



Research Paper

Examining expanded differences in slot machine pars: An analysis of revenue impacts and player sensitivity to “price”

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ABSTRACT

This work addressed critical questions related to the optimization of all-important slot revenues, as they are critical to the success of most of the world's casinos. Using data collected from a live gaming floor, the obfuscated pars of otherwise identical reel slot machines were manipulated to determine whether differences in game-level performance and/or player detection would occur. The results indicated significant and substantial increases in revenue for the high-par games, with no clear signs of play migration toward their paired low-par counterparts. These results were produced in spite of the clear disincentive to play the high-par games. This work extended the extant literature by increasing the time to discover the difference in pars to a full year, and by expanding the difference in the pars of the paired games. Additionally, this was the first study to examine the impacts of par differences on actual casino win.

1. Introduction

For most casino operators around the world, slot revenues are critical to the overall success of the operation. As a result, management is often under constant pressure to optimize them. Of course, there are some notable exceptions. For instance, Asian casinos are far more reliant on table game win, and Las Vegas Strip casinos produce a more diverse revenue stream. But at the global level, these hotel-casino resorts represent the exception, rather than the rule. In the usual case, casino venues are heavily reliant on slot win.

Due to the importance of slot win, any insight regarding its production takes on increased value. At the center of this issue is a series of studies that have challenged popular industry assumptions related to the role of the casino advantage (hereinafter, par) in the production of slot win (Lucas, 2019; Lucas and Spilde, 2019a, 2019b). This work has addressed both short- and long-term issues related to increasing the obfuscated pars of reel slot games. The results of these field studies have challenged the ability of players to detect differences in the pars of otherwise identical reel slots. Additionally, the findings from these paired designs have demonstrated significantly increased revenue production from the high-par games.

This work expands the existing series of field studies by increasing the difference in the pars of otherwise identical games, and increasing the sample of daily observations to a full year. Both of these extensions

further address (1) the popular notion that players can detect differences in the concealed pars of reel slots; and (2) the performance effects of increased par gaps, in terms of average, daily, game-level revenue. The results of this study stem from the most extreme manipulation of pars to date, i.e., within the existing field study research stream.

For operators, the findings offer critical business intelligence related to revenue optimization and a deeper understanding of “price” sensitivity, assuming par is a proxy for price. For game makers, the results will inform about the ability of par to influence game-level revenue performance, which directly impacts game sales, revenue sharing agreements with operators, lease fees for games, and financial returns on game development costs. This work exists within an interesting space, as par (or price) is unmarked, and can only be inferred from playing the reel slots. Therefore, little is known about the effects of increased pars at the level examined in this study.

2. Literature review

2.1. Industry positions

The trade literature is replete with concerns for player sensitivity to changes in pars over time (Frank, 2017; Gallaway, 2014; Legato, 2019), among frequent players (Legato, 2019; Meczka, 2017), and with respect to player experience/involvement (Meczka, 2017). These trepidations

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are anchored in premonitions of a mass exodus of play, once the increased pars are detected. This is especially true for operators catering to a frequently visiting clientele. It is believed that given sufficient time to discover such increases, short-term revenues gains will fade (Frank, 2017; Legato, 2019). More specifically, Frank (2017) contends that six to nine months is enough time for a repeat clientele to discover increased pars; however, he does not specify the magnitude of the detectable increases. While these concerns are very real, it is difficult to assess the validity of their foundations. Few if any methodological details are provided regarding the origins of these arguments. In the rare case that they are disclosed, there are considerable issues with the analytical approaches that support the conclusions.

Within the industry, opinions differ between operators and game makers on the impacts of increased pars. Some game makers claim that these “price” hikes have damaged gaming revenues (Applied Analysis, 2015; Rutherford, 2015). Still, some operators defend the games and continue to offer them (Rutherford, 2015; Legato, 2019), suggesting that the higher pars are optimizing revenues. The current study sought to address the critical issues advanced here, with the aim of improving the overall understanding of the par dynamic on the slot player experience.

Prior to a review of the academic literature, some additional background on the core issue may be helpful to those less familiar with the casino business. Many operators believe that lower pars on reel slots lead to greater play time, also known as time on device (TOD), which in turn leads players to conclude that such games offer greater gaming value (Dunn, 2004; Frank, 2017; Legato, 2019; Lucas et al., 2007). This belief is the driving force behind popular advertising claims related to “loose” slots (Legato, 2019). All of this is anchored in the assumption that players can perceive differences in pars, and that pars serve as an effective proxy for price within the domain of reel slots. Therefore, concerns related to player sensitivity to price shocks are common among those who subscribe to this thinking. It follows that these subscribers would use par as the primary positioning variable for their casino’s slot mix, taking great care to keep pars sufficiently low, or at least congruent with those offered by their direct competitors.

2.2. Simulations and lab studies

Harrigan and Dixon (2010) simulated play on two reel slots, with Game A featuring a 15 % par and Game B a 2 % par. Both pay tables were associated with the same licensed game title. Two thousand virtual players engaged each game with a \$1000 bankroll and fixed wagers of \$3.75 per spin. Each player wagered until reaching a credit balance of zero. The mean number of spins per player was greater for the 2 % game, but this difference was dramatically reduced when the median was the measure of central tendency. The authors were careful to note the mean differences were driven by tail events, i.e., outcomes produced by the few players who were lucky enough to hit some top-award jackpots in the process of losing their initial bankrolls. While the increased mean number of spins could signal the potential for player detection of different pars, it would apply to only a few players. Further, these mean differences were produced in part by requiring each virtual player to continue wagering until reaching a zero-credit balance. In a live setting, some players who hit big jackpots would not continue play.

Dixon et al. (2013) used the same two games as Harrigan and Dixon (2010) to determine whether live subjects would be able to detect differences in the pars of the games. Study participants played each of the two equal-appearing games over 30 different play sessions, with each one comprised of 500 spins on each game. Like Harrigan and Dixon, a constant-wager constraint was employed. The researchers captured the payback percentages for each session, on each game, for each participant. These outcomes were used to test the null hypothesis of equal payback percentages, with the results indicating a significant difference for six of the seven participants. That is, the mean payback

percentage was significantly greater for the game with the 2 % par, for six of the seven subjects who completed all 30 sessions. By way of survey, all seven participants were able to correctly identify the 2 % game upon completion of the study. While the par gap of 13 percentage points was substantial (i.e., 15 % vs. 2 %), the results supported the idea that players could detect a difference in the house edge, over time.

Lucas and Singh (2011) simulated games across several par gaps and different levels of pay table variance. Holding the latter constant, their results generally indicated that players would not have the wherewithal to detect a difference in the pars of the games, based on the outcomes of play. Their simulated play sessions ranged from 250 to 2000 spins, and assumed a constant wager. Further, they used the results from the same simulations to conclude that a single player would not be likely to detect a difference in the pars of the paired games, even after playing each of them on 10,000 consecutive days. Their simulations included paired games with par differences ranging from three to nine percentage points (e.g., a 12 % game vs. 3 % game would represent a nine percentage-point gap). Unlike, Harrigan and Dixon (2010), Lucas and Singh did not require their virtual players to gamble until losing an initial bankroll.

2.3. Field study research

Lucas and Brandmeir (2005) failed to find a significant difference in the performance of 38, \$5.00 reel slots located in an Atlantic City casino, after increasing the pars of the games from 5.0% to 7.5%. Their performance measurements consisted of daily, theoretical, win-per-unit (TWPU) observations. These daily TWPU measures were collected over a period of several months in year one and again within the same period of the following year. The mean TWPU was \$582 with the pars set at 7.5 % and \$528 under the 5.0 % par condition. Again, this difference was not statistically significant. The year-over-year design did open the door to the possibility of both endogenous and exogenous forces influencing the results, across time.

Next, results from a series of studies found high-par games to significantly outperform their paired low-par counterparts (Lucas, 2019; Lucas and Spilde, 2019a, 2019b). These researchers manipulated pars of reel slots within a paired design, holding the following variables constant: game title, visible pay table, bank membership, end location (within the bank of games), credit values, minimum and maximum bets, and cabinet design. Further, the performance metrics of the games were assessed within the same time frame, eliminating the possibility for the seasonality or temporal effects associated with the Lucas and Brandmeir (2005) study.

Not only were the observed differences in the theoretical win of the games statistically greater for the high-par slots within all 13 two-game pairings, they were also economically significant (Lucas, 2019; Lucas and Spilde, 2019a). Additionally, these revenue gains were not compromised by eventual play migration. That is, there was no evidence that players were able to detect the differences in the pars of the paired titles over time, in spite of the substantial increases. Of course, the pars on reel slots are obfuscated, so the primary means of detection would be based on evaluating the outcomes from play on each game. The results of the paired-samples t-tests and time series regression analyses, along with time series plots, all supported this conclusion.

Staying within the field study research stream, ten of these two-game pairings analyzed daily outcomes over a six-month sample, with two additional pairings providing nine months of daily observations. The researchers noted that management from the host casinos felt six months was more than enough time for their frequently-visiting clientele to detect the differences in the pars (Lucas and Spilde, 2019a, 2019b). Still, there are those who believe more time is needed for such players to identify and respond to par gaps (Frank, 2017; Legato, 2019), hence the 12-month sample analyzed in this study. Not only did we extend the sample duration, but we also expanded the par gap of the paired games to nearly 11 percentage points. These conditions

represented the most extreme experimental parameters to date, within the field study research stream.

2.4. Cognitive biases

Sundali and Croson (2006) advanced a framework for cognitive biases found to affect gambling decisions, which included the gambler's fallacy, hot outcome, hot hand, and stock of luck. With descriptions dating to Laplace (1820), the gambler's fallacy is a belief in negative autocorrelation within a non-autocorrelated random sequence of outcomes (Sundali and Croson, 2006). Conversely, those prone to hot outcome bias believe in positive autocorrelation within the same series of random outcomes. For example, if a roulette ball were to settle in a red pocket on four consecutive spins, a gambler succumbing to hot outcome bias would overestimate the likelihood of the ball settling in the red pocket on the next spin, whereas someone prone to the gambler's fallacy would underestimate the same likelihood.

The hot hand bias is also a belief in positive autocorrelation within a non-autocorrelated sequence of random outcomes (Sundali and Croson, 2006). But in this case, the affected gambler is overestimating the likelihood of outcomes based on a series of historical results produced by a person/gambler, as opposed to a casino game. In this case, the source of the bias is connected to the gambler, i.e., the hot hand. Its opposite condition has been described as stock of luck, which is based on the notion that gamblers have a finite amount of luck (Sundali & Croson). In this case, afflicted gamblers would underestimate the chances of a bettor winning a wager that followed a string of wins produced by that same bettor, i.e., a belief in negative autocorrelation within in non-autocorrelated series of random outcomes. This condition is based on the idea that the gambler placing the wager has run out of luck.

It could be argued that all of these forms of cognitive bias stem from what Gilovich et al. (1985) referred to as representativeness heuristics. These decision-making short cuts are associated with Tversky and Kahneman's (1971) law of small numbers, which demonstrated the tendency of people to rely on outcomes produced from insufficient samples as effective, useful, or reasonable representations of their associated population parameters. But there are some subtle differences worth noting. With hot hand and stock of luck, the bias stems from over-reliance on a small sample, but it is also associated with a gambler, as opposed to a game. Therefore, the origin of these two forms of bias has also been connected to Langer's (1975) illusion of control, which describes the belief that the gambler producing the wins or losses is somehow influencing the likelihood of the outcomes.

Lab studies have provided support for the tendency of subjects to engage in the gambler's fallacy (Ayton and Fischer, 2004), hot hand, (Gilovich et al., 1985), hot outcome (Edwards, 1961), and stock of luck (Leopard, 1978). Within the domain of live casino gaming, Croson and Sundali (2005) observed market-level evidence of the gambler's fallacy and hot hand biases within a sample of roulette players. Using the same sample, Sundali and Croson (2006) observed the presence of these conditions at the level of the individual gambler. Both studies included observations from live wagering in a casino, with the opportunity for gamblers to incur real losses. Also within the field, behavior consistent with both hot hand and stock of luck has been observed among blackjack players (Keren and Wagenaar, 1985). To the best of our knowledge, no published studies have attempted to identify cognitive bias in the wagering behavior of slot players.

Given the extent to which these cognitive biases have been observed in both the lab and the field, they provide a plausible explanation for the inability of players to detect differences in the pars of reel slots, within the existing series of paired-design field studies (i.e., Lucas, 2019; Lucas and Spilde, 2019a, 2019b). Specifically, all forms of bias would impair the gambler's ability to identify the true house advantage of the game. In turn, this impairment would impact rational play migration toward the low-par game within each pairing. While we do not

directly examine the presence of these cognitive biases, they may be helpful in understanding the results of this study and the previous work in this stream.

2.5. Hypotheses

In step with existing field studies, the following hypotheses were advanced to test for differences in the performance of the paired slot machine titles (Lucas, 2019; Lucas and Spilde, 2019a, 2019b). While H₀₁ (T-win) appears in all of these papers, H₀₂ (Actual Win) is included here for the first time. The limitations of using actual win as a performance measure have been duly noted (Cardno et al., 2015), but the inclusion of H₀₂ was substantiated by our sample size. With a full year of daily observations, the issue with the volatility of actual win as a performance measure was thought to be sufficiently diminished.

$$H_{01}: \mu_{(TW, High)} - \mu_{(TW, Low)} = 0$$

$$H_{02}: \mu_{(AW, High)} - \mu_{(AW, Low)} = 0$$

Within H₀₁, " $\mu_{(TW, High)}$ " indicated the mean daily theoretical win for the high-par game within a specific pairing, while " $\mu_{(TW, Low)}$ " represented the same for the low-par game within the same pairing. This same structure held for H₀₂, with "AW" representing actual win. Both H₀₁ and H₀₂ were tested with data from each of the two-game pairings.

To clarify, theoretical win is the casino's expected value, given the dollar value of the wagers placed and the programmed house edge (i.e., par) on those same wagers. Actual win represents the net dollar value of (1) the dollar amount of wagers placed; and (2) the dollar amount of payouts resulting from those wagers. Of course, some period of time must be declared in the calculation of T-win and actual win (e.g., day, month, year, etc.). The daily actual win data in this study were obtained from the gross win meters of the experimental games.

While H₀₁ and H₀₂ both tested for differences in game-level revenue production, the question of detection over time remained. This was the aim of H₀₃. Hypotheses such as this one have been advanced in existing studies to identify whether play migration was present, i.e., the occurrence of shifts in play to the low-par games (Lucas, 2019; Lucas and Spilde, 2019a, 2019b).

$$H_{03}: B_{TREND (TW)} = 0$$

Here, " $B_{TREND (TW)}$ " represented the regression coefficient for a trend variable, within a time series regression analysis designed to explain the change in the daily T-win difference over time. This difference referred to the daily change in the performance of the paired games. H₀₃ was tested for each two-game pairing.

Due to the duration of the sample, H₀₄ was advanced to test for differences in the actual daily win, for each pairing. The rationale for this hypothesis was directly in line with the justification for H₀₂. That is, with 365 daily observations, the test of H₀₄ provided a meaningful comparison of results against those from H₀₃. Additionally, this data reflected the outcomes that the gamblers actually experienced, as opposed to the ones they "should" have experienced. While there are limitations related to the use of actual win data (Cardno et al., 2015), this lens provided its own value and unique perspective.

$$H_{04}: B_{TREND (AW)} = 0$$

Consistent with the premise of H₀₃, " $B_{TREND (AW)}$ " was the regression coefficient for a trend variable, within a time series regression analysis (i.e., TSRA). This TSRA was designed to explain the change in the daily actual win difference, over time. That is, the daily difference in the performance of the paired games. H₀₄ was also tested for each two-game pairing.

3. Methodology

3.1. Data sources and samples

The data were collected from a tribal hotel-casino resort operating in the western United States. Daily observations of performances metrics were collected beginning on February 8, 2018 and ending on February 7, 2019, for a total of 365 data points for each of the following measures: T-win, actual win, and coin-in. These measurements were collected for each individual game in the study.

The resort offered fewer than 800 hotel rooms and several levels of dining options, ranging from quick-casual to gourmet restaurants. With more than 1800 slot machines and 50 table games, this resort also offered live poker and bingo. Other nongaming amenities included a fitness center, spa, multiple bars, and multiple live entertainment venues. More specific information could not be provided to protect the requested anonymity of the host property.

The host casino was located near a major metropolitan area, with several direct and nearby competitors. Given this location and market condition, there was a critical reliance on patronage from local residents, making this resort an ideal site for this study. Such patrons visit frequently, making them more likely to discover differences in the pars of the games over the sample period. That is, more likely than a tourist-based clientele, engaging the games on a relatively infrequent basis, with a considerable duration between visits.

3.2. Experimental games and configuration

The game titles analyzed in this study were Precious Jade (TRI-1) and Lucky Tree (TRI-2), both of which were manufactured by Scientific Games. Within each paired title, the following game characteristics were held constant: \$0.01 credit value, visible pay table (i.e., reward schedule), minimum and maximum wager, number of betting lines, bonus and/or free game features, and cabinet design. None of the games offered a progressive jackpot, to ensure congruent top awards. Other than very slight and unavoidable changes in the pay table variance, the paired titles differed only in terms of the pars. Table 1 summarizes the par differences for each of the paired titles.

To mitigate location bias at the bank level, the paired games were located on the opposite ends of the same small array of games (See Fig. 1). Assignment within the bank was randomized for the first game. From there, that game's paired counterpart was located on the opposite end of the same lateral side of the bank. For the second pairing, the high- and low-par game positions were transposed with respect to the first pairing, such that the high-par game appeared on the opposite end of the bank. This was done to prevent both high-par games from occupying a position on the same end of the bank (i.e., to address the potential for end bias).

The experimental game titles did not appear in other locations on the slot floor. This was done to increase play on these games, by restricting their availability. This tactic has been employed in previous studies of this kind (Lucas, 2019; Lucas and Spilde, 2019a).

Given all of these experimental design features, there was no reason for gamblers interested in these titles to play the high-par games. In fact, from Table 1, there was a profound disincentive to play the high-

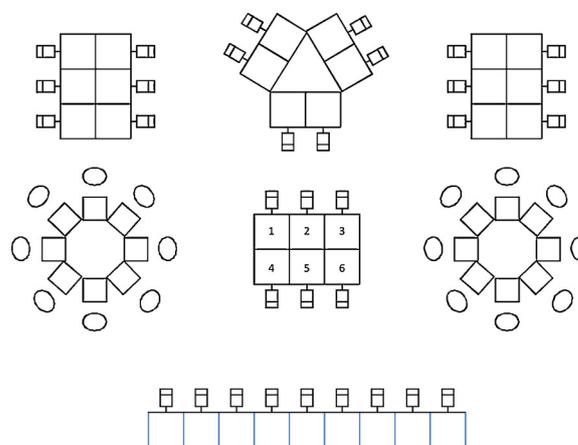


Fig. 1. Illustration of the experimental bank configuration. Note. Unit 1: Precious Jade, 14.77 % par; Unit 2: Precious Jade, 10.05 % par; Unit 3: Precious Jade, 4.02 % par; Unit 4: Lucky Tree, 3.80 % par; Unit 5: Lucky Tree, 9.80 % par; Unit 6: Lucky Tree, 14.54 % par.

par games. Further, there was no clear advantage in the location of the individual games, and the low-par options were in plain sight, located less than three feet from their high-par counterparts. If players were able to detect differences in the pars, then we would expect to see clear signs of increased performance on the low-par games. This design afforded no rational motive for players to risk and lose their bankrolls to the high-par games.

3.3. Data analysis

H₀₁ and H₀₂ were tested by way of paired-samples t-tests, consistent with the existing studies in this research stream (Lucas, 2019; Lucas and Spilde, 2019a, 2019b). Two-independent-samples t-tests were not employed by these researchers due to significant correlation in the observed performance metrics, at the pairing level. This condition violated the requisite assumption of independence. The same significant correlation coefficients were found in our data; hence the dependent-samples approach. Specifically, the Pearson correlation coefficients for the Precious Jade and Lucky Tree pairings were both significant at 0.01 alpha (two-tailed), across the 365 daily T-win observations (i.e., $r = 0.57$ and 0.58 , respectively).

The alpha for the tests of both H₀₁ and H₀₂ was 0.05, but because the tests were repeated for each pair, a Bonferroni adjustment was made. This reduced the effective alpha to 0.025 (i.e., $0.05 \div 2$). These tests were conducted in SPSS, version 24.

H₀₃ and H₀₄ were tested via time series regression analysis. The criterion variables were expressed as the daily percentage change in T-win and actual win, respectively. A separate model was constructed for each criterion variable, within each pairing. All percentage changes were computed from the perspective of the low-par game. For instance, the daily T-win generated by the low-par game was subtracted from the daily T-win produced by the high-par game. This difference was then divided by the daily T-win from the low-par game. This percentage change was computed for each day, for each pairing. The same process was applied to the actual win data.

For measuring the stability of play across time (i.e., play migration), the percentage-change approach mitigated seasonality effects. For example, the total amount of money that customers bring to the casino often declines during the holiday season. If the daily dollar difference in T-win were used as the criterion variable, such conditions could falsely signal play migration. This occurs due to the drop in denominator value (i.e., the T-win on the low-par game). The percentage-change method is not prone to this possible misrepresentation.

After a review of time series plots, a linear trend variable (TREND)

Table 1

Par Comparisons within Two-game Pairings.

2-Game Pairing	Game Theme/Title	Par 1	Par 2	Par Diff. (% Pts.)	Par Inc. (%)
TRI-1	Precious Jade	4.02 %	14.77 %	10.75	267.41
TRI-2	Lucky Tree	3.80 %	14.54 %	10.74	282.63

Notes. Par Diff. (% Pts.) = Par 2(100) - Par 1(100).

Par Inc. (%) = Par Diff. (% Pts) ÷ Par 1(100).

was created by assigning a value of 1 to the first day of the data set, and increasing that value by 1 on each successive day. If H₀₃ or H₀₄ were rejected, it would suggest the presence of significant play migration over the 365-day sample period (i.e., the presence of a non-constant mean difference). To the contrary, a failure to reject these hypotheses would support the absence of significant play migration, or an inability of frequent players to rationally respond to the par increases.

The time series plots also identified outlier dates, with respect to criterion variable values. Binary variables were created for these dates on an as-needed basis, to address the occasional departures from a constant variance over the sample periods. Autoregressive (AR) and moving average (MA) terms were also added as needed, to address serial correlation among the criterion variable values and error terms, respectively. This approach was consistent with prior research (Lucas, 2019; Lucas and Spilde, 2019a, 2019b).

The alpha for the two-tailed tests of H₀₃ and H₀₄ was 0.05, but because the tests were repeated for each pairing the alpha was reduced to 0.025 (i.e., 0.05 ÷ 2). This resulted from the same Bonferroni correction applied to both H₀₁ and H₀₂. Although the data were screened in SPSS, version 24, the formal tests of H₀₃ and H₀₄ were conducted in EViews, version 10.

4. Results

Prior to the testing of H₀₁ and H₀₂, descriptive statistics for the daily differences were examined for each pairing, as summarized in Table 2. This table contains both an all-cases section and an outliers-omitted section, for each pairing. Outliers were identified and omitted based on examinations of histograms and Q-Q plots of daily differences for each variable, within each pairing. Three variables were included in Table 2, although coin-in differences were not formally tested. Coin-in represents the total dollar value of the wagers placed in a game, over a specified period of time, with no regard for resulting payouts. Therefore, it is not a revenue measure. Still, inclusion of the coin-in data affords the reader a more complete picture of the performance outcomes, and provides a useful frame for interpreting the results of the hypotheses.

From Table 2, the daily difference in T-win emerged as the most stable performance metric. Because of the 365-day sample size, the mean actual win (A-win) and mean T-win were reasonably similar. To the contrary, the standard deviation for actual win remained noticeably greater making it a less attractive, but still useful, candidate for measuring performance. Most importantly, all of the mean differences for T-win and A-win were positive and of considerable magnitude. This indicated superior performance by the high-par games, as these differences were computed by subtracting the value for the low-par game

Table 2
Descriptive Statistics: Daily Differences by Two-game Pairing.

2-Game Pairing	Mean	Median	Std. Dev.	Min.	Max.
TRI-1: Precious Jade (n = 365)					
Coin-in: 14.77 % - 4.02 %	-65.23	-51.00	1,117.01	-5,055.50	8,271.00
T-win: 14.77 % - 4.02 %	161.37	118.72	155.04	-48.36	1,584.76
A-win: 14.77 % - 4.02 %	167.57	114.00	358.37	-1,145.08	1,581.89
TRI-1: Outliers Omitted					
Coin-in: 14.77 % - 4.02 % (n = 357)	-31.80	-38.50	788.36	-2,828.50	2,492.50
T-win: 14.77 % - 4.02 % (n = 363)	155.04	118.21	128.11	-48.36	623.46
A-win: 14.77 % - 4.02 % (n = 364)	171.20	117.15	352.13	-830.14	1,581.89
TRI-2: Lucky Tree (n = 365)					
Coin-in: 14.54 % - 3.80 %	-178.99	-80.50	1,091.38	-10,294.00	3,627.00
T-win: 14.54 % - 3.80 %	188.30	160.53	124.40	-119.66	752.66
A-win: 14.54 % - 3.80 %	168.77	135.61	548.26	-2,238.13	3,637.00
TRI-2: Outliers Omitted					
Coin-in: 14.54 % - 3.80 % (n = 362)	-119.02	-73.00	845.79	-3,435.00	3,627.00
T-win: 14.54 % - 3.80 % (n = 362)	184.07	160.34	115.68	-119.66	568.46
A-win: 14.54 % - 3.80 % (n = 363)	150.95	134.92	494.07	-2,238.13	2,079.21

Notes. All statistics represent daily values, expressed in US dollars. "A-win" represents actual win.

Table 3
Results of Paired-Samples t-tests on Daily T-win for each Game Pairing.

2-game Pairing (Pars)	Mean Diff.	S.E. Diff.	t	p	df
All Cases:					
TRI-1 (14.77 % - 4.02 %)	\$161.37	\$8.12	19.885	< 0.0005	364
TRI-2 (14.54 % - 3.80 %)	\$188.30	\$6.51	28.917	< 0.0005	364
Outliers Omitted:					
TRI-1 (14.77 % - 4.02 %)	\$155.04	\$6.72	23.056	< 0.0005	362
TRI-2 (14.54 % - 3.80 %)	\$184.07	\$6.08	30.275	< 0.0005	361

Notes. All positive mean differences indicate a greater mean for the game with the greater casino advantage. All monetary values are expressed in terms of US dollars.

from that of the paired high-par game.

Although not shown in Table 2, there were only ten days on which the low par game recorded more T-win than the high-par game, within the TRI-1 pairing. For TRI-2, there were only six of these days. This established a consistency in the daily differences, with respect to sign.

The coin-in differences were not surprisingly negative, but it was somewhat unexpected that the absolute values were less than the T-win differences for each pairing. This was noteworthy, as \$1 of bankroll can be expected to generate \$20 in coin-in, on a game with a 5 % par, over the long run (i.e., \$1 ÷ 0.05 = \$20). In this case, the ratio of long-run coin-in to T-win would be 20:1. Of course, this assumes that the \$1 is wagered until lost, and that the game retains its par on each spin (i.e., the outcomes follow a geometric distribution). If the game featured a 15 % par, the expected coin-in would be \$6.67, under the same assumptions (i.e., \$1 ÷ 0.15 = \$6.67). In any case, it's much easier for a game to generate \$1 of coin-in than \$1 of T-win. In this regard, the Table 2 data suggested that the pairing-level differences in coin-in were relatively slight in comparison to the corresponding T-win differences.

Table 3 contains the results of H₀₁. With the outliers-omitted, the null was rejected for both TRI-1 and TRI-2 ($\mu = \$155.04$, $df = 362$, $p < 0.0005$ and $\mu = \$184.07$, $df = 361$, $p < 0.0005$, respectively). This indicated a significant difference in the daily means of the paired games, in favor of the high-par units.

Table 4 displays the outcomes of H₀₂ for each pairing. In the outliers-omitted condition, the null was rejected for both TRI-1 and TRI-2 ($\mu = \$171.20$, $df = 363$, $p < 0.0005$ and $\mu = \$150.94$, $df = 362$, $p < 0.0005$, respectively). Like the T-win results from H₀₁, the mean daily A-win was also significantly greater for the high-par games. This similarity in results was likely due to the considerable number of daily observations (i.e., $n = 365$).

Prior to the formal tests of H₀₃ and H₀₄, time series plots of the

Table 4
Results of Paired-Samples t-tests on Daily Actual Win for each Game Pairing.

2-game Pairing (Pars)	Mean Diff.	S.E. Diff.	t	p	df
All Cases:					
TRI-1 (14.77 % - 4.02 %)	\$167.57	\$18.81	8.909	< 0.0005	364
TRI-2 (14.54 % - 3.80 %)	\$168.77	\$28.78	5.865	< 0.0005	364
Outliers Omitted:					
TRI-1 (14.77 % - 4.02 %)	\$171.20	\$18.51	9.250	< 0.0005	363
TRI-2 (14.54 % - 3.80 %)	\$150.94	\$26.00	5.805	< 0.0005	362

Notes. All positive mean differences indicate a greater mean for the game with the greater casino advantage. All monetary values are expressed in terms of US dollars.

Table 5
Summary of Results for Linear Trend Variables. DV: Percentage Change in Daily Win Metrics (n = 365).

Difference Series	Trend Coefficient	S.E. B	t	p
TRI-1 (T-win)	-0.0011	0.0032	-0.3348	0.7380
TRI-2 (T-win)	0.0003	0.0008	0.3939	0.6939
TRI-1 (A-win)	0.0010	0.0051	0.1942	0.8462
TRI-2 (A-win)	-0.0095	0.0194	-0.4889	0.6252

Notes. "Difference Series" refers to the percentage change in the daily win metric for the specified game pairing (e.g., (High-Par T-win - Low-Par T-win) ÷ Low-Par T-win).

such conspicuous play migration had occurred, the slope of these trend lines would have been clearly negative. Of course, a negative slope would presume rational play migration toward the low-par games.

As expected, the A-win plots reflected an increased level of the variance in daily game-level outcomes. Due to the extreme nature of a few data points, the change effects for most of the days were considerably subdued by the y-axis scaling protocol. This was most evident in the Lucky Tree plot, where the majority of the daily changes were small enough to be obscured by the trend line. Still, it was difficult to make a case for clear signs of play migration, with a nearly horizontal trend line.

Table 5 lists the test results for both H₀₃ and H₀₄. Consistent with the time series plots, the trend variables for both TRI-1 (T-win) and TRI-2 (T-win) failed to produce a statistically significant effect (B = -0.0011, p = 0.7380 and B = 0.0003, p = 0.6939, respectively). Both results supported the presence of a horizontal trend line over the sample periods (i.e., a constant mean difference, or stationary mean). Staying within Table 5, the outcomes from the A-win data for both TRI-1 and TRI-2 were consistent with those generated by the T-win observations (B = 0.0010, p = 0.8462 and B = -0.0095, p = 0.6252, respectively). In summary, the data failed to reject H₀₃ and H₀₄ in either pairing.

In terms of assessing play migration, the time series plots may actually be more telling than the results of the hypothesis tests. This is not to say that the tests of H₀₃ and H₀₄ were not helpful. Taken together, along with the elevated performance of the high-par games, it was difficult to conclude that players were detecting a difference in the pars of the paired games, from play alone.

5. Discussion

Both H₀₁ and H₀₂ were rejected in favor of the high-par games, signaling significant and meaningful increases in both T-win and A-win within both pairings. With respect to H₀₃ and H₀₄, neither null hypothesis was rejected, indicating a stable mean difference over the 365-day sample periods. These results failed to support the presence of play migration. Alternatively stated, in spite of the clear disincentive to play the high-par games, gamblers continued to consistently engage them over the year-long sample. Further, there was no evidence of their collective ability to recognize the superior value of the low-par games, which were located less than three feet away.

It is important to note that the results of all our hypothesis tests were produced with the greatest par gaps and sample durations of any field study to date. Still, our findings were consistent with the previous studies in this research stream, regarding the significant increases in revenue on the high-par games, and the lack of evidence for the ability of players to detect differences in the pars (Lucas, 2019; Lucas and Spilde, 2019a, 2019b). Our findings also aligned with Lucas and Brandmeir (2005), regarding the elevated performance of the high-par games, and with Lucas and Singh (2011), in that players would not be likely to discover pars from play alone.

The presence of cognitive bias serves as a plausible explanation and/or contributor to the results of this study, as well as those from prior field work. With strong support for cognitive bias in live casino

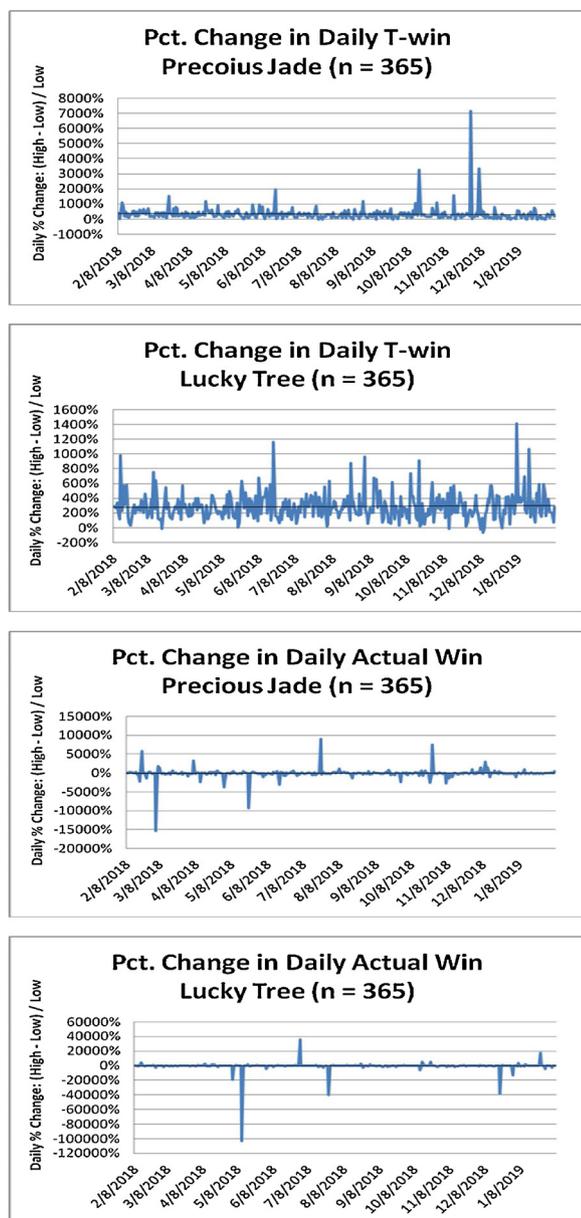


Fig. 2. Time series plots of daily percentage differences in win metrics for each pairing.

daily percentage change in both T-win and A-win were reviewed to assess the stationarity of each difference series. Per Fig. 2, these plots failed to indicate any evidence of problematic play migration, as all illustrated nearly horizontal trend lines over the 365-day samples. If

gaming environments (Croson and Sundali, 2005; Sundali and Croson, 2006; Keren and Wagenaar, 1985) and within controlled lab studies (Ayton and Fischer, 2004; Gilovich et al., 1985; Edwards, 1961; Leopard, 1978), it would seem reasonable that such bias would manifest among slot players. If this were the case, it would certainly impact the occurrence of rational play migration to the low-par games. While we do not expressly examine the presence of cognitive bias in this study, our results are consistent with its effects (i.e., regarding impaired judgment of past and future outcomes).

Our results were also in line with Tversky and Kahneman’s (1971) facetiously-dubbed law of small numbers. Given the typically minimal number of trials (i.e., spins) produced by slot players on a single visit, and the remarkable amount of variance in the outcome distributions of modern slots (Singh et al., 2013), any reliance on such results could impair the gambler’s ability to make valid judgments about pars. With the strong tendency to invoke the law of small numbers well established (Gilovich et al., 1985; Tversky and Kahneman, 1971), slot play would seem to offer an ideal opportunity for its application.

The outcomes produced in this study were not consistent with those from the problem gambling research (Dixon et al., 2013; Harrigan and Dixon, 2010). This may have been due to the difference in the par gaps. More specifically, the results from the problem gambling studies were generated from simulations and experiments featuring a par gap of 13 percentage points, just over two points beyond the gaps examined herein. Additionally, there were differences in the experimental designs, especially with respect to the controlled lab study in Dixon et al. Also, Harrigan and Dixon required each virtual player to continue play until losing all of their credits. There were also differences in the criterion variables and game titles. Notwithstanding all of these variations, it is not unusual for the results of lab studies to differ from those produced in the field.

5.1. Managerial implications

Most importantly, our results demonstrated the potential of greatly increased pars, with respect to revenue generation. At the pairing level, the high-par games in TRI-1 and TRI-2 recorded percentage increases in T-win of 242.8 % and 243.1 %, from the level of the low-par games (see

Table 6). These revenue gains were computed from the outliers-omitted condition. Within this same section of Table 6, the percentage increases in A-win for the high-par games in TRI-1 and TRI-2 were 343.5 % and 143.6 %, respectively. It is interesting to compare these results against the corresponding changes in coin-in, from the level of the low-par games. More specifically, these substantial revenue gains were produced with very modest declines in coin-in (i.e., 2.1 % for TRI-1 and 6.2 % for TRI-2). For additional perspective, the high-par games in TRI-1 and TRI-2 produced annual increases in T-win of \$56,585.95 and \$67,185.55, respectively. Both of these gains were computed with data from the outliers-omitted condition. Such substantial gains warrant serious consideration, regarding decisions related to par settings. Not many operators would be able to consider these increases as trivial. All of these conclusions were also supported by the outcomes reflected in the all-cases scenario of Table 6.

While we do not know the extent to which our results can be extrapolated, it would seem more than reasonable to expand future studies with regard to the number of pairings. It remains possible, if not likely, that operators could be leaving money on the table. This would be especially true for those employing an EDLP strategy in reel slots, whereby it is assumed that gamblers can infer price from play alone. This popular approach is described and endorsed by operators in Legato (2019).

Most casinos make hay when the sun shines, i.e., the bulk of their revenues occur on holidays, weekends, and special-event days. When the business is there, higher pars may help operators claim a greater share of the aggregated wagering volume. It is dangerous to assume that players will eventually lose their gaming bankrolls over repeated visits. Additionally, many in the industry suggest and/or presume that budget-constrained players gamble until their entire trip bankroll is lost (Frank, 2017; Legato, 2019; Meczka, 2017). This can also be a dangerous assumption, as all players face some time constraint. Moreover, life’s many curve balls will make their claim on would-be bankrolls, by way of unplanned expenses. These gaming bankrolls also exist within the competitive space of other leisure services, vacations, etc. All of these factors may limit the casino’s future access to these funds, challenging the inevitability-of-loss assumption.

It is presumptuous to suggest that players are hopelessly dedicated

Table 6
Game-level Performance Comparisons by Pairing.

Pairing	Outliers-Omitted Scenario						
	Mean Daily T-win			Mean Daily Coin-in			Annual Rev. Gain Per Game
	Low-Par Game	High-Par Game	Incr.	Low-Par Game	High-Par Game	Decr.	
TRI-1	\$63.84	\$218.87	242.8%	\$1,504.01	\$1,472.22	-2.1%	\$56,585.95
TRI-2	\$75.73	\$259.80	243.1%	\$1,930.10	\$1,811.08	-6.2%	\$67,185.55
Pairing	Mean Daily A-win			Mean Daily Coin-in			Annual Rev. Gain Per Game
	Low-Par Game	High-Par Game	Incr.	Low-Par Game	High-Par Game	Decr.	
	TRI-1	\$49.84	\$221.04	343.5%	\$1,504.01	\$1,472.22	-2.1%
TRI-2	\$105.14	\$256.09	143.6%	\$1,930.10	\$1,811.08	-6.2%	\$55,096.75
Pairing	All-Cases Scenario						
	Mean Daily T-win			Mean Daily Coin-in			Annual Rev. Gain Per Game
	Low-Par Game	High-Par Game	Incr.	Low-Par Game	High-Par Game	Decr.	
TRI-1	\$63.95	\$225.32	252.3%	\$1,590.74	\$1,525.51	-4.1%	\$58,900.05
TRI-2	\$75.96	\$264.26	247.9%	\$1,996.40	\$1,817.41	-9.0%	\$68,729.50
Pairing	Mean Daily A-win			Mean Daily Coin-in			Annual Rev. Gain Per Game
	Low-Par Game	High-Par Game	Incr.	Low-Par Game	High-Par Game	Decr.	
	TRI-1	\$49.64	\$217.21	337.6%	\$1,590.74	\$1,525.51	-4.1%
TRI-2	\$86.64	\$255.41	194.8%	\$1,996.40	\$1,817.41	-9.0%	\$61,601.05

Notes. “Incr.” and “Decr.” represent the percentage change from the level of the low-par game, within each pairing. “Annual Rev. Gain Per Game” reflects the annualized daily revenue gains on the high-par games. All monetary values are expressed in terms of US dollars.

to gambling until their budgets are exhausted. It is strategically advantageous to consider personal budgets as mere guidelines, with re-allocations of funds occurring frequently. Even the total dollar value of individual gaming budgets can be difficult to pinpoint, as big wins and quick losses can both expand *a priori* “budgets.” This is not to say that no limits apply to par increases, but viewing gaming budgets as fluid or dynamic would support the idea of winning the bankroll when you have a shot at it.

The effects of increased pars are visible at the aggregated or long-term level (i.e., at the casino level), but not at the short-term or individual player level (Lucas and Singh, 2008, 2011). This important distinction is due to the extreme amount of variance present in the modern slot machine’s outcome distribution (Singh et al., 2013). This is how Lucas and Singh (2008) were able to demonstrate how play time could be increased with *increases* in pars, provided such increases were accompanied by corresponding decreases in the pay table variance.

Some confusion may exist within the industry due to conflating the long-term and short-term perspectives of slot play (see Legato, 2019 and Meczka, 2017). They are not the same. In this case, they are importantly different. Individual players must detect a difference in the signal (par) through a remarkable amount of noise, i.e., variance (Singh et al., 2013). This is not an issue for management, as game performance is an aggregation of many individual player experiences, representing many different possible outcomes. This is why management should not fear increasing the pars, within the limits of the extant research (Lucas, 2019; Lucas and Spilde, 2019a, 2019b). These results suggest that it is too difficult for individual players to detect the differences in reel slot pars.

The findings of the current field study further support this conclusion by way of elevated performance on the high-par games, with no indication of play migration across (1) the greatest difference in pars within a paired-design field study; and (2) an extended sample of 365 days. Further, these results were produced in a casino that catered to a frequently-visiting local clientele, within a competitive marketplace.

The difficulty of detection is also affected by the gambler’s wagering behavior on a typical visit. For example, many gamblers play multiple titles, with single-spin wagers that vary considerably. Without restricting play to a constant-wager condition on a single game, the challenge of detecting par becomes considerably more difficult. Given these conditions, it is not clear how a player could make any accurate conclusions about a game’s par from play alone. With play on multiple units, the task of identifying pars would only become more difficult.

While Table 6 demonstrates minimal declines in coin-in at the pairing level, many operators believe that such declines will eventually produce noticeable decreases in time on device (TOD) (Frank, 2017; Legato, 2019). But it’s important to remember that game-level results are produced by the outcomes of many players, including big winners and lots of losing players. Therefore, the mean of these aggregated outcomes will not present a meaningful proxy for the average TOD, or the average number of spins, per individual player. This is especially true for the players most likely to rely on such a measure, i.e., the losing players. This is because a disproportionate amount of the game-level coin-in comes from winning players, many of whom are essentially outliers. Moreover, the biggest winners inherit the capacity to contribute substantially more coin-in, by recycling their winnings. So any attempt to understand TOD should not consider the coin-in from winning players. This is why previous researchers have employed pulls per losing player (a.k.a. PPLP) as a criterion variable, when attempting to understand the effects of independent variables on TOD (Kilby and Fox, 1997; Lucas and Singh, 2008; Lucas et al., 2007). Again, these players are the ones most likely to invoke this measure of gaming value, as it’s reasonably safe to assume that winning players will be satisfied with their gaming results.

Customer complaints about losing too often and insufficient TOD are to be expected for nearly all of the licensed pars, as slot machines are designed to create a few lucky winners from an abundance of less

fortunate losing players. Therefore, these complaints alone do not provide compelling evidence that players are sensitive to par settings. Fears of gamblers making fine distinctions between pars may be further allayed by the results of Lucas and Singh (2011). More specifically, their simulation of session-level play found that individual players did not produce statistically different results on games with differing pars. This finding may help diminish par sensitivity concerns, as it is not possible to perceive a difference that does not exist.

5.2. Limitations & future research

Despite agreement with the prior field study research, our results cannot be generalized beyond the scope of the experiment, with respect to game titles, par gaps, number of experimental games, and sample duration. For example, any study that involved a greater number of experimental games, within a single slot floor, would inform about the ability to produce our results on a broader scale. That said, there is no obvious reason to believe that detection would be more likely in the expanded format, as the game-level ratio of signal to noise would remain dominated by noise (i.e., variance). All the same, such studies would make adoption of the results more likely, provided they produced outcomes similar to those featured in this work.

Of course, the par gap cannot increase indefinitely. At some point, players would surely notice the difference; however, it would be difficult to find existing games with par differences greater than those examined in this paper. Still, this limitation should be noted as it could affect the creation of new games with pars in excess of those examined in the extant research.

Great care must be taken regarding any causal inference. The findings of this study and those from the previously cited field studies (Lucas, 2019; Lucas and Spilde, 2019a, 2019b) all support greater performance for the high-par games, with no evidence of significant play migration. But this does not prove that the higher par settings will always produce increased revenue, or that players will not detect price shocks associated with higher pars. Further, just because the data do not support the popular counter argument does not prove that it is wrong, i.e., the argument in favor of player sensitivity to pars. Be that as it may, the empirical evidence seems to be accumulating on one side of this debate, at least within the live gaming environments.

Because of the field study design, we could not control for differences in the portfolio of players that engaged each of the experimental games. There could have been differences in the quality of players across the games; however, the 365-day sample helped guard against importantly different player portfolios. This extended duration provided ample opportunities for players in this heavy repeater market to encounter all of the experimental games. Also, the removal of outliers eliminated material impacts from premium players on any of the individual games. Lab studies could control for such limitations, but they cannot feature own-money wagering with the potential for real loss, within a live casino setting. This is due to IRB constraints related to the ethical treatment of subjects. Therefore, such studies create a different level of attenuation, as the subjects are not risking their own money and the lab does not feature the distractions and game-choice options featured in the live gaming environment. Moreover, the field is the setting in which operators must offer the pars that they select.

Finally, more work needs to be done on the effects of pay table volatility on the individual gaming experience, especially at the session level. The extant research clearly indicates that this variable has a more profound effect than par, on the player’s session-level outcomes (Lucas and Singh, 2008). But there is a need for more definitive work in this area. For example, how can the location of the variance in the pay table be manipulated to produce more tailored experiences for players with limited bankrolls (e.g., more time on device). It is possible that some of these questions have been addressed by game designers, but the answers have not been effectively disseminated to the operators, or the academic literature.

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