written as:

\[
e = \frac{py - wlx}{wlx} = \frac{p(I - A)x - wlx}{wlx}.
\] (1)

Following the New Interpretation, the monetary expression of labor time (MELT) can be used to convert between magnitudes measured in money and units of labor time. Letting \( \mu \) denote the MELT, the value rate of surplus value \( e_v \) is written as

\[
e_v = \frac{(p(I - A)x - wlx)/\mu}{wlx/\mu}.
\] (2)

Note that equations (1) and (2) are equivalent \( e \equiv e_v \), thus the money rate of surplus value is the same as the value rate of surplus value. This result matches Marx’s (1981, 241-242, 275) assumptions in his presentation of the transformation of values into prices of production, and this result holds only in the New Interpretation (Cogliano 2013).

In what Foley (2000) calls the ‘dual-system’ approach to Marx, the money rate of surplus value and the value rate of surplus value do not necessarily need to match. Let the \( 1 \times n \) vector \( \lambda \) represent the standard dual-system embodied labor values, with \( \lambda = l(I - A)^{-1} \), and let \( b \) denote the \( n \times 1 \) subsistence bundle of workers. In this approach, the value rate of surplus value \( e_\lambda \) is written as:

\[
e_\lambda = \frac{\lambda(I - A - bl)x}{\lambda blx}.
\] (3)

The price rate of surplus value is given in equation (1).

Equations (1) and (3) are equivalent if \( p = \phi \lambda \) for some scalar \( \phi > 0 \). If prices \( p \) are prices of production \( p = (1 + r)(pA + wl) \) with a uniform profit rate \( r \), then if \( r = 0, p = \phi \lambda \). However, for any \( r > 0, p \neq \phi \lambda \). Equations (1) and (2) can also be equal if the organic compositions of capital are equal across industries. Letting \( A_{*j} \) denote a column of the \( A \) matrix, if \( A_{*j}/l_j \) are equal across all \( j = 1, \ldots, n \), then \( p = \phi \lambda \) and \( e \equiv e_\lambda \).

The cases of \( r = 0 \) and/or a uniform composition of capital are unrealistic for capitalist economies, thus \( p \neq \phi \lambda \) in general—as has been shown in the vast literature on the transformation problem. The main conclusion of this appendix is that, in general, the money and value rates of surplus value can be shown to be equal only in the New Interpretation.
2 Alternative Assumptions on the Frequency of EQRSSV

2.1 Rates of Surplus Value Over the Business Cycle

As previously discussed, it is possible assume that $e$ equalizes at different frequencies when imputing surplus value production at the industry level. The calculations in the paper assume that $e$ equalizes annually. An alternative would be to assume that $e$ equalizes over business cycles. Using NBER recessions approximated to the nearest year, the average $e$ from one peak in the business cycle to the next can be found. These peak-to-peak $e$ (within the available data) are shown below in Figure 1. The peak-to-peak $e$ can be used in place of annual $e$ to impute the surplus value production by industry. The imputed surplus value production is restated as percentages of total surplus value production over the time series from 1990-2015 and compared to realization over the same horizon in Figure 2. Figure 3 shows surplus value production versus realization for productive industries from 1990-2015. Since only surplus value production changes with a different assumption about $e$ the data on surplus value realized remains unchanged from the corresponding figures in the paper. Hence, a figure for surplus value realization by unproductive industries is not shown for this exercise. Overall, the results of this alternative assumption about $e$ do not substantially differ from those arrived at through assuming $e$ equalizes annually.

Figure 1: Rate of Surplus Value, Peak-to-Peak $e$, & Profit-Wage Ratio for the U.S. 1990-2015
Figure 2: Total Surplus Value Produced vs. Total Surplus Value Realized 1990-2015 (Peak-to-Peak RSV)

Figure 3: Percent of Total Surplus Value Produced vs. Percent of Total Surplus Value Realized 1990-2015 (Peak-to-Peak RSV)
2.2 Four-year Moving Average Rates of Surplus Value

Another possibility for imputing surplus value production at the industry level is to use moving averages of $e$. A four-year moving average of $e$ is shown in comparison to the annual $e$ in Figure 4. The surplus value imputation is done using the four-year moving average $e$ and shown below in Figures 5 and 6. The four-year moving average $e$ is a shorter time series than the annual $e$, thus the imputation of surplus value cannot be performed for all years in the data. For this exercise the time period is restricted to 1992-2014. Figure 5 shows total surplus value production by industry for 1992-2014 and compares this to total surplus value realization. Figure 6 shows surplus value production versus realization by productive industries in each year from 1990-2014. Because surplus value realized by unproductive industries remains unchanged from the results reported in the paper this chart is omitted here. Using a different assumption about the frequency of the equalization of $e$ does not make much difference to the overall picture of surplus value production and realization, or which industries are net producers of surplus value versus net realizers.

Figure 4: Rate of Surplus Value vs. Profit-Wage Ratio with 4 Year Moving Average RSV
Figure 5: Total Surplus Value Produced vs. Total Surplus Value Realized 1992-2014 (4 Year Moving Average RSV)

Figure 6: Percent of Total Surplus Value Produced vs. Percent of Total Surplus Value Realized 1992-2014 (4 Year Moving Average RSV)
2.3 Time Series Average Rate of Surplus Value

The figures below use the average \( e \) over 1990-2015 to estimate surplus value produced by productive industries. Figure 7 shows total surplus value production by industry and compares this to total surplus value realization. Figure 8 shows surplus value production versus realization by productive industries in each year from 1990-2015. The realization of surplus value by unproductive industries is unchanged from the results reported in the paper and omitted here. This alternative assumption on the frequency of the equalization of \( e \) makes little qualitative difference in the results and provides further confirmation of the robustness of the empirical approach.

Figure 7: Total Surplus Value Produced vs. Total Surplus Value Realized 1990-2015 (Average \( e \) Over Time Period)
Figure 8: Percent of Total Surplus Value Produced vs. Percent of Total Surplus Value Realized 1990-2015 (Average \( e \) Over Time Period)

References


