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**US Debt Sustainability Under Low Interest Rates
and After the Covid-19 Shock**

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ABSTRACT

US federal debt held by the public rose from 35 percent of GDP in 2007 to 72 percent in 2015, reflecting the Great Recession as well as rising health and social spending, and then to 79 percent in 2019 following the 2017 tax cuts. In 2020-21 federal assistance to address the Covid-19 shock will reach over 25 percent of 2021 GDP. The prolonged trend of declining real interest rates since the early 1980s has reduced the real economic burden associated with a given debt to GDP ratio. In part because of new attention to interest rate versus growth rate dynamics, economic thinking has correspondingly shifted toward less concern about debt levels. Nonetheless, Congressional Budget Office projections adjusted for the \$1.9 trillion American Rescue Plan indicate that the interest burden will rise from the recent 1½ percent of GDP to 2.3 percent by 2030 and 8.4 percent by 2050, as the debt ratio rises to 116 percent of GDP and 203 percent respectively. Feedback from the rising debt ratio boosts the real interest rate in the CBO's model to about 1 percent by 2030 and 2½ percent by 2050. This study first estimates the ex-post realized real interest rate on federal debt over the past six decades. For projecting the debt burden, it suggests a prudential benchmark at the 33rd percentile in the past distribution, a real rate of 1 percent on the 10-year treasury note. Simulations of the CBO-based projections using this rate, which is higher in the 2020s but lower thereafter, find an interest burden of 4.7 percent of GDP by 2050, far above the average of 1.9 percent over the past six decades. At a real rate of 1 percent, classic Maastricht fiscal targets would shift to about 130 percent of GDP for the debt ceiling and 4 percent for the deficit. However, even these more lenient targets would not be met unless primary deficits are cut to about 1 to 2 percent of GDP, well below their current path rising from about 3 percent of GDP to 4.5 percent by the 2040s as social security and especially health outlays escalate.

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Introduction

The worst pandemic in 100 years has hit the United States at a time when economic thinking has become more comfortable with high public debt, facilitating an aggressive response in public spending for relief has reached a remarkable 27 percent of GDP.² One reason for the receptiveness to large spending has been the perception that the fiscal stimulus in the Great Recession of 2007-09 erred on the side of excessive caution and left an aftermath of unduly slow growth. More fundamentally, low real interest rates in recent years have shifted the center of gravity in macro-economic analysis away from concern about chronic deficits and rising debt toward a certain benign neglect of debt as a problem. Paul Krugman recently wrote:³

... there is now widespread agreement among economists that debt is far less of a problem than conventional wisdom asserted. Among other things, while the level of federal debt may seem high, low interest rates mean that the burden of servicing that debt is actually very low by historical standards.

The political sphere has been quick to seize on the new view.⁴

Among economists, the new nonchalance about debt is by no means unanimous. The Congressional Budget Office emphasizes the risks associated with debt on track to exceed 200 percent of GDP by 2051.⁵ Treasury Secretary Janet Yellen stated in 2017 that the United States was on an “unsustainable debt path”. In confirmation hearings, she reiterated that “There are long-term budget challenges,” while emphasizing that in the short-term and with low interest rates, “The most important thing we can do today to set us on a path to fiscal sustainability is to defeat the pandemic, provide relief to the American people, and make long-term investments that will benefit future generations” (Yellen, 2021, p. 26).

² The IMF (2021, p. 3) estimates that the assistance legislated in March and April of 2020 amounted to \$3.1 trillion (14.8 percent of GDP). Another \$900 billion in federal spending was legislated in December 2020, and an additional \$1.9 trillion in March 2021, for a total of \$5.9 trillion, or 26.9 percent estimated 2021 GDP.

³ Paul Krugman, “Four Rules That Should Guide Bidenomics,” *New York Times*, January 15, 2021.

⁴ Thomas Friedman writes: “[a] powerful consensus ... has taken hold on both parties: That we are in a new era of permanently low interest rates, so deficits don’t matter as long as you can service them, and so the role of government in developed countries can keep expanding – which it has with steadily larger bailouts, persistent deficit spending, mounting government debts and increasingly easy money out of Central Banks to finance it all.” “Capitalism and Socialism, American-Style,” *New York Times*, January 27, 2021. Also see George Will, “Dubious Noises about the Debt,” *Washington Post*, December 27, 2020.

⁵ The CBO (2021f, p. 5) states: “Debt that is high and rising as a percentage of GDP boosts federal and private borrowing costs, slows the growth of economic output, and increases interest payments abroad. A growing debt burden could increase the risk of a fiscal crisis and higher inflation as well as undermine confidence in the U.S. dollar ... “. The Center for a Responsible Federal Budget has consistently been among the most vocal critics of the outlook for rising public debt (see for example CRFB, 2021). In contrast, Furman and Summers (2020) argue that “CBO’s forecasts of net interest as a share of GDP have been systematically biased toward being too high” and that “policymakers should put relatively little weight on projections for ten years or more in the future” (pp. 28, 31).

A core problem with ignoring debt because of low interest rates is rollover risk. Upon maturity, a federal note or bond must be refinanced with new borrowing. As the debt to GDP ratio climbs, at some point financial markets are likely to demand higher interest rates --as seen in the “taper tantrum” of 2013 when rates reacted sharply to the announcement that the Federal Reserve would wind down “quantitative easing” purchases of federal debt. Even though the United States could always inflate away its real debt because it borrows in its own currency, the prospect of a need to do so would widen the inflation risk premium. Moreover, because of likely political intolerance to inflation high enough to make a sharp reduction in outstanding real debt, financial markets could begin to incorporate a credit-default risk premium.

The causes of the near-zero real 10-year Treasury rate in recent years remain the subject of debate. The hypotheses include rising inequality (combined with a higher marginal saving rate of the rich), the rise in China’s contribution to the global pool of savings, heightened risk aversion in the wake of the Great Recession (and now the pandemic) and a corresponding rise in demand for safe assets, a long-term decline in growth, a shift to new technologies requiring less capital, and others.⁶ As discussed below, the Congressional Budget Office has attempted to model the various influences causing the low rates, and finds that even taking such trends into account, the path of rising debt relative to GDP brings the real 10-year rate back up to 1 percent by 2030, 1.8 percent by 2040, and 2.6 percent by 2050 (CBO, 2020b).

This study takes the approach of setting a “prudential” floor for the real interest rate in projecting federal debt. It first calculates the ex-post realized real interest rate (RRIR) on the 10-year Treasury note over the past six decades. It then sets the prudential floor for the RRIR at 1 percent, the 33rd percentile in the past distribution of real rates. Using a simple accounting model that estimates the interest obligation on each future vintage of new borrowing, I find that the 1 percent real rate causes a path of interest burden that is back up to the historical average by 2025 and reaches about 2½ times the historical average by 2050. (The CBO places the interest/GDP burden even higher by 2050, at over four times the historical average). A sizable persistent primary deficit of about 3 percent of GDP, rising to over 4 percent, thus yields a dangerously high long-term debt burden whether measured by the ratio of interest or debt to GDP.

This study first sets the analytical framework by considering the renewed attention to interest-rate versus growth-rate debt dynamics (set forth in detail in Appendix A). It then examines the long-term trend toward lower interest rates, using a measure of the ex-post real realized interest rate (RRIR). The discussion then considers a prudent benchmark real rate for fiscal projections, followed by a focus on the crucial role of the primary deficit in the long-term projections. The different paths of the debt burden when measured by interest payments versus debt stock relative to GDP are then shown. The analysis then presents simulations re-estimating the CBO long-term debt projections using the prudent benchmark real rate and taking account of the \$1.9 trillion American Rescue Plan adopted in March 2021. Three sets of interest rate paths (CBO, prudential 1 percent real rate, and zero percent real rate) are

⁶ N. Gregory Mankiw, “The Double-Edged Sword of Low Interest Rates,” *New York Times*, December 6, 2020.

combined with three sets of primary deficit paths (CBO baseline, limit to 2 percent of GDP, limit to 1 percent) to explore the range of plausible outcomes.

The discussion then examines more closely the influence of rising debt on the real interest rate, a crucial feature of the CBO projections. An additional test is run incorporating the impact of rising debt on the real interest rate (estimated in Appendix D). The implications of low interest rates for Maastricht-type debt targets are then examined. The study concludes with overall policy implications. Appendix A sets forth the interest-rate versus growth-rate analytics of debt dynamics. Appendix B develops an ex-post measure of the realized real interest rate (RRIR). Appendix C reports the simulation model developed to replicate the long-term CBO projections, for use in sensitivity analysis. Appendix D uses the aggregate production function approach to estimate the parameter for feedback from debt to the interest rate.

Real Interest Rate versus Growth and Debt Sustainability

Low real interest rates in recent years have focused attention on the analysis of public debt sustainability in the “ $r - g$ ” framework: the race between debt buildup from interest paid on outstanding debt and growth in the GDP base available to service the debt. In 2003 I set forth such a framework in analyzing debt sustainability for Argentina after its default in 2001 (Cline, 2003, Annex A). The primary (non-interest) fiscal surplus needed to keep the debt to GDP ratio from rising, as a fraction of GDP, turned out to be the real interest rate minus the real growth rate, multiplied by the initial ratio of debt to GDP, and all divided by unity plus the real growth rate plus inflation.⁷ With low real interest rates equal to or less than the growth rate, the principal policy inference has been that public debt is not much of a problem.⁸ Appendix A sets forth the basic equations of the “ $r-g$ ” debt dynamics.

A major practical problem with such an inference is that when there are large structural primary deficits, a zero contribution to a rising debt ratio from the standpoint of interest on inherited debt does not suffice to prevent brisk escalation in the debt ratio. A central implication of this fact is that policy initiatives that have the effect of locking in new ongoing primary deficits can cause debt problems even when the interest rate is at or below the growth rate. The discussion of structural primary spending trends below considers the magnitude of the primary deficit problem.

The Long-term Trend toward Lower Real Interest Rates

Despite the low interest rates in recent years, it would be imprudent to ignore actual levels of real interest rates in recent decades in designing fiscal policy. The best measure of

⁷ The formulation for Argentina added a term for depreciation of the currency multiplied by the share of debt in foreign currency. A decade later I set forth a similar formulation in examining the euro area debt crisis (Cline, 2014, pp. 39-40).

⁸ Blanchard (2019) provided a particularly influential analysis to this effect. He concluded: “Put bluntly, public debt may have no fiscal costs (p. 1197).”

federal borrowing costs is the rate on the 10-year note. Average residual maturity on federal debt has been about 6 years since 2013, about the length that would be expected if all debt were issued with 10-year maturities.⁹

Measuring the real interest rate is not straight-forward. Treasury Inflation-Protected Securities (TIPs) were not introduced until the late 1990s, and data series on 10-year TIPs only begin in 2003. Moreover, TIPs account for only about 9 percent of outstanding debt (CBO 2020a, p. 6). However, a correct measure of the real interest rate eventually realized on a nominal 10-year note cannot be calculated simply by deducting the inflation rate from the 10-year interest rate for the year of issuance. The reason is that the real interest burden depends on the subsequent time path of inflation in comparison to the steady year-of-issuance nominal interest payments.

I calculate the 10-year “realized real interest rate” (RRIR) as the internal rate of return on an investment purchased in year 0, paying annually over each of the subsequent 10 years the average coupon rate of 10-year government obligations issued year 0, and returning principal in the 10th year. The stream of return is deflated by the PCE (personal consumption expenditure) price index.¹⁰

Figure 1 shows the resulting estimates of realized real interest rate on the 10-year Treasury note, from 1962 through 2011 actual (solid line) and then expected for 2012-20 (dashed line). (The actual realized rate for the Treasury note purchased in 2015, for example, will not be known until 2025). The expected path for 2012-20 applies CPE inflation as projected in CBO (2020b).

The importance of deflating the stream of payments by actual realized inflation, rather than simply subtracting the current-year inflation rate in the year of issuance to arrive at the real interest rate, is illustrated by both the late 1960s-early 1970s, and the early 1980s. In 1970, for example, the nominal 10-year rate was 7.35 percent. PCE inflation was 4.68 percent. Deducting the current year inflation to arrive at the real rate would yield 2.67 percent as the real rate. However, because of much higher inflation over the years that followed, the realized real interest rate for 10-year notes issued in 1970 turned out to be only 0.63 percent. In contrast, for a 10-year note issued in 1981, the nominal rate was 13.92 percent. PCE inflation in 1981 was 8.96 percent. The naïve real rate calculated as the difference would have been 4.96 percent; the RRIR instead turned out to be 9.59 percent. The slowdown in inflation over the life

⁹ CBO (2020a, pp. 8-9). If all debt were originally 10-year maturity, and if outstanding debt were constant, average residual maturity would be 5 years. With rising debt, the average residual maturity is longer. Note however that average residual maturity fell as low as 4 years in 2008 because of heavy borrowing in Treasury bills in the Great Recession (p. 8).

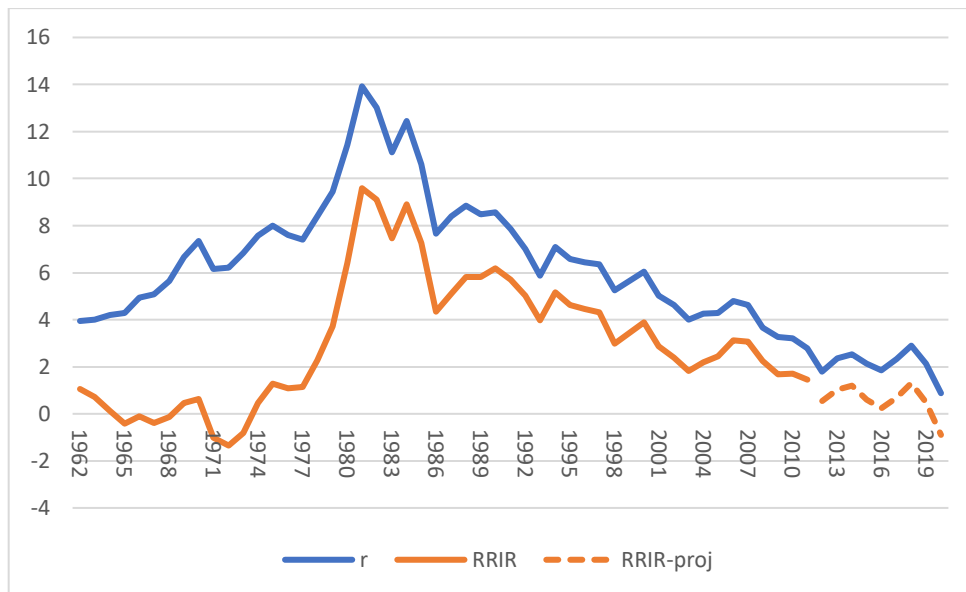
¹⁰ Thus, the real interest rate r^* is calculated as: $r^* = IRR(-1 + r \sum_{t=1}^{10} \frac{1}{p_t} + \frac{1}{p_{10}})$ where “IRR” is the internal rate of return (equating present value to zero), r is the nominal coupon interest rate (expressed as a fraction), t is the year, and p is the price index normalized to unity in the year of purchase. (The Excel function IRR is used in this calculation.)

of the 10-year note meant that the real return was much higher than implied when deflating by inflation in the year of issuance.

A time series for the Treasury Inflation-Protected (TIP) 10-year rate is available beginning in 2003 (FRED, 2021b). Figure 2 compares the RRIR reported in figure 1 to the 10-year TIP rate, over the period 2003-2020. In this period, for the sub-period for which the outcome is already known (solid line), the RRIR stood at an average of 2.20 percent, whereas the TIP rate averaged 1.71 percent. On this basis, the TIP may understate the real cost of Treasury borrowing by about 50 basis points. The difference makes sense from the consideration that the TIP is a lower-risk investment than a Treasury note or bond, because it does not have inflation risk.

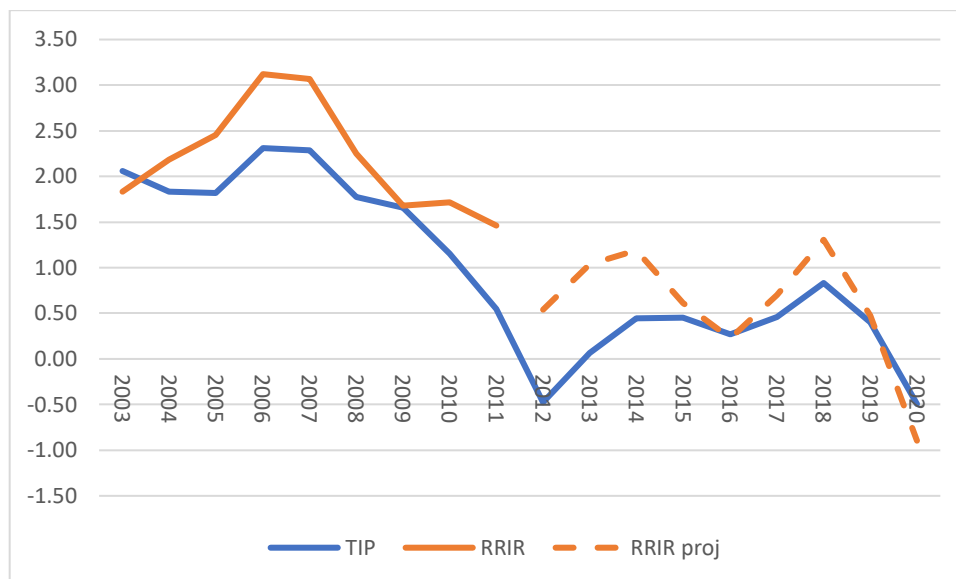
Figure 1

Nominal Rate (r) and Realized Real Interest Rate (RRIR)
on 10-year Treasury Note Purchased in Year Indicated



Source: FRED (2021a), CBO (2020b); author's calculations

Figure 2
Real Interest Rate on 10-year Treasuries: TIP versus RRIR



Source: Figure 1; FRED (2021b)

Identifying a Prudent Benchmark for the Real Interest Rate

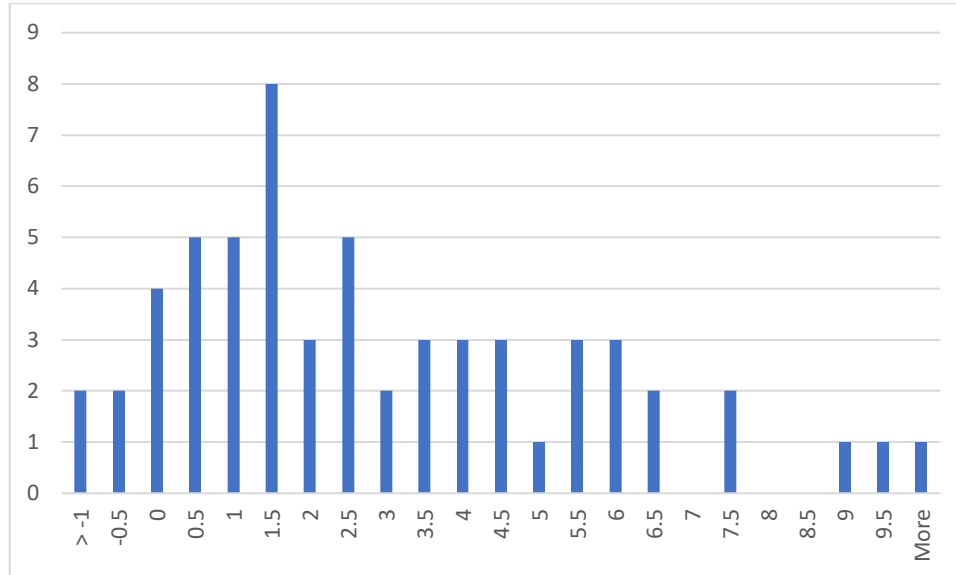
Across the full 59 years for estimated Realized Real Interest Rates shown in figure 1, the average RRIR is 2.69 percent; the median is 2.18 percent. Figure 3 shows a histogram of the RRIR estimates, with the number of years as the height of each column, placed into bins of 0.5 percentage point width. The mode is 8 years with RRIR between 1.0 percent and 1.5 percent. The 25th percentile year shows the RRIR at 0.61 percent; the 75th percentile, 4.34 percent. The 33rd percentile RRIR is 1.05 percent.

A prudent benchmark for the realized real interest rate for the 10-year note, and thus for federal debt, over the next three decades would be 1 percent, the 33rd percentile for the past. Selecting this threshold takes some account of the persistent downward trend since the early 1980s, while limiting the risk of myopia associated with projecting the most recent experience into the indefinite future. The RRIR for 2020 was -0.9 percent, the third-from-lowest over the 59-year period in Figure 1.

In February 2021, the CBO (2021a) projected that the rate of PCE inflation would be an average of 1.9 percent in fiscal years 2021-25 and 2.1 percent in FY2026-30. It projected that the nominal rate on the 10-year Treasury note would average 1.6 percent in FY2021-25 and 3.0 percent in FY2026-31. The resulting implied real interest rates would average -0.3 percent in FY2021-25 and 0.9 percent in FY2026-2030. In the first five-year period, these projections are substantially below a prudential benchmark of 1 percent.

Figure 3

Frequency Distribution of Realized Real Interest Rate on 10-Year Treasuries
(1962-2020 annual averages; percent)



Source: See figure 1

Mind the Primary

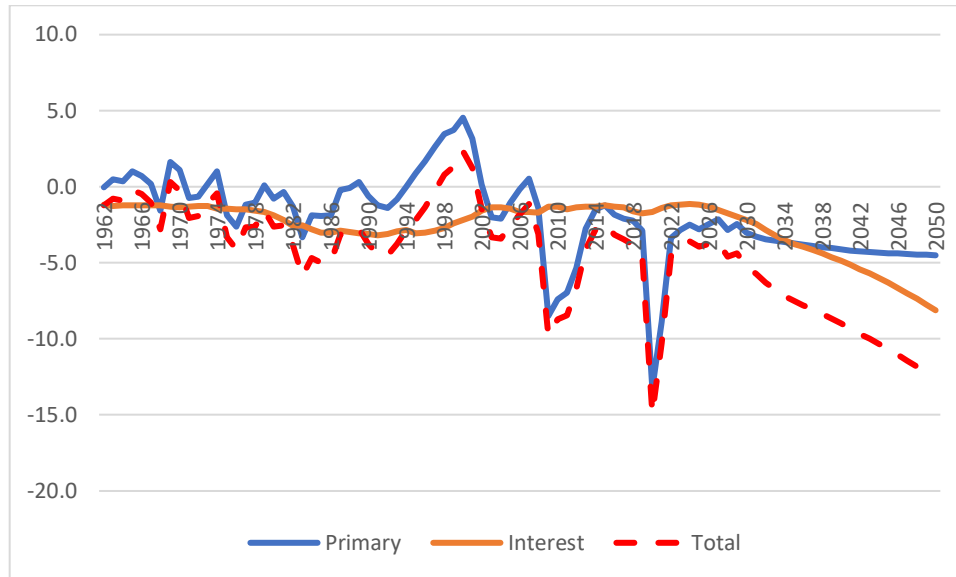
Concern about the level of the real interest rate stems from the first element in the “ r minus g ” driver of debt dynamics. The primary (non-interest) fiscal balance needed to keep the debt to GDP ratio from rising, on the left-hand side of the stability equation, equals the existing debt ratio multiplied by “ $r - g$ ” (interest rate minus growth rate).¹¹ Most of the recent policy discussion has focused on the fact that “ r ” is low. However, a potentially larger problem is that there is a sizable structural component of the primary deficit.

As shown in Figure 4, from 1962 through 2007 the primary balance was usually only a small deficit or a surplus. The primary deficit exceeded the interest deficit only 13 percent of the time. Moreover, the primary balance was in surplus in 20 of these 46 years, especially in 1995-2001. In contrast, since 2007 the primary deficit has exceeded the interest deficit in all but two of the eleven years. The CBO projects that the primary deficit will continue to be larger than the interest deficit for the next 15 years. (Note that although figure 4 shows the large surge in primary spending in fiscal 2020 associated with pandemic expenses, its primary spending is understated by about 6 percent of GDP for fiscal 2021 and about 3 percent for FY2022 because it excludes likely spending under the American Rescue Plan Act.)

¹¹ More precisely, the initial coefficient is the existing debt ratio divided by unity plus the inflation rate plus the growth rate.

Figure 4

Actual and Projected Primary Balance, Interest Payments, and Total Fiscal Balance as Percent of GDP, 1962-2050



Source: CBO (2021d)

The large primary deficits were unavoidable in 2008-09 in the face of revenue loss and the need for fiscal stimulus in the Great Recession, and again in 2020-21 with the covid-19 shock. However, structural trends have worsened the primary balance and are on track to continue doing so.

As shown in table 1, mandatory expenditures on social security rose from 4.1 percent of GDP in 2007 to 5.2 percent in 2020 and are projected to reach an average of 6.3 percent of GDP by the 2040s. Mandatory spending on major health programs (mainly Medicare and Medicaid) stood at 4.5 percent of GDP in 2007, rose to 6.6 percent in 2020, and is projected to average 9 percent of GDP by the 2040s. From 2019 to 2050 spending on social security and major health care programs is projected to rise from 10.8 percent of GDP to 17.0 percent, with about half of the increase coming from an aging population and the other half from “excess cost growth” in health care (CBO, 2020d, p. 3).

Table 1

Fiscal Year Outcomes and Projections
(percent GDP)

	07	19	20	21	22-31	32-41	42-51
Revenue	18.0	16.3	16.3	16.0	17.5	17.7	18.2
Individual income taxes	8.1	8.1	7.7	7.7	9.1	9.6	10.1
Payroll (social) taxes	6.1	5.9	6.2	6.0	5.9	5.8	5.7
Corporate income taxes	2.6	1.1	1.0	0.7	1.3	1.2	1.2
Other	1.2	1.2	1.4	1.4	1.3	1.1	1.3
Outlays excluding interest	17.4	19.2	29.6	24.9	20.3	21.6	22.7
Social Security	4.1	4.9	5.2	5.2	5.6	6.1	6.3
Major health programs	4.5	5.6	6.6	5.8	6.3	7.8	9.0
Other mandatory	1.7	2.4	10.0	6.3	2.3	2.1	2.0
Discretionary	7.3	6.3	7.8	7.6	6.1	5.5	5.5
Primary balance	0.6	-2.9	-13.3	-8.9	-2.8	-3.9	-4.5
Net Interest	-1.7	-1.8	-1.6	-1.4	-1.6	-4.0	-7.0
Deficit (-)	-1.1	-4.6	-14.9	-10.3	-4.4	-7.9	-11.5
Debt held by public	35.2	79.2	100.1	102	107 ^a	145 ^a	202 ^a

a End-period

Source: CBO (2021d, series 51134, 51119)

The overall result including other non-interest spending is that the primary balance is on track to swing from a surplus of 0.6 percent of GDP in 2007 to a deficit averaging about 3.5 percent of GDP in the 2020s and 2030s, and 4.5 percent of GDP in the 2040s. In 2000-2019 real growth averaged 2.14 percent (CBO 2021b). Over this period the 10-year RRIR averaged 1.75 percent (figure 1). The average ratio of debt to GDP was 55 percent (CBO, 2021d). Average inflation was 1.84 percent annually (personal consumption expenditure, PCE, index; FRED (2021c)). So a slight primary deficit (0.21 percent of GDP) would have been consistent with holding the debt to GDP ratio unchanged.¹² Instead, the average primary deficit was 2.05 percent of GDP (CBO, 2021d).

Going forward, the CBO (2020e) projects average real GDP growth in 2021-2050 at 1.85 percent. Starting from a 2020 debt to GDP ratio of 100 percent, and applying the prudential benchmark of 1 percent for the real interest rate, the primary balance needed to hold the debt ratio constant would be a deficit of 0.8 percent of GDP. Instead, even excluding 2021 because of the pandemic spending surge, the average primary balance is projected by the CBO at a

¹² See Appendix A, equation A5).

deficit of 3.8 percent of GDP. Compared with the past two decades, the benefit of a reduction in the real interest rate from 1.75 percent to the benchmark 1 percent, or 0.75 percentage point, is more than offset by the increase in the primary deficit from 2.05 percent of GDP to 3.8 percent, or by 1.75 percentage point. The slightly more advantageous race between the real interest rate and growth is swamped by the rise in the primary deficit.

Interest Burden versus Debt Ratio

The heart of the new benign neglect of debt is the proposition that the rise in the ratio of debt to GDP has overstated the rising burden of the debt, because the price of the debt – the interest rate – has fallen. By implication, a measure of debt burden based on interest payments rather than debt is needed.

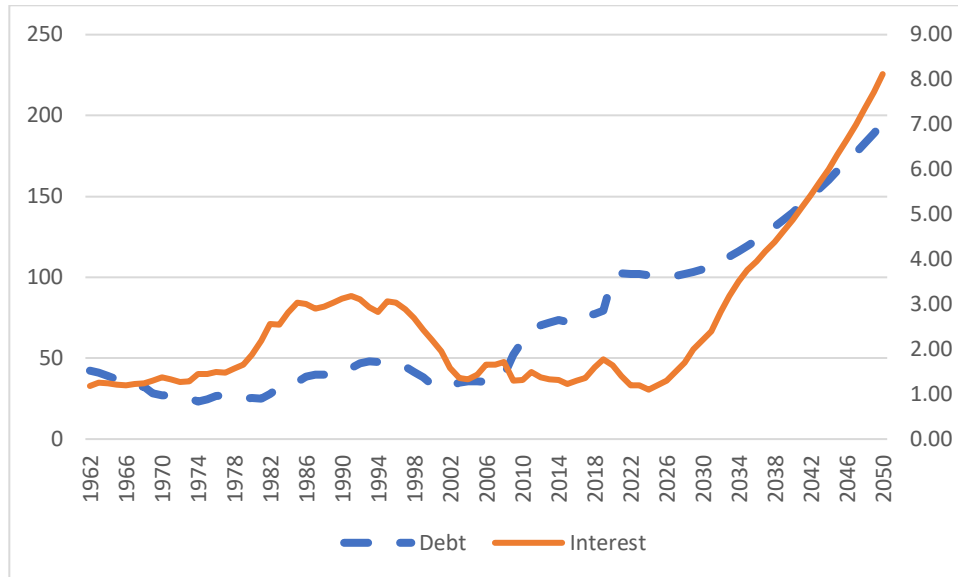
It is certainly true that the path of interest payments as a percent of GDP has been far more benign than the path of debt as a percent of GDP in recent years. Figure 5 shows federal debt held by the public as a percent of GDP on the left axis (dashed line) and net interest payments as a percent of GDP on the right axis.¹³ Whereas the debt ratio has risen from 25 percent of GDP in 1981 to 79 percent in 2019, 100 percent in 2020, and a projected 102 percent in 2021 (before the \$1.9 trillion American Rescue Plan legislation), the interest burden actually fell from a high plateau of about 3 percent of GDP in 1985-96 to a low plateau of about 1.5 percent of GDP in 2002-2020.

The CBO projections show a further widening of the gap between the debt burden as measured by the ratio of debt to GDP versus interest payments relative to GDP during 2021-26, thanks to the legacy of debt borrowed at extremely low real interest rates in 2012 and after (see figure 1 above). In the CBO projections, real interest rates (deflating by the PCE) However, for the period 2027-2050 the CBO's projections show a sharp upward turn for the *interest ratio*, which rises from about 1.6 percent of GDP in 2027-28 to 8.1 percent by 2050 – far above the previous peak of 3.1 percent in 1991. The interest-burden path catches up once again with the debt-ratio path by the early 2040s, and thereafter surpasses it.

¹³ Data are from CBO (2021d, series 51134) for 1962-2019; CBO (2021c) for 2020-31; and CBO (2021f) for 2032-50.

Figure 5

Federal Debt Held by the Public as Percent of GDP (left scale) and Net Interest Payments as percent of GDP (right scale), FY 1962-2050



Source: CBO (2021c, d, f)

Debt Simulations based on the CBO Long-Term Projections

CBO Long-term Projections

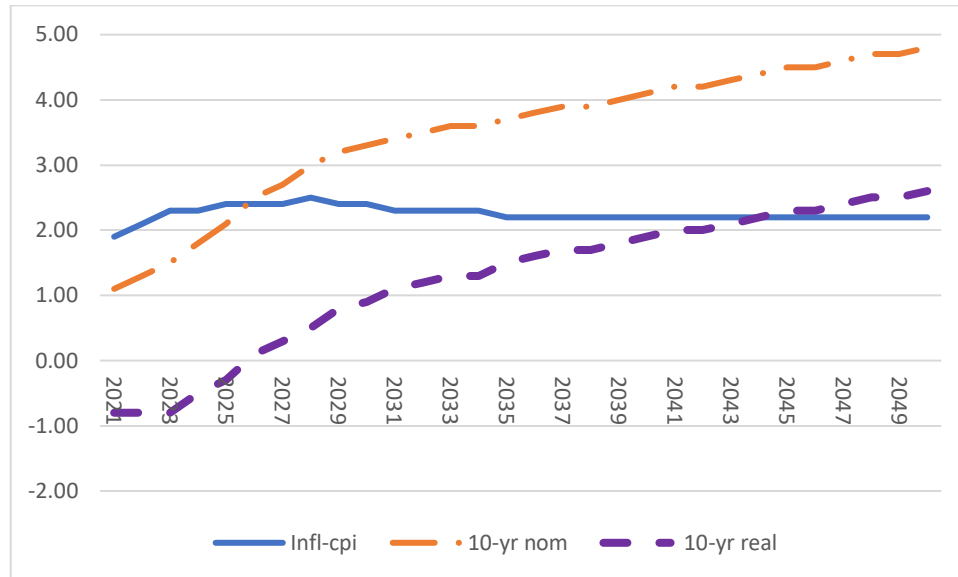
For debt sustainability analysis, the bottom line is that by 2050 the CBO's long-term projections indicate a sharp escalation in the burden of debt, whether measured by the ratio of debt to GDP or the ratio of interest payments to GDP. A key question, however, is whether a new normal of "low interest rates forever" would mean that the long-term debt burden would be much lower than that projected by the CBO on either metric.

A major influence boosting the interest burden in CBO's long-term projections is the rise in the real interest rate. As shown in Figure 6, the real rate on the 10-year Treasury note is placed at about -0.5 percent in 2021-23, rises to zero only by 2025, but then proceeds to rise steadily, reaching 1 percent by 2029, 1.8 percent by 2040, and 2.6 percent by 2050.¹⁴ As discussed below, the driving force in the rising long-term real interest rate is the CBO's modeling in which a rising ratio of debt to GDP causes the real interest rate to rise.

¹⁴ Note that the real rate in figure 5 deflates using the consumer price index, and so is about 30 basis points lower than the real rate deflating by the PCE index (as in Figure 1).

Figure 6

Consumer Price Inflation and Interest Rate on the 10-year Treasury Note
(percent, nominal and real)



Source: CBO (2021f).

Appendix C develops an “accounting” or “direct effects” simulation model to examine the sensitivity of the CBO projections to the interest rate. An economic, as opposed to accounting, model could incorporate feedback from alternative projections of interest rates to alternative levels of GDP, inflation, and primary deficits. The central tests here boost the interest rate above the CBO baseline in the 2020s, but place it lower than that baseline in the 2040s and 2050s. The lesser increase in the interest rate would tend to boost economic growth in the 2030s and 2040s by raising capital investment and output potential, reinforcing the finding below that the CBO estimates may understate the burden by 2030 but overstate it by 2050.

Impact of the Prudential Benchmark Interest Rate

Figure 7 shows the CBO projections of net interest payments as a percent of GDP for 2020-30 (chart A) and for the full period 2020-50 (chart B) (CBO 2021c, f). The path “sim0” shows the replication of the CBO projections using the model set forth in Appendix C.¹⁵ The first

¹⁵ These projections take account of the \$900 billion pandemic relief legislation of December 2020 but not the pending \$1.9 trillion American Rescue Plan Act (CBO, 2021d). Note that the new projections reduce the interest adjustment parameter ψ in Appendix C from 0.77 in 2021-30 and 0.886 in 2031-2050 to 0.73 and 0.873, respectively.

major sensitivity simulation replaces the CBO’s time path of real interest rates with this study’s benchmark 1 percent rate for 2025-2050, with an initial phase-in for 2022-24.¹⁶ The nominal rate equals the real rate plus the CBO’s baseline for PCE inflation.¹⁷

Figure 7
Net Interest Payments as Percent of GDP: CBO and Simulated
A. 2020-2030 B. 2020-2050



Source: CBO (2021a, c, f) and author’s calculations

As shown in chart 6A, with the return to 1 percent real for the 10-year Treasury note by 2025, *net interest payments reach 1.8 percent of GDP by 2026, almost the same as the 1.9 percent average over 1962-2019.* The interest burden reaches 2.4 percent of GDP by 2030.

Whereas the prudential 1 percent real interest rate path places the interest burden significantly above the CBO path during the 2020s, by the 2030s and especially 2040s the CBO projections rise above the 1-percent benchmark path, as shown in chart 6B. The interest burden reaches 8.2 percent of GDP in 2050. *Even if the real 10-year rate is held instead at the prudential minimum of 1 percent real, the interest burden reaches 4.7 percent of GDP by 2050, or 2.5 times as high as the average over the past six decades.*

Sensitivity Tests

To examine the influence of the interest rate and the primary balance on the debt burden, Figure 8 shows the paths of the debt to GDP ratio (chart A) and net interest to GDP ratio (chart B) under three alternative scenarios for each of these two key variables, resulting in nine (3²) cases. The base case, scenario “s11”, is that of the CBO long-term projections, altered by *adding \$1.9 trillion to pandemic relief spending to the primary deficit in fiscal 2021 (about*

¹⁶ The real rate is set at 0.25 percent in 2022, 0.5 percent in 2023, and 0.75 percent in 2024.

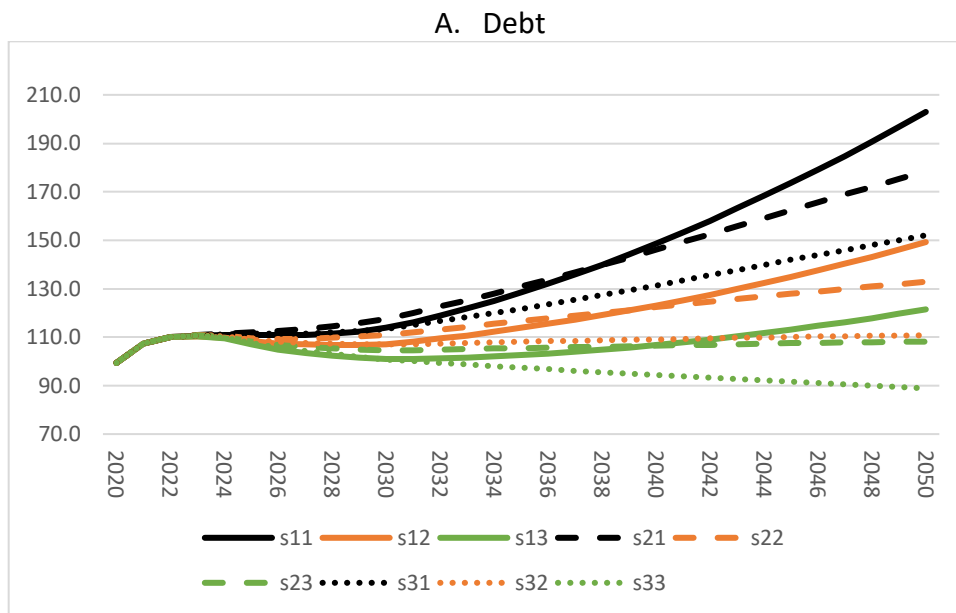
¹⁷ For 2041-50 only consumer price inflation is available in the CBO projections. In this period the nominal rate is set at the rate of CPI inflation plus unity minus 0.29 percentage point (the average annual excess of CPI inflation over PCE inflation in the 2020-30 projections).

two-thirds) and FY2022 (the rest).¹⁸ As shown in table 1 above, the primary deficit stood at 2.9 percent of GDP in FY2019 but surged to 13.3 percent in 2020 from pandemic relief spending and revenue loss. For 2021, without additional relief spending the primary deficit was to fall to 8.9 percent of GDP. Setting the new baseline to include an extra \$1.9 trillion spending, the primary deficit would instead reach 14.6 percent of GDP in FY2021 and 6.6 percent in FY2022.¹⁹

For each of the other scenarios (“s”), the first digit refers to the interest rate and the second refers to the primary balance. For the first digit (interest rate), “1” refers to the CBO baseline; “2” indicates the prudential benchmark 1 percent real interest rate; and “3” refers to a real interest rate of zero. For the second digit (primary balance), “1” refers to the CBO baseline, “2” refers to a primary deficit of 2 percent of GDP (the average outcome in 2000-2019), and “3” refers to a primary deficit of 1 percent of GDP.²⁰ In figure 8, the solid lines refer to case 1 (CBO) for the interest rate path; the dashed lines, case 2 (real interest rate at 1 percent by 2025 and after); and the dotted lines, case 3 (real interest at zero by 2025 and after). Table 2 reports the results of the simulations for 2020, 2025, 2030, 2040, and 2050.²¹

Figure 8

Ratio of Debt to GDP and Net Interest to GDP under Alternative Scenarios (percent)



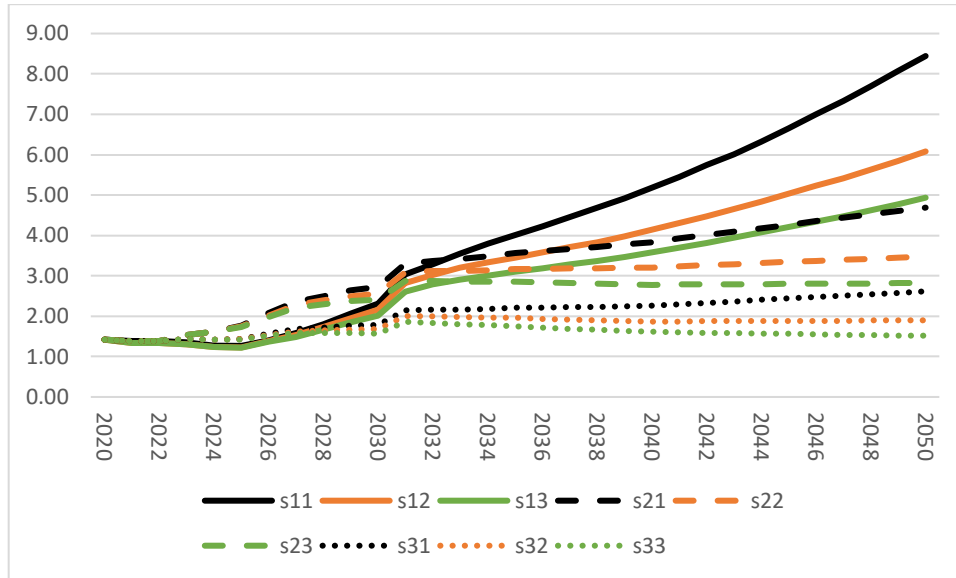
¹⁸ The CBO estimates that 64 percent of the \$1.9 trillion American Rescue Plan would be spent in FY2021, another 22 percent in FY2022, and the remainder spread over the next six years. CBO (2021e).

¹⁹ The CBO’s February 2021 projections for 2021-31 (CBO, 2021c) provide the first decade, and the CBO’s March 2021 long-term projections (CBO, 2021f) are applied for 2032-50.

²⁰ The change from the CBO baseline to the alternative scenarios is phased in from 2022 to 2025 for the real interest rate, and from 2024 to 2025 for the primary deficit.

²¹ The simulation estimates for 2020 are slightly lower than the actual outcomes (which were 100.1 percent for the debt ratio and 1.64 percent for the interest ratio; CBO (2021c).

B. Interest



Source: Author's calculations. Scenarios: see text.

Table 2

Debt and Interest as a Percent of GDP at Benchmark Years
Under Alternative Scenarios

	2020	2021	2025	2030	2040	2050
Debt ratio						
s11	99.4	107.3	111.0	116.1	148.5	203.0
s12	99.4	107.3	108.8	108.9	124.3	151.3
s13	99.4	107.3	107.3	101.6	107.9	123.4
s21	99.4	107.3	111.9	120.0	146.1	178.5
s22	99.4	107.3	109.6	112.0	122.3	132.9
s23	99.4	107.3	108.1	104.7	106.4	108.2
s31	99.4	107.3	111.4	114.9	131.3	152.1
s32	99.4	107.3	109.0	107.2	109.1	110.8
s33	99.4	107.3	107.5	100.1	94.4	88.9
Interest ratio						
s11	1.41	1.38	1.27	2.31	5.18	8.44
s12	1.41	1.33	1.23	2.16	4.14	6.08
s13	1.41	1.33	1.22	2.02	3.58	4.94
s21	1.41	1.38	1.77	2.73	3.84	4.69
s22	1.41	1.38	1.75	2.56	3.20	3.47
s23	1.41	1.38	1.74	2.41	2.78	2.82
s31	1.41	1.38	1.44	1.79	2.26	2.62
s32	1.41	1.38	1.43	1.68	1.87	1.90
s33	1.41	1.38	1.43	1.57	1.61	1.51

Source: Author's calculations. Scenarios: see text.

The broad picture that emerges from the simulations is that the debt burden is on a rising long-term path, whether measured by the debt to GDP ratio or the ratio of interest payments to GDP. Only the most extreme favorable case (s33) shows the debt ratio lower in 2050 than at present. For the interest/GDP metric, all of the scenarios except the two most favorable (s32, s33) show the burden higher by 2050 than the average 1.9 percent over the past six decades. A moderately optimistic variant of the “prudential interest rate benchmark” case (1 percent real interest rate, 2 percent of GDP primary deficit, case s22) still leaves the interest burden at 3.5 percent of GDP by 2050.

The results do show a strong influence of controlling the primary balance on curbing the long-term explosion of the debt ratio and the interest ratio. In the CBO baseline (adjusted to include the American Rescue Plan, case s11), the average primary balance in 2022-2050 is a deficit of 3.9 percent of GDP. When the CBO interest rates are unchanged but the primary

deficit drops to 2 percent of GDP (case s12), by 2050 the debt ratio reaches only 151 percent of GDP instead of 203 percent. If the primary deficit is cut further to 1 percent of GDP, again with no change from the CBO interest rate path (case s13), by 2050 the debt ratio is only 123 percent. *A principal message of the simulations is thus that curbing the primary deficit will be needed to curb the debt burden over the longer term.*

As shown in chart B of figure 8 (as well as figure 7 above), another key message is that the low ratio of interest payments to GDP in 2021-25 is a transitory phenomenon that depends heavily on the CBO's assumption of negative real rates until 2025. The recently popular view that low interest rates mean policy makers do not need to worry about rising debt implicitly endorses the CBO's near-term projections but ignores the CBO's longer-term projections.

Impact of Rising Debt on the Interest Rate

The prevailing nonchalance about debt thanks to near-zero real interest rates implicitly assumes either that there is no influence of rising debt on the interest rate the government must pay, or that the long-term forces that have reduced the rates will strengthen and reduce rates further by enough to offset a rise in rates that could be expected from rising debt. However, leading quantitative estimates suggest that further rate reductions by secular forces are likely to be far smaller than the upward pressure on rates if the debt rises by anywhere near as much as the CBO long-term projections indicate.

Gamber (2020, pp. 26; 39; 49-50) summarizes these opposing forces in the CBO model as follows. Declining labor force growth reduces the marginal product of capital and the interest rate. A rising domestic and foreign saving rate increases the supply of saving and drives down the interest rate. Falling total factor productivity growth reduces the return to capital and the interest rate. A rise in the risk premium, or rising preference for safe assets, reduces the interest rate the government needs to pay on borrowing. Against these forces, however, a rise in the ratio of public debt to GDP boosts the rate the government must pay. In addition, a rise in the capital share of income raises the rate of return to capital and the interest rate. Table 2 shows the impact parameter for each of these influences, and indicates the contribution of each one to the change from a base period of 1995-2004 to either 2032-35 or 2046-50.

Table 3
CBO Model Forces Contributing to Real Interest Rate Change
from 1995-2004 Base (percentage points)

Influence (percent)	Impact Parameter ^a	Effect by 2032-35	Effect by 2046-50
Labor force growth	1.56	-1.05	-0.84
Private domestic and foreign saving rate	-0.59	-1.24	-2.42
Total factor productivity growth	2.37	-1.78	-1.78
Risk premium	-1	-0.47	-0.27
Debt-to-GDP ratio	0.025	1.84	3.74
Capital share of income	0.43	0.89	0.94
Total impact		-1.81	-0.63
Benchmark real 10-year Treasury rate '95-04		2.96	2.96
Real 10-year Treasury rate forecast		1.15	2.33
Nominal 10-year Treasury rate forecast		3.38	4.56

a. Percentage point change in interest rate for one percentage point change in influence variable

Source: Gamber (2020)

The largest secular force reducing the interest rate is estimated to be the rising saving rate, from a 1995-2004 benchmark of 21.9 percent to 24 percent by 2032-35 and 26 percent by 2046-50. The rise in middle-aged workers relative to younger workers has increased the share of the labor force in the phase of high saving for retirement, and increasing lifespans increase the needed saving rate. Moreover, a slowdown in expected income growth has reduced the “time preference” discount rate.²² The higher saving rate reduces the real interest rate by 1.24 percentage point from benchmark by 2032-35 and by 2.42 percentage point by 2046-50. Next is total factor productivity growth, which was 1.86 percent annually in the benchmark period but is projected at 1.11 percent in both of the forecast periods.

The slowdown in productivity growth reduces the interest rate by 1.78 percentage point. The other large contributor to the falling interest rate is the slowdown in labor force growth, from 0.95 percent annually in 1995-2004 to 0.28 percent by 2031-35, but partially recovers to 0.41 percent by 2046-50. The four negative secular forces on interest rates (including risk premium) add up to a reduction from the benchmark interest rate by 4.54 percentage points by 2032-35. By 2046-50, their combined interest-reducing force reaches 5.31 percentage points.

There is a large countervailing source of upward pressure on the interest rate from the large increase in the ratio of debt to GDP, however. As set forth in Appendix D, a standard Cobb-Douglas (1928) model of production indicates that if the capital stock is reduced, the marginal product of capital will rise, driving up the interest rate. Fiscal deficits that divert saving

²² The classic Ramsey (1928) discount rate equals the rate of “pure” time preference for myopia plus the product of the elasticity of marginal utility and the expected growth rate of income.

away from private investment will have this effect. With this effect represented by the ratio of public debt to GDP, Gamber and Seliski (2019) find that a rise in debt by 1 percent of GDP boosts the 10-year Treasury rate by 2.5 basis points, based on a review of literature as well as new statistical estimates. Rachel and Summers (2019, p. 22) place this impact even higher, at 3.5 basis points as their inferred consensus estimate in the literature.

Gamber (2020, p. 49) places the average benchmark (1995-2004) debt ratio at 38 percent of GDP, and cites the CBO (2020e) projections of an average of 112 percent in 2032-35 and 188 percent in 2046-50. Applying the impact parameter of 2.5 basis points per percentage point in the debt ratio, the effect of rising debt is to boost the interest rate above the benchmark by 1.84 percentage point in 2032-35 by 3.74 percentage point in 2046-50. The other influence raising the rate is the rise in the capital share, which shows the responsiveness of output to additional capital.²³ An increase in this share from 33.2 percent in the benchmark period to 35.26 percent in 2032-35 and 35.38 percent in 2046-50 contributes an additional 0.89 percentage point to the interest rate in the first forecast period and 0.94 percent in the second.

By 2032-35 the CBO projects the net effect of the secular changes to reach -1.81 percentage point change in the real 10-year rate from the 1995-2004 benchmark. By 2046-50 the further increment in the positive change from (rising debt ratio and capital share) exceeds the further reduction from the four negative secular forces, reducing the net change from benchmark to -0.63 percentage point. With the benchmark real rate at 2.96 percent, the real 10-year rate stands at 1.11 percent by 2032-35 and 2.33 percent by 2046-50.

A major question in the estimates of secular trends is the extent to which interest rates in the 2010s were only temporarily low because of the long shadow of the Great Recession. Gamber (2020, p. 13) notes estimates indicating that the three episodes of quantitative easing reduced the 10-year Treasury rate by about 100 basis points. Gauti B. Eggertsson has argued that the secular trend estimates are quite sensitive to assumptions, and that a plausible alternative to a secular stagnation hypothesis (constant shortfall of private investment demand from private saving supply) is the view that “the fall observed in natural rates is only temporary and due to the global financial crisis and its aftermath.”²⁴ Indeed, by early 2021 Summers was arguing that the stimulus of the American Rescue Plan was so large that it could raise inflationary pressures and interest rates (and by implication, real interest rates).²⁵

Overall, the Gamber (2019) estimates judge that *the near-zero real interest rate in recent years has been a temporary phenomenon because upward pressure on the interest rate from rising public debt will swamp further downward pressure from demographic, productivity, and other secular factors*. He moreover hints that unbridled increases in the debt ratio as in the

²³ In the Cobb-Douglas function, the capital share is the “output-elasticity,” or proportionate responsiveness, to capital.

²⁴ Comment on Rachel and Summers, 2019b, p. 58.

²⁵ Lawrence H. Summers, “The Biden stimulus is admirably ambitious. But it brings some big risks, too.” *Washington Post*, February 4, 2021.

CBO long-term baseline could begin to add a sovereign-default risk premium to the rate demanded by investors.²⁶

Incorporating feedback from debt to the interest rate

A major implication of the main calculations in Table 2 and figure 8 above is that if even a moderate floor of 1 percent is set for the real 10-year Treasury rate, the idea that looking at the interest burden instead of the debt burden makes the debt problem disappear is misguided. Yet more specific attention to the influence of the debt ratio on the interest rate suggests that the prudential 1 percent real rate used in this study could well err on the low side.

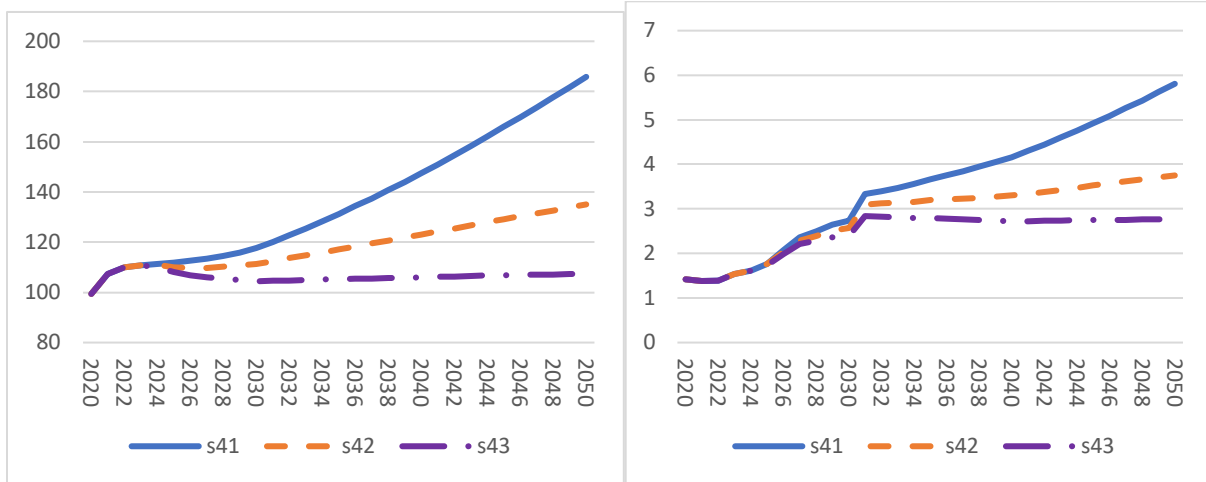
In Appendix D, I find that at the present level of the capital/output ratio (3.43), the impact of a one-percentage point increase in the debt to GDP ratio on the interest rate would be 1.88 basis points with no crowding out, or 0.94 basis points with 50 percent crowding out. Figure 9 shows what happens to the debt/GDP and interest/GDP projections if in addition to setting a prudential floor of 1 percent for the real interest rate, the calculations incorporate such a feedback to the interest rate. In particular, in the main projection scenario s21, debt held by the public rises by 67 percentage points of GDP from 2025 to 2050. Such an increase would cause the real interest rate to rise from the prudential benchmark 1 percent to 1.63 percent.

Figure 9 shows the results of incorporating feedback from the debt ratio to the interest rate. The first scenario digit, “4”, refers to this incorporation in addition to the projections of the main scenario of this study (“s2”). As before, the second digit refers to the primary deficit (1= baseline, 2= ceiling of 2 percent of GDP, 3= ceiling of 1 percent of GDP). As shown in figure 9, adding feedback from the debt ratio to the interest rate boosts the debt ratio by 2050 from 178 percent of GDP (s21 in figure 8 and table 2) to 186 percent (s41), and correspondingly raises the interest/GDP ratio from 4.7 percent to 5.8 percent. Although the qualitative finding of a baseline less severe than that projected by the CBO would remain unchanged, the finding of a major increase in both the debt and interest burden despite the recent comfort from low interest rates would be sharpened further. The figure also shows once again the strong influence of cutting the primary deficit on achieving greater fiscal sustainability on both the debt/GDP and interest/GDP metrics.

²⁶ “Projections of such levels of debt raise the possibility that rates could go much higher than CBO forecasts if investors began to have doubts about the ability or the willingness of the public to meet significantly higher debt-service obligations or of the Federal Reserve to keep inflation near its 2 percent target.” Gamber (2020, p. 21).

Figure 9

Effect of Incorporating Feedback from the Debt Ratio to the Interest Rate:
Debt and Interest Payments as Percent of GDP



Source: author's calculations

Reconsidering Debt Targets

Maastricht Mutatis Mutandis

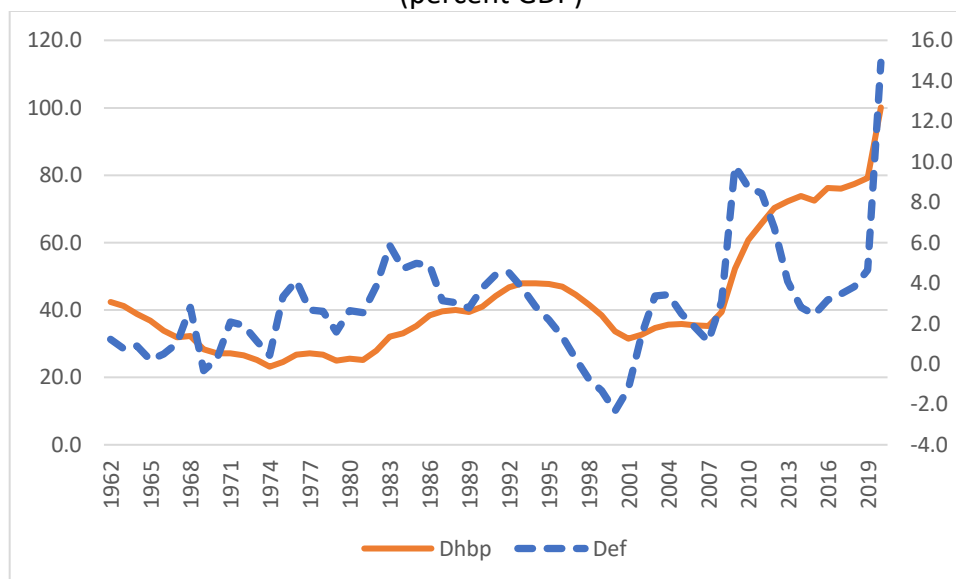
A central policy question is whether traditional fiscal targets should be changed to take account of lower interest rates. The predominant fiscal target framework for advanced economies has been the Maastricht criteria, adopted in February 1992 by the European Monetary Union (INSEE, 2021). The Maastricht criteria set a target of no more than 60 percent of GDP for the ratio of public debt to GDP, and a parallel target of a fiscal deficit no more than 3 percent of GDP. Implicitly these targets represented a normal state in which the nominal interest rate would be something like 5 percent, the real growth rate 3 percent, and the inflation rate 2 percent. Under these conditions, and with the primary balance at zero, nominal debt would grow at 5 percent (the interest rate). Nominal GDP would also grow at 5 percent (real growth plus inflation), so the debt ratio would remain unchanged. For its part, the fiscal deficit would also remain unchanged at 3 percent of GDP, because the 5 percent interest rate applied to the debt at 60 percent of GDP would generate a deficit of 3 percent of GDP. With the interest rate equal to the growth rate, the debt ratio would remain unchanged with a primary balance of zero (the “r-g” condition, Appendix A).

In the actual circumstances of the time, the three fiscally best-performing EMU economies (Germany, France, and the Netherlands) had the following average outcomes in the 6 years leading up to Maastricht (1986-91): real growth, 3.38 percent, inflation, 2.04 percent,

and government long-term interest rate, 7.85 percent.²⁷ So the real interest rate stood at about 5.8 percent. Applying equation A5) from Appendix A, holding the debt ratio at 60 percent of GDP would have required primary surpluses amounting to 1.54 percent of GDP.²⁸

It is informative to consider the US fiscal experience against the Maastricht criteria. As shown in figure 9, the debt ratio remained comfortably below 60 percent of GDP for the half-century preceding the Great Recession (figure 5 above). However, the Great Recession boosted debt from 35 percent of GDP in 2007 to 61 percent in 2010, and the combination of the tax cuts in the Trump administration and the Covid-shock recession brought the debt ratio to 100 percent of GDP at the end of fiscal 2020. For their part, fiscal deficits reached successive recession-cum-stimulus peaks at 5.9 percent of GDP in 1983, 8.7 percent in 2008, and 14.9 percent (likely to be followed by an even larger deficit in 2021).

Figure 9
US Fiscal Deficit (right scale) and Federal Debt Held by Public (left)
(percent GDP)



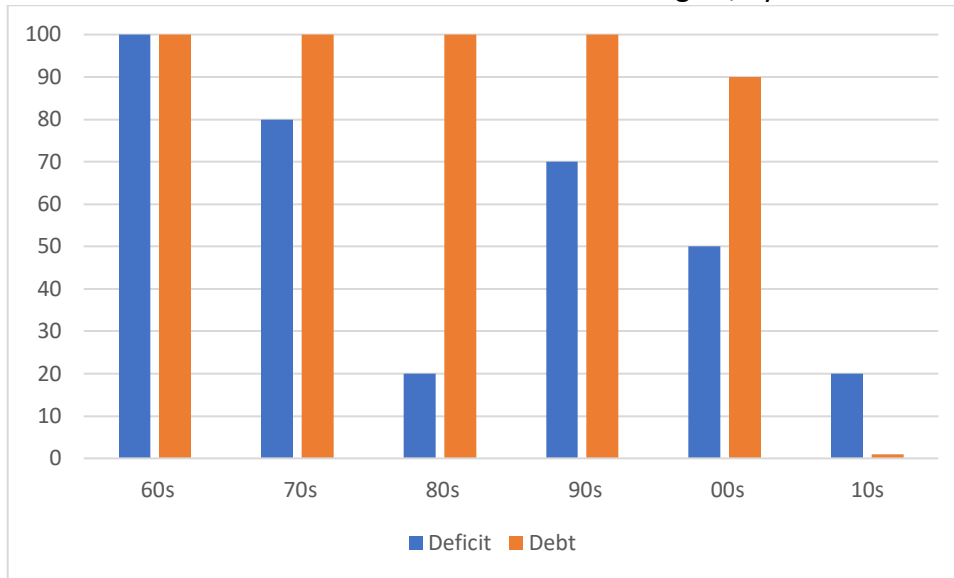
Source: CBO (2021d)

Figure 9 shows the corresponding percent of years in each of the past six decades that the US has met Maastricht fiscal targets. The debt targets were fully met in the first four decades, 90 percent met in the oughts, but completely missed in the 2010s (from 2011 to 2020). The fiscal target was met in only 2 of 10 years, in the 2010s, the same outcome as in the 1980s.

²⁷ Calculated from IMF (1992; 2021).

²⁸ But Germany came relatively close to a nominal interest rate of only 5 percent in 1986-88 (averaging 5.9 percent; IMF (1992).

Figure 10
Percent of Years US Achieved Maastricht Targets, by Decade



Source: CBO (2021d)

If one recognizes the long-term decline in interest rates but adopts a prudential benchmark of 1 percent for the real 10-year Treasury rate (the 33rd percentile in the distribution over the past 6 decades), a *mutatis mutandis* adjustment of Maastricht fiscal targets might be as follows. Considering that the median realized real interest rate (RRIR) was 2.2 percent over the past six decades, and under a main premise that the burden of debt is proportionate to the real interest rate, a *first approximation shift in the debt target would be to 132 percent of GDP* ($= 60 / [1/2.2]$). This approach could risk erring on the side of an excessive debt ceiling, however, because as the real interest rate approaches zero it would imply that the ratio of debt to GDP can safely reach infinity. Yet an infinite debt ratio is almost certainly associated with an infinite probability of debt default because rollover becomes impossible.

A similar parametric shift for the Maastricht fiscal deficit ceiling might be as follows. If the benchmark real interest rate declined by 1.2 percentage point (from median 2.2 percent to prudential floor 1.0 percent), by implication the increase in the inflationary “water” in the interest burden amounts to 1.2 percentage point. So a ceiling of 3 percent of GDP for the deficit could arguably be boosted to *4.2 percent of GDP deficit ceiling*. Such a target would have been met 50 percent of the time in the 2010s instead of only 20 percent (figure 10). The deficits projected by the CBO (2021f) would come close to meeting such an amended Maastricht target in the 2020s, but would far exceed this target in the 2030s and 2040s (table 1 above).

In the simulations shown in figure 8 above and summarized in table 2, the baseline using a prudential 1 percent real interest rate (scenario s21) yields debt and interest burdens that would exceed these adjusted Maastricht targets. By 2050, debt reaches 178 percent of GDP.

Interest reaches 4.7 percent of GDP, and adding the CBO's baseline 2050 primary deficit of 4.5 percent of GDP would place the fiscal deficit at 9.2 percent of GDP. The adjustment path of s22, limiting the primary deficit to 2 percent of GDP (compared to the average of about 4 percent over 2022-50 in the CBO baseline), would approximately meet the adjusted-Maastricht debt target (the debt ratio would reach 133 percent in 2050) but not the adjusted fiscal target (the deficit in 2050 would stand at 5.5 percent of GDP, comprising 2 percent primary and 3.5 percent interest). The more aggressive adjustment path of s23, *cutting the primary deficit to 1 percent of GDP, would decisively meet the adjusted debt target* (holding debt to 108 percent of GDP in 2050) and would *also meet the adjusted fiscal target* (yielding a 2050 deficit of 3.8 percent of GDP, from 1 percent primary plus 2.8 percent interest).

Prudential ceiling for the interest burden

In evaluating the interest burden, the 33rd-percentile "prudential" approach would yield the following result. From 1962 to 2019, the average ratio of net interest payments to GDP (shown in figure 5) was 1.91 percent; the median ratio, 1.57 percent; the maximum, 3.19 percent (in 1962); and the minimum, 1.18 percent (in 2019). If prudence suggests only being willing to accept the *67th percentile* of the past distribution as a benchmark for the interest burden going forward, that limit would be *2.2 percent of GDP* for the desired limit to the interest burden. Of the adjustment scenarios considered here, s22 and s23, neither one achieves this target, with the more aggressive adjustment case s23 placing the interest burden at about 2.8 percent of GDP by 2040 and 2050. This level would represent the 76th percentile of the past distribution of the interest burden, arguably a still-responsible prudential outcome.

The right "real" interest/GDP ratio

A key finding of this study is that even with a new environment of low interest rates, the medium- and long-term outlook for the interest burden of US debt indicates that a major fiscal adjustment will be needed in which primary deficits are curbed from their present trajectory. However, Furman and Summers (2020) have suggested that only the "real" interest burden should be considered in designing an interest-burden metric. They indicate that this ratio should be calculated by multiplying the current year's inflation rate multiplied by the previous year's debt, and subtracting the product from the interest payments in the numerator for comparison to GDP in the denominator. There are four problems with this approach.

First, by seeking a "real" value for the interest burden numerator while applying a nominal value to the GDP denominator, the approach would show a downward bias in measuring the burden over time as inflation boosted the denominator but not the numerator.

Second, if one seeks the "real" ratio of interest payments to GDP, surely that concept can be captured by taking the time series of nominal interest payments and deflating each year's amount into constant-dollar equivalents, and then doing the same thing for GDP in the denominator. The result would be real annual interest payment divided by real GDP. But

abstracting from the difference between the GDP deflator and the consumer price index (CPI) or personal consumption expenditure (PCE) used to deflate interest payments, each year's resulting ratio of real interest payments to real GDP will be identical to each year's ratio of nominal interest payments to nominal GDP.

Third, the Furman-Summers approach mixes stock adjustment in the numerator with flow in the denominator. Conflating stocks and flows is one of the critiques they themselves raise in questioning the meaning of the debt to GDP ratio, the usual metric of debt burden, because debt is a stock and GDP is a flow.

Fourth (albeit less fundamentally), even in their framework it would only be appropriate to deduct the inflationary erosion on the maturities coming due in the year in question, not of the entire outstanding stock of debt.

In sum, a focus of attention on the interest burden, to accompany attention to the debt to GDP ratio, is important, but should consider the ratio of all (net) interest payments compared to GDP, without reducing the numerator by subtracting inflationary erosion of the debt stock.

Macro-modeling versus accounting projections

The fiscal projections in this study are subject to the limitation that they do not seek to incorporate induced effects on the level of real output (or inflation) when the primary deficit or interest rate is changed. In broad terms a lower primary deficit would tend to reduce output working through the demand multiplier. Reducing the interest rate (e.g. well below the long-term path in the CBO projections to "only" 1 percent real) would tend to have the opposite effect output by spurring more investment. However, because the underlying CBO projection against which the sensitivity tests are conducted applies a baseline for GDP that exceeds (2021-25) or adheres (thereafter) to the growth of potential output, refinement to incorporate macro feedbacks would seem unlikely to alter the basic findings.²⁹

Conclusion and Policy Implications

Despite the low interest rates of recent years, there is no fiscal free-lunch forever. Although the low rates warrant taking the interest burden into account in addition to the debt burden in designing fiscal policy, a prudent benchmark for the real interest rate at 1 percent yields debt paths (and interest burden paths) that are well above past averages by 2030 and reach extremely high levels by 2050. Yet 1 percent is arguably a prudential floor, as it is the 33rd percentile of the real rate on the 10-year Treasury note over the past six decades based on the ex-post Realized Real Interest Rate (RRIR) measure developed in the first section of this study.

²⁹ The CBO (2021c, p. 13) places potential growth at 1.9 percent annually.

A fundamental analytical problem with nonchalance about rising debt ratios on grounds of low interest rates is that there are strong theoretical and empirical reasons to think that a rising ratio of debt to GDP causes an increase in the interest rate, making the combination of permanently low rates alongside ever-rising debt ratios an oxymoron (barring a fully offsetting further deepening of secular trends bringing rates down).

A more prosaic yet central problem with a benign neglect of dramatically rising debt ratios is that the analytical construct that has popularized the downplaying of debt – so-called “ r minus g ” dynamics – assumes away debt problems caused by large primary deficits rather than high interest rates and/or low growth. The implicit argument is that if the interest rate (r) is not much higher than the growth rate (g), the debt to GDP ratio can always be stabilized by swinging the primary deficit to a surplus equal to the difference between r and g . But reducing the primary deficit from its prospective long-term path of 3 to 5 percent of GDP to zero -- or even a deficit of 1 or 2 percent of GDP -- will be a major political challenge, especially in an environment in which there is a broad-based demand for more rather than less spending for social, anti-poverty, infrastructural and environmental purposes. A direct implication is that increases in such “permanent” spending categories should be paid for through permanent revenue increases rather than by additional borrowing.

An exercise adjusting the 1990s “Maastricht” fiscal targets to fit the benchmark 1 percent real interest rate boosts the permissible debt ceiling from 60 percent to 132 percent of GDP, and raises the fiscal deficit limit from 3 percent to 4.2 percent of GDP. The central simulations show that with the 1 percent real interest rate, it would be necessary to limit the primary deficit to 1 percent of GDP to meet both of these targets, far below the CBO’s baseline average of about 4 percent during 2022-50.

An ultimate challenge is that in the absence of reducing prospective primary deficits, perceived default risk could eventually begin to raise the cost of federal borrowing. The ability to inflate away debt rather than default because the debt is denominated in US currency is no guarantee against perceived default risk if investors judge that the potential inflation needed would be politically unacceptable.

Appendix A³⁰

“r Minus g” Once Again

It is well-known by now that the primary surplus required to keep the ratio of GDP to debt from rising is, approximately, the interest rate minus the growth rate. A closer look shows that the rule also involves interaction with the level of the debt to GDP ratio as well as the real interest rate, real growth rate, and rate of inflation.

Let D = debt, Y = GDP, r = the real interest rate, g = the real growth rate, and π = the rate of inflation, and subscript t designate the year. Let def be the fiscal deficit, and θ be the primary surplus as a fraction of GDP. Let λ be the ratio of debt to GDP.

Debt in year t then equals debt in the previous year plus the fiscal deficit in the current year:

$$A1) D_t = D_{t-1} + def_t$$

The deficit equals interest minus the primary surplus. Interest equals the interest rate multiplied by the previous year's debt stock; the primary surplus equals the primary parameter θ multiplied by GDP. Thus:

$$A2) D_t = D_{t-1} + (int_t - prim_t) = D_{t-1} \times (1 + r + \pi) - \theta Y_t$$

Dividing both sides by current year GDP, and then replacing current-year GDP by previous year GDP augmented by real growth and inflation yields the following current-year ratio of debt to GDP:

$$A3) \lambda_t = \frac{D_{t-1}(1 + r + \pi)}{Y_t} - \theta = \frac{D_{t-1}(1 + r + \pi)}{Y_{t-1}(1 + g + \pi)} - \theta = \lambda_{t-1} \frac{(1 + r + \pi)}{(1 + g + \pi)} - \theta$$

If the ratio of debt to GDP is successfully held constant, such that $\lambda_t = \lambda_{t-1} = \lambda$, then:

$$A4) \lambda = \lambda \frac{(1 + r + \pi)}{(1 + g + \pi)} - \theta$$

Rearranging and simplifying, the primary surplus ratio required to hold the debt ratio constant is then obtained as:

$$A5) \theta = \lambda \frac{(1 + r + \pi)}{(1 + g + \pi)} - \lambda = \lambda \left(\frac{(1 + r + \pi)}{(1 + g + \pi)} - 1 \right) = \lambda \left(\frac{1 + r + \pi - 1 - g - \pi}{(1 + g + \pi)} \right);$$

³⁰ For earlier statements, see Cline (2003, Annex A) and Cline (2014, pp. 39-40)

$$\theta = \frac{\lambda}{1 + g + \pi} \times (r - g)$$

The primary surplus that stabilizes the debt ratio then equals the real interest rate minus the real growth rate (the final expression in parentheses), multiplied by the (constant) debt to GDP ratio and divided by unity plus the nominal growth rate. Stabilizing the debt ratio at a higher level accordingly requires a larger primary surplus (twice as large, if the stable plateau is 120 percent of GDP instead of 60 percent). The stabilizing primary surplus is further affected by growth and inflation, but not by much.³¹

It is important to emphasize that the formulation just shown implicitly represents a steady state in which the interest rate on the whole stock of debt also equals the interest rate on this year's new debt, and the nominal interest rate equals a constant real rate (r) plus inflation (π). This framework constrains the influence higher inflation can have on the debt burden (or interest) burden, because inflationary erosion of past debt is almost fully offset by a higher interest rate on the full stock of debt.

³¹ For example, with $r = .025$, $g = .015$, $\pi = 0.025$, and $\lambda = 1.2$, boosting inflation to 10 percent ($\pi = 0.10$) would only reduce the needed primary surplus from $\theta = 0.0144$ (1.44 percent of GDP) to 0.0135. A plausible increase in growth (for example, to $g = 0.025$) would have even less impact on θ .

Appendix B

Calculating the Realized Real Interest Rate

When the nominal interest rate is constant over time and the inflation rate is constant over time the real interest rate is constant over time. However, US historical experience has shown wide variations of both inflation rates and interest rates over time. Under these circumstances, especially for long-term debt the real interest rate that actually obtains over the tenure of the debt can be far different from the simple difference between the nominal interest rate and the rate of inflation in the year of issuance. That rate may be called the “ex-ante” or “naïve” real interest rate.

For purposes of analyzing the real burden of interest (and hence of debt), it is important to calculate the ex-post realized real rate for historical series on debt that has matured, and for debt that remains outstanding, the best estimate of the likely ex-post realized real interest rate based on informed projections of the likely path of inflation over the remaining years to maturity. This rate may usefully be called the “realized real interest rate,” or RRIR.

The RRIR can be calculated as the internal rate of return on an investment project that comprises purchase of the debt claim in year 0, annual coupon earnings at the nominal rate on the debt in question over the period to maturity, and repayment of principal in the final year. The RRIR is then the time discount rate that sets the present value of this investment project to zero.

Let the price index be “ p_t ” in year t , normalized to unity in year $t = 0$. Let the coupon interest rate on a 10-year Treasury note be c . Then the real cash flow on purchase of the note for an initial amount of \$1 will be: $-1 + c/p_1 + c/p_2 + \dots + c/p_9 + c/p_{10} + 1/p_{10}$. The discounted present value of this stream of real cash flow, discounting at real rate λ , will be:

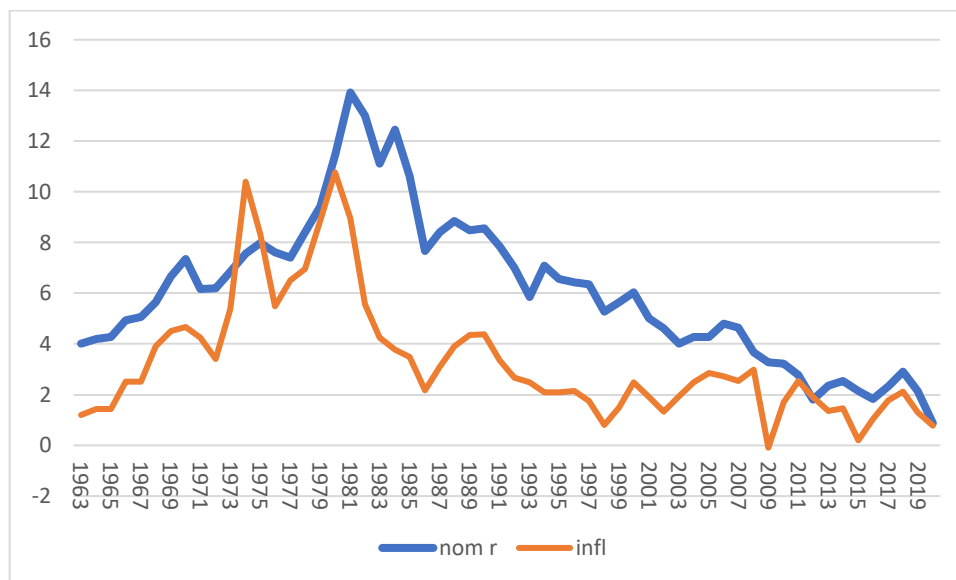
$$B1) PV = -1 + c \sum_{t=1}^{10} \frac{1}{p_t(1 + \lambda)^t} + 1/[p_{10} \times (1 + \lambda)^{10}]$$

On the right-hand sided, the first term is the initial outlay of \$1; the second term is the sum of the stream of coupon payments deflated by the price index and discounted at rate λ . The final term is the repayment of principal in the 10th year, deflated and discounted.

When the discount rate is set to λ^* such that $PV = 0$, the rate λ^* is the internal rate of return on this real stream of investment earnings. This rate is the Realized Real Interest Rate (RRIR) measure applied in this study. In implementation it is calculated using the “IRR” function in Microsoft Excel.

Figure B.1 shows the annual average nominal 10-year Treasury interest rate for 1963 to 2020, and the annual inflation rate (using the personal consumption expenditure index).

Figure B.1
Annual Inflation and Nominal Interest Rate on the 10-year Treasury Note
(percent)

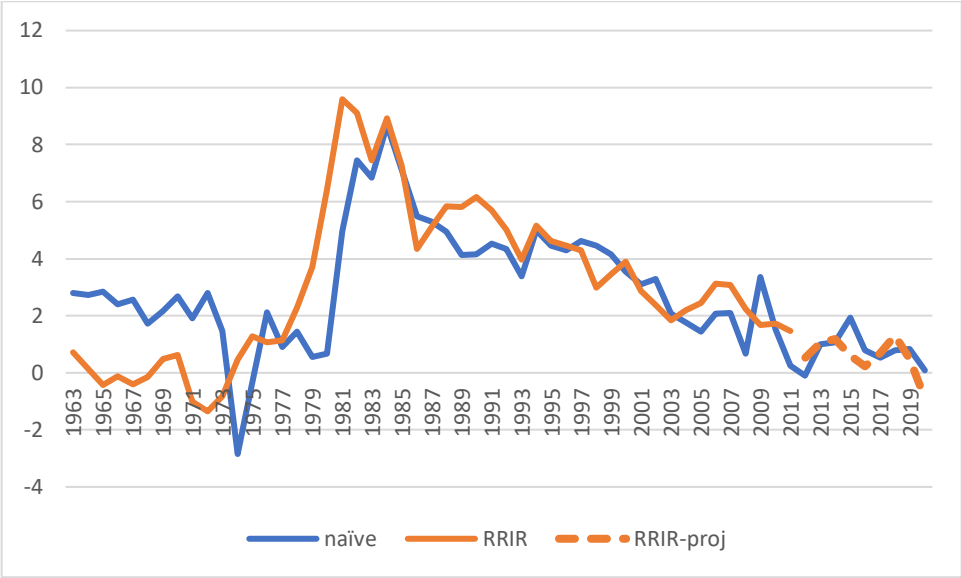


Source: FRED (2021a)

Figure B.2 shows the corresponding measure of the “naïve” 10-year rate when calculated as the rate for the year in question minus the inflation rate for that year. It also shows the RRIR, realized real interest rate calculated as the internal rate of return on realized real payment stream. For the years after 2011 this rate is “projected” because the final payments have not yet been received. The inflation rates needed to calculate the realized outcomes in the projected series are from CBO (2020b).

Because of the surprise of the high future inflation encountered by those who purchased 10-year notes in the early 1960s, the naïve real rates on notes issued in those years were far higher than the realized real rates that actually eventuated. Conversely, notes issued in 1979 through 1982 turned out to have substantially higher realized real interest rates than naïve real interest rates because high inflation in the year of issuance was followed by falling inflation. In 1979, the interest rate minus the inflation rate was only 0.5 percent, but the eventual realized real return on the 10-year note purchased in that year turned out to be 3.7 percent. In 1981, the corresponding comparison was 4.96 percent naïve real versus 9.59 percent RRIR.

Figure B.2
 Real Interest Rate on the 10-year Treasury Note: Naïve and RRIR
 (percent)



Source: calculated from FRED (2021a) and CBO (2020b)

Appendix C

An Accounting Model of Federal Debt Accumulation and Interest Obligations

Total US federal debt at the end of 2019 amounted to \$22.7 trillion, or 107 percent of GDP (CBO sept 20-30, pp.6, 8). Of the total, debt held by the public was \$16.8 trillion (79.2 percent of GDP). The remaining \$5.9 trillion was held by federal trust funds and other government accounts. Of these, the largest were Social Security (\$2.9 trillion), civil service retirement (\$960 billion), military retirement (\$827 billion), and Medicare (\$303 billion) (CBO 2020a, p. 24). As a small offset to debt held by the public, the federal government held \$1.84 trillion in financial assets, primarily comprising student loans (\$1.26 trillion) and Treasury's operating cash balance (\$382 billion; p. 23).

There is an inherent legacy structure to the future path of debt and interest obligations. In any given year, the interest coming due is the sum of the interest obligations attributable to the remaining maturities of each of the past several years' borrowings. In particular, the interest obligation cannot be measured by simply applying this year's interest rate to last year's total stock of debt. The first central component of a projection model is thus a tractable summary form of calibrating the vintages, maturities, and year-specific interest rates of outstanding debt in order to arrive at the current year's inherited interest obligations.

For the United States, the vintages and coupon rates of outstanding debt are not readily available.³² However, the CBO (2020a, p. 9) does report the "average remaining maturity of marketable debt," which provides guidance for the vintage components of the stock of debt. This average remaining maturity fell from 5.6 years at the end of 2001 to 4 years at the end of 2009, and then rose back to 5.6 years by the end of 2019. The first parameter in the model here is " μ " for average remaining maturity. For simplicity the projections use $\mu = 5$ years.

The second central challenge for a projection model is identifying the "representative interest rate" that best serves as the overall interest rate variable. It turns out that the 10-year Treasury note interest rate provides a relatively good approximation of the weighted average interest rate on outstanding debt, applying the rate of the year in question for each outstanding vintage.

A third question concerns the treatment of debt held by the public as opposed to total debt. Net interest in the outlays excludes interest to federal trust funds. Working in the other direction, there are modest interest earnings from federal credit programs (mainly student loans).³³ Application of the interest rate to federal debt held by the public is the correct approach because only that debt generates an interest outflow from the government sector. To address the complication of modest interest earnings, a typical "shrinkage" parameter " ψ " can

³² In contrast, publicly available data on individual outstanding Italian government securities, with detail on coupon and maturity, were used in my 2012 analysis of sustainability of Italian public debt (Cline, 2012).

³³ In FY2020 interest earnings were \$43 billion (CBO 2020a, p. 13).

be applied to the gross interest on debt held by the public to arrive at the net interest estimate for outlays. This factor may also incorporate the possible influence of average original maturity being less than 10 years, and hence the term premium being lower.

The resulting accounting-based projection model is then as follows. For debt, D_t is debt held by the public at the end of year t , and def_t is the deficit in year t .

$$C1) D_t = D_{t-1} + def_t$$

The deficit equals net interest in year t , int_t minus the primary surplus, $prim_t$ (revenues minus all non-interest spending).

$$C2) def_t = int_t - prim_t$$

Net interest payable is calculated as the shrinkage factor ψ multiplied by the sum of interest payments due on each vintage of new borrowing over the past 5 (μ) years:

$$C3) int_t = \psi \sum_{k=t-1}^{t-\mu} \rho_k B_k$$

where ρ_t is the 10-year Treasury interest rate (nominal) in year t and B_t is new borrowing in year t .

New borrowing in turn equals the deficit plus amortization (amz_t) of debt to be rolled over:

$$C4) B_t = amz_t + def_t$$

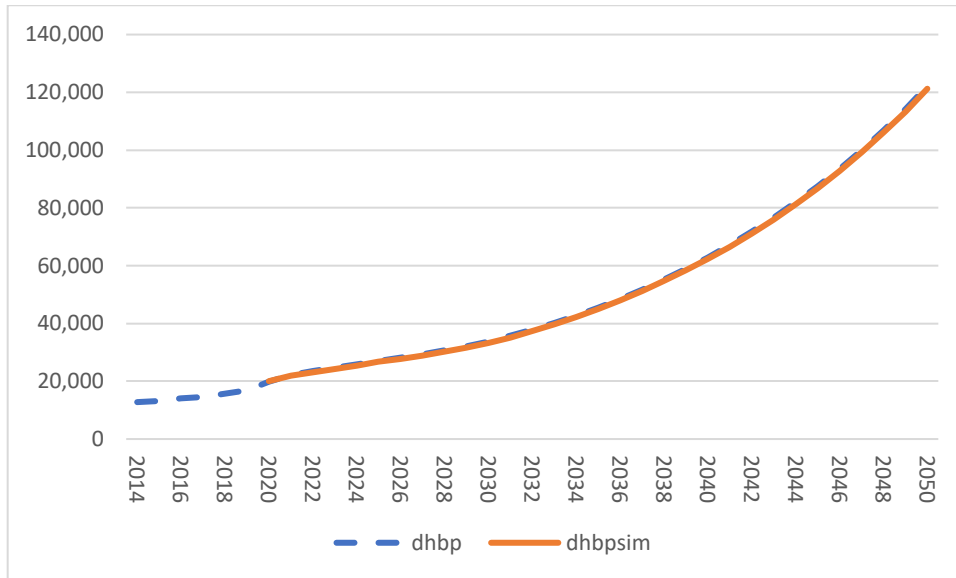
Amortization is the stock of debt held by the public at the end of the previous year divided by the average residual maturity μ .

$$C5) amz_t = D_{t-1}/\mu$$

With projections using this model, two alternative measures of debt burden can be calculated: debt/GDP and interest/GDP. The projections hold constant the future streams of GDP and primary balances projected by the CBO (2020e).

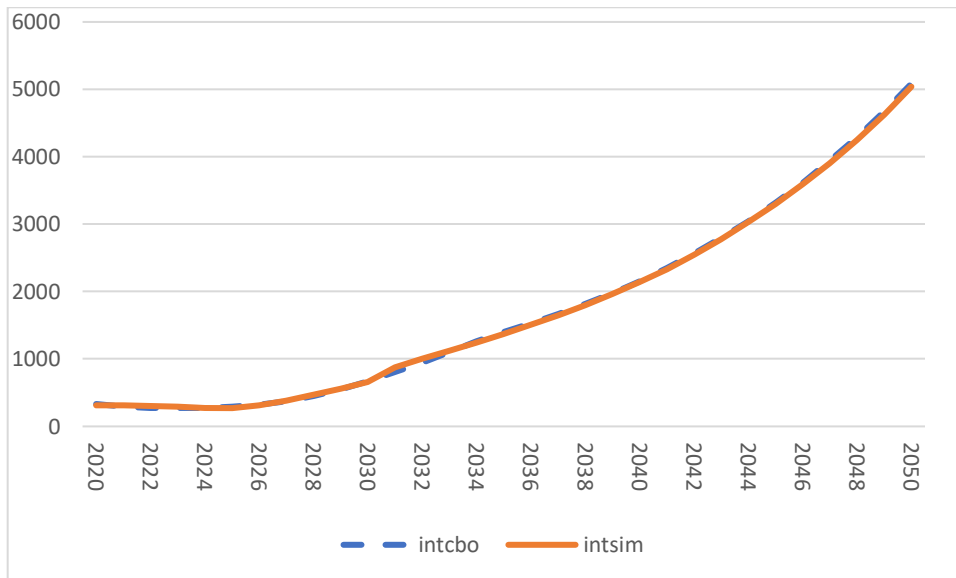
The only parameter to be calibrated is ψ . During 2021-30, the value of this shrinkage parameter needed to bring the average ratio of simulated to CBO-projected interest payments down to unity is $\psi = 0.77$; thereafter the required value for this parameter is 0.886. As shown in figures C.1 and C.2, when applying the CBO's assumed baseline of the 10-year Treasury rate, the model successfully replicates the CBO long-term forecasts of debt held by the public and net interest payments (with the model estimates indicated as "sim").

Figure C.1
Federal Debt Held by the Public: CBO Baseline and Model Replication
(\$ billions)



Source: CBO (2020e); author's calculations

Figure C.2
Net Interest Payments: CBO Baseline and Model Replication
(\$ billions)



Source: CBO (2020e); author's calculations

Appendix D

Crowding Out and the Impact of Rising Debt on the Interest Rate

If a fiscal deficit diverts saving that otherwise would go to investment, the consequence will be less capital available to work with labor in the production process. Less capital than in the no-deficit baseline will boost the marginal product of capital, and thus the interest rate markets are prepared to pay for capital.

In the work-horse Cobb-Douglas aggregate production function, the exponent of capital, α , is the elasticity of output with respect to capital, and the exponent on labor, $(1 - \alpha)$, is the elasticity with respect to labor.³⁴ The two exponents also show the corresponding factor shares in output, with factor payments equal to amount of the factor multiplied by the factor's marginal product.

Let K = capital stock, k = the ratio of capital to output, MP_K = marginal product of capital, and Y = output (GDP). Payment to capital equals the amount of capital multiplied by its marginal product. Payment to capital also equals its factor share multiplied by output. So:

$$D1) MP_K \times K = \alpha Y$$

By definition, it is also the case that output equals the amount of capital divided by the capital/output ratio, or:

$$D2) Y = \frac{K}{k}$$

Replacing Y in D1) by the right-hand side of D2), and simplifying, yields:

$$D3) MP_K = \frac{\alpha}{k}$$

A key feature of the Cobb-Douglas production function is thus that the marginal product of capital equals the factor share of capital divided by the capital-output ratio.

Consider the impact on the marginal product of capital if there is a change in capital amounting to 1 percent of GDP. From equation D3), the percent change (" $\% \Delta$ ") in marginal product of capital will be the percent change in the numerator of the right-hand side, minus the percent change in the denominator. The numerator is a constant, so its percent change is zero.

³⁴ In the Cobb-Douglas production function, $Y = AK^\alpha L^{(1-\alpha)}$, where Y is output, A is a constant, K is the stock of capital, and L is the amount of labor applied to production. Cobb and Douglas (1928). The elasticity is the percent change in the dependent variable resulting from a 1 percent change in the independent variable.

For the denominator “ k ”, the percent change will equal the percent change in capital stock (K) minus the percent change in output (Y).

$$D4) \% \Delta MP_K = \% \Delta \alpha - \% \Delta k = 0 - [\% \Delta K - \% \Delta Y]$$

When capital is increased by an amount equal to 1 percent of GDP, the increase will amount to only $1\%/k_0$ (where k_0 is the capital/output ratio before the change) when expressed as a percent of the capital stock. The percent change in output will equal the percent change in capital stock multiplied by the output elasticity of capital. From $D4$), the percent change in marginal product of capital then becomes:

$$D5) \% \Delta MP_K = - \left[\frac{1\%}{k_0} - \alpha \frac{1\%}{k_0} \right] = - \left[\frac{(1 - \alpha)1\%}{k_0} \right]$$

The change in the marginal product of capital will then be (and denoting the initial value with subscript 0):

$$D6) \Delta MP_K = MP_{K_0} \times \% \Delta MP_K = \frac{\alpha}{k_0} \times \left[- \frac{(1 - \alpha)1\%}{k_0} \right] = \frac{\alpha(\alpha - 1)1\%}{k_0^2}$$

The final right-hand side in equation $D6$) is the same result as that obtained by Gamber and Seliski (2019, p. 18) using a Cobb-Douglas framework and applying a sequence of differentials.

In the US economy, the capital share (and output elasticity of capital) is typically placed at $\alpha = 0.33$. The US capital stock for 2017 is estimated at \$66.94 trillion (FRED, 2021c). GDP in 2017 was \$19.543 trillion (BEA, 2021). The capital/output ratio (k) was 3.425, so from equation $D3$) the marginal product of capital was 0.0964, or almost 10 percent. Applying these values of α and k_0 to equation $D6$) yields the result that an increase in the capital stock by 1 percent of GDP would reduce the marginal product of capital by 0.0188%, or 1.88 basis points.

Correspondingly, reducing the capital stock from its baseline by 1 percent of GDP as a consequence of an extra 1 percent of GDP fiscal deficit with complete crowding out would boost the marginal product of capital by 1.88 basis points. This outcome is similar to the range found in previous Cobb-Douglas based estimates (2.1 to 3.5 basis points, for 5 studies in the period 1995 to 2009 as reported by Gamber and Seliski, 2019, p. 24).

If the potential crowding out is offset by additional private saving or borrowing from abroad, the impact on the marginal product of capital will be less. Correspondingly, the increase in the interest rate caused by crowding out will be less.

References

BEA (Bureau of Economic Analysis). 2021. *National Income and Product Accounts. Table 1.1.5 Gross Domestic Product*. Washington (February)

Blanchard, Olivier. 2019. "Public Debt and Low Interest Rates," *American Economic Review*, vol. 109, no. 4, pp. 1197-1229.

Brookings (Brookings Institution). 2019. *Brookings Papers on Economic Activity, Spring 2019*. Washington (December)

CBO (Congressional Budget Office). 2020a. *Federal Debt: A Primer*. Washington (March). www.cbo.gov/publication/56165.

CBO. 2020b. *An Update to the Economic Outlook: 2020 to 2030*. Washington (July). www.cbo.gov/publication/56442.

CBO. 2020c. *The Effects of Pandemic-Related Legislation on Output*. Washington (September). www.cbo.gov/publication/56537

CBO. 2020d. *An Update to the Budget Outlook: 2020 to 2030*. Washington (September). www.cbo.gov/publication/56517. Supplementary tables: www.cbo.gov/data/budget-economic-data#4

CBO. 2020e. *The 2020 Long-Term Budget Outlook*. Washington (September). www.cbo.gov/publication/56516. Supplementary tables: <https://www.cbo.gov/data/budget-economic-data#1>

CBO. 2020f. *Federal Net Interest Costs: A Primer*. Washington (December). www.cbo.gov/publication/56780.

CBO. 2021a. *An Overview of the Economic Outlook: 2021 to 2031*. Washington (February). www.cbo.gov/publication/56965.

CBO. 2021b. *Historical Data and Economic Projections*. Washington (February). <https://www.cbo.gov/about/products/budget-economic-data#11>

CBO. 2021c. *The Budget and Economic Outlook: 2021 to 2031*. Washington (February) <https://www.cbo.gov/publication/56970>.

CBO. 2021d. *Budget and Economic Data*. Washington (February). <https://www.cbo.gov/data/budget-economic-data#2>

CBO. 2021e. *Estimated Budget Effects of the American Rescue Plan Act of 2021*. Washington (February). <https://www.cbo.gov/publication/57012>

CBO. 2021f. *The 2021 Long-Term Budget Outlook*. Washington (March). <https://www.cbo.gov/publication/56977>

Cline, William R. 2003. *Restoring Economic Growth in Argentina*. 2003. World Bank Research Working Paper 3158. Washington: World Bank (October) <https://openknowledge.worldbank.org/handle/10986/18043>

Cline, William R. 2012. *Interest Rate Shock and Sustainability of Italy's Sovereign Debt*. Policy Brief 12-5. Washington: Peterson Institute for International Economics (February)

Cline, William R. 2014. *Managing the Euro Area Debt Crisis*. Washington: Peterson Institute for International Economics (June)

Cobb, C. W., and P. H. Douglas. 1928. "A Theory of Production," *American Economic Review*, vol. 18 (supplement), pp. 139-165.

CRFB (Committee for a Responsible Federal Budget). 2021. *Analysis of CBO's March 2021 Long-Term Budget Outlook*. Washington (March)

FRED (Federal Reserve Bank of St. Louis). 2021a. 10-Year Treasury Constant Maturity Rate (DGS10). St. Louis (January)

FRED. 2021b. 10-Year Treasury Inflation-Indexed Security, Constant Maturity (DFII10). St. Louis (January)

FRED. 2021c. Capital Stock at Constant National Prices for United States (RKNANPUSA666NRUG). St. Louis (February)

Furman, Jason, and Lawrence Summers. 2020. *A Reconsideration of Fiscal Policy in the Era of Low Interest Rates*. Cambridge, MA (November). Available at: <https://www.piie.com/system/files/documents/furman-summers2020-12-01paper.pdf>

Gamber, Edward N. 2020. *The Historical Decline in Real Interest Rates and Its Implications for CBO's Projections*. Congressional Budget Office Working Paper 2020-09. Washington (December).

Gamber, Edward, and John Seliski. 2019. *The Effect of Government Debt on Interest Rates*. Congressional Budget Office Working Paper 2019-01. Washington (March)

IMF (International Monetary Fund). 1992. *International Financial Statistics Yearbook*. Washington.

IMF. 2020. *World Economic Outlook Database October 2020*. Washington.

IMF. 2021. *Fiscal Monitor Update*. Washington (January)

INSEE (Institut National de la Statistique et des Études Économiques). 2021. "Convergence criteria" (Maastricht Treaty). Paris (January).

Rachel, Lukasz, and Lawrence H. Summers. 2019a. "On Falling Neutral Real Rates, Fiscal Policy, and the Risk of Secular Stagnation." *Brookings Papers on Economic Activity*. BPEA Conference Drafts. Washington: Brookings Institution (March). <https://www.brookings.edu/wp-content/uploads/2019/03/On-Falling-Neutral-Real-Rates-Fiscal-Policy-and-the-Risk-of-Secular-Stagnation.pdf>

Rachel, Lukasz, and Lawrence H. Summers. 2019b. "On Secular Stagnation in the Industrialized World," *Brookings Papers on Economic Activity, Spring 2019*, pp. 1-54. Washington: Brookings Institution (December)

Ramsey, F. P. 1928. "A Mathematical Theory of Saving," *Economic Journal*, vol. 138, no. 52 pp. 543-59.

Yellen, Janet. 2021. *Finance Committee Questions for the Record*. US Senate Committee on Finance. Washington (January). <https://www.finance.senate.gov/hearings/hearing-to-consider-the-anticipated-nomination-of-to-be-the-honorable-janet-l-yellen-to-secretary-of-the-treasury>