EG02041 Advanced Mathematics

Class 7: First-Order Differential Equations

8.3 Autonomous Differential Equations and Stability

Equilibrium Values and Stability

In an autonomous equation, the right-hand side is only function of y

$y' = 6y^2 - y^3$ is autonomous

$y' = x - 6y$ is not autonomous

Autonomous differential equations are often used to predict the population growth. If *Y* represent the size of the population, it seems reasonable to assume that the rate of the population size not dependent on time.

Equilibrium Values:

One model of population growth (decay) sets the relative rate of growth as a constant so that population satisfies the autonomous differential equation:

$y' = ky$ for same constant of k

Definition:

Let C be an equilibrium value of autonomous differential equation, with an initial value $y(0) = y_o$ that is close to C

1) If y diverges from C for large value, x, we say that C is unstable
2) If y coverage to C for large value of x, we say that C asymptotically stable
3) If slightly diverge above coverage C is semistable.

Example 1: Find the equilibrium value of differential equation $y^2 - y - 2 = 0$, and assess the stability of each.

Example 2: Determine the equilibrium value of $y' = y^4 - 4y^2$ and assess the stability of each.

Logistic Growth Model

A population that grows with a constant per capita rate K satisfies the inhibited growth model

$$y = ky$$

We have seen the non-zero solutions:

$$y' = ky(1 - y/L)$$

Where k and L are positive constants. The per capita growth rate is $k(1 - y/L)$, which decrease from k to 0 as y increase from 0 to L. The physical significant of L will become apparently shortly.

Example 5: Determine the equilibrium value of the logistic growth model and assess the stability of each.

Example 6: *Bacteria growth*. Many bacteria strains are used by the dairy industry to produce different types of fermented milk and yogurts. During a fermentation experiment, the population y of bacteria lactobacillus fermented after t hour in a wheat media satisfied the differential equation.

$$y' = 0.532y(1 - \frac{y}{1900})$$

Initially 8 million bacteria per ml were present. Determine the number of bacteria per ml after 5 hours