# STOCK RETURNS AND THE MILLER-MODIGLIANI VALUATION FORMULA: REVISITING THE FAMA-FRENCH ANALYSIS

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**Abstract**

Fama and French (2006) use the dividend discount model to develop the joint role of three variables – expected profitability, expected investment and current *BM* – in predicting future stock returns. One reported empirical result is anomalous. The valuation model establishes that the comparative static relation between returns and expected investment is negative, yet it appears to be positive and insignificant. We show that the posited valuation relations between expected returns and the three variables apply at the firm level, and not at the per-share level at which they were tested. Once the variables are measured at the firm level, all the Fama-French predictions are validated.

Keywords: Book to market, profitability, investment, expected return

JEL codes: G12, G32

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## 1. Introduction

The relations between profitability, investment, *BM* and expected returns have been studied extensively in recent decades.[1] Fama and French (2006) provide an important contribution by examining these return regularities together using the dividend-discount model. More specifically, the following valuation formula should hold:

$$V(t) = \sum_{\tau=0}^{\infty} \frac{E\left(Y(t+\tau)\right) - E\left(\Delta B(t+\tau+1)\right)}{(1+\rho)^{\tau+1}}, \qquad (1)$$

where $V(t)$ is the market value of the firm's equity at the beginning of period t, $Y(t)$ is income after interest and taxes assumed to be received at the end of period *t*, $B(t)$ is the book value of equity at the beginning of period *t*, $\Delta B(t+1) \equiv B(t+1) - B(t)$ and $\rho$ is the discount rate. Dividing through by the book value of equity, it follows immediately that future stock returns should be positively correlated with both current *BM* and *expected* future profitability relative to current book value, and negatively correlated with *expected* future growth in book equity again relative to current book value. Since the two causes of growth in book equity are investments by equity-holders in the form of either retained earnings or purchases of new share issues, the terms 'expected future investment' and 'expected future growth in book equity' are used interchangeably.

The Miller-Modigliani valuation formula in (1) is a tautology. It should hold irrespective of whether investors are rational or suffer from cognitive biases. In other words, the relation between rational assessments of expected profitability and investment, current *BM*, and true expected returns should hold independently of the mechanism used by investors to form their expectations. Therefore, the fact that not all of its predictions appear to be true in the data—Fama and French report a positive insignificant coefficient for expected investment—seems puzzling.

[1] For example, Rosenberg et al. (1985) and Fama and French (1992) document a positive relation between current BM and future returns; Titman et al. (2004) and Cooper et al. (2008) document a negative relation between current investment and future returns; Haugan and Baker (1996) document that profitability is positively correlated with future returns, even after controlling for BM; Chen et al. (2011) suggest a new three-factor model in which investment and profitability are the main explanatory variables.

We argue that the limited success of the empirical portion of Fama and French is largely related to their measures of expected profitability and expected investment. We show that valuation formula (1) applies at the firm level and not, as in the Fama-French empirical investigation, at the per-share level. This is because changes in the number of shares—due either to new issues or repurchases—are likely to mitigate the correlation between the expected change in investment per-share and expected returns.[2] For example, consider a firm that issues equity. Whether book equity per share increases or decreases will depend on the firm's *BM* at the time of the share issue, and an expected increase in book equity need not imply an expected increase in book equity per share. Our empirical work demonstrates that the coefficient on the expected change in investment is significantly changed when the variables are estimated at the firm level rather than at the per-share level. We confirm that the coefficient of expected investment is small and insignificant at the per-share level.[3] However, at the firm level the coefficient is negative and significant, as predicted by the valuation formula. Further, in a robustness test we examine whether the predictions of the valuation formula hold in portfolios formed on the basis of *BM* and confirm that the negative correlation between expected investment and realized returns holds in both low and high-*BM* portfolios.

Fama and French (2006) considered various measurement errors in the empirically determined proxies for expected profitability and expected investment as one potential explanation. Fama and French (2008a) argue that the limited success of the valuation formula is because *BM* captures information about both expected cash flows and discount rates. Two variables, the evolution of *BM* and net issuance of equity are used to try and disentangle these two components of the information in *BM*. However, Fama and French (2008a) do not then examine the relation between expected future investment and expected returns. They consider only the relation between current investment and expected returns. Our study differs in that we examine the performance of the valuation formula directly and do so at the correctly-specified firm level.

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[2] Both Fama and French (2005) and Lyandres et al. (2008) show that firms which increase their investments are likely to be large issuers of equity.

[3] We also confirm that Fama-French per-share result that the coefficients on both BM and expected profitability are significantly positive, and that the coefficients on size is significantly negative.

The remainder of the paper is organized as follows. Section 2 examines the valuation formula at the firm and per share levels. Section 3 describes the methodology, data, and summary statistics. Section 4 presents our empirical results and Section 5 concludes.

## 2. Valuation formula at the per-share and the firm levels

We follow the notation used in Miller and Modigliani (1961) with upper case used to denote firm level variables and lower case for per-share variables. The price of a share at the start of period *t*, $p(t)$, can be written as

$$p(t) = \frac{E(d(t)) + E(p(t+1))}{1+\rho},$$

where $d(t)$ is the dividend per share paid at end of period *t* to holders of record at the start of period *t* and $\rho$ is the discount rate or internal rate of return. Multiplying by $n(t)$, the number of shares at the start of period *t*, gives the total value of equity as

$$\begin{aligned} V(t) &= n(t)\left(\frac{E(d(t) + p(t+1))}{1+\rho}\right) \\ &= \frac{E(D(t)) + E((n(t+1) - m(t+1))\, p(t+1))}{1+\rho} \\ &= \frac{E(D(t) - m(t+1)\, p(t+1)) + E(V(t+1))}{1+\rho}, \end{aligned} \qquad (2)$$

where $D(t)$ is the total dividend $n(t)d(t)$, and $m(t+1)$ is the number of shares issued / repurchased at the end of period *t* at the ex-dividend closing price of $p(t + 1)$.

Now consider the link between the end of period total dividends paid in excess of net equity issued in the numerator of (2) and the implications of clean surplus accounting:

$$D(t) - m(t+1)\, p(t+1) = Y(t) - \Delta B(t+1).$$

Substituting for $D(t) - m(t+1)\, p(t+1)$ in the numerator of (2) and iterating gives the valuation formula in expression (1). Dividing both sides of (1) by the book value of equity at the start of period *t* gives

$$\frac{V(t)}{B(t)} = \sum_{\tau=0}^{\infty} \frac{E\left(Y(t+\tau)/B(t)\right) - E\left(\Delta B(t+\tau+1)/B(t)\right)}{(1+\rho)^{\tau+1}}. \tag{3}$$

We refer to (3) as the Miller-Modigliani valuation formula. Expected future net income relative to current book equity is a measure of profitability. Expected future changes in book equity relative to current book equity are a measure of investment. Thus the valuation formula involves four terms: expected profitability, expected investment, *BM*, and the expected return on the stock. Conditioning on two of the four, a monotone relation must hold between the other two. Ceteris paribus, there is a positive relation between expected profitability and expected future stock returns; a negative relation between expected investment and expected future stock returns; and a positive relation between current *BM* and expected future stock returns.

Next, we examine the valuation formula at the per-share level. Rewriting (3) and dividing both the numerator and denominator on the right-hand-side by the number of shares at the start of period *t* gives

$$\frac{V(t)}{B(t)} = \frac{\sum_{\tau=0}^{\infty}\left(E\left(Y(t+\tau)/n(t)\right) - E\left(\Delta B(t+\tau+1)/n(t)\right)\right) \Big/ (1+\rho)^{\tau+1}}{B(t)/n(t)}.$$

Note that $Y(t+\tau)/n(t) \neq Y(t+\tau)/n(t+\tau) \equiv y(t+\tau)$ since the number of stocks can change between *t* and $t+\tau$. Multiplying and dividing each element in the summation in the numerator by the number of shares at the start of the corresponding period, $n(t+\tau)$, gives

$$\frac{V(t)}{B(t)} = \sum_{\tau=0}^{\infty} \frac{E\left(\frac{n(t+\tau)}{n(t)}\frac{y(t+\tau)}{b(t)}\right) - E\left(\frac{n(t+\tau)}{n(t)}\frac{\Delta b(t+\tau+1)}{b(t)}\right)}{(1+\rho)^{\tau+1}}, \tag{4'}$$

where $y(t)$ is per-share earnings received at the end of period *t*, $b(t) \equiv B(t)/n(t)$, and $\Delta b(t+1)$ is the per-share change in book equity from the beginning of period t to the beginning of period $t+1$. Absent share issuance and repurchase, $n(t+\tau) = n(t)$ for all $\tau$ and the relation simplifies to that empirically investigated in Fama and French (2006):

$$\frac{V(t)}{B(t)} = \sum_{\tau=0}^{\infty} \frac{E\left(y(t+\tau)/b(t)\right) - E\left(\Delta b(t+\tau+1)/b(t)\right)}{(1+\rho)^{\tau+1}}. \qquad (4'')$$

To see how a per-share analysis can differ from a firm level analysis consider a firm with $n(t)$ shares outstanding and book equity per share of $b(t)$. Assume that nothing will happen over the coming period except that the firm will issue $m(t+1)$ new share at the end-of-period market price of $p(t+1)$ and invest the proceeds. The growth in book equity over the period will be $\left(m(t+1)\,p(t+1)\right)/\left(n(t)\,b(t)\right)$ and is clearly positive. Whether book equity per-share increases or decreases will depend on *BM* at the time of the share issue. The growth in book equity per-share of $\left(\frac{m(t+1)\,p(t+1)+n(t)\,b(t)}{m(t+1)+n(t)} - b(t)\right) \Big/ b(t)$ will be either negative or positive as a function of whether the end-of-period *BM* exceeds or is less than unity. Thus per-share investment need not even have the same sign as firm level investment. It is perhaps not surprising that it is the coefficient on per-share investment that turns out to have the opposite sign to that expected in a firm level analysis of the valuation formula.

To illustrate the problem with relation $(4'')$, consider two growth firms with the same current value of $BM < 1$, the same (positive) expected future firm level profitability, and but for the coming year the same expected future firm level investment. In the coming year the expected firm level investment of firm A is higher than that of firm B. The Miller-Modigliani valuation formula of equation (3) makes clear that the expected return on firm A must be lower than that on firm B as its firm level investment is higher. Now suppose that firm A will finance the coming year's incremental firm level investment by issuing shares and suppose a researcher seeks to investigate the per-share relation in $(4'')$. This relation fails to properly account for firm A's stock issuance. Firm A's per-share profitability will be lower than that of firm B in all years after the coming year. Firm A's per-share expected investment will be higher than that of firm B in the coming year, but lower in all years thereafter. As a result, the flawed relation $(4'')$ does not make clear which firm should have the higher expected return.

It is particularly instructive to rewrite $(4')$ as

$$\frac{V(t)}{B(t)} = \sum_{\tau=0}^{\infty} \frac{E\left(\frac{y(t+\tau)}{b(t)}\right) - E\left(\frac{\Delta b(t+\tau+1)}{b(t)}\right) + E\left(\frac{m(t+\tau+1)}{n(t+\tau)}\frac{p(t+\tau+1)}{b(t)}\right)}{(1+\rho)^{\tau+1}}. \qquad (4''')$$

Expressing the valuation formula as $(4''')$ makes clear that regression analyses based on a linearization of $(4'')$ suffer from the omission of the expected new issues term in $(4''')$. Holding constant *BM*, per share expected profitability, and per share expected investment, relation $(4''')$ shows that the omitted term will be positively related to expected future stock returns.

## 3. Methodology, data and distributional statistics

We follow Fama and French (2006) (henceforth FF) in first estimating a series of regressions in which various accounting and market variables are used to predict future investment and future profitability. We refer to these as the first-stage regressions:

$$\begin{aligned} \Delta A(t+1)/A(t) &= \lambda_0 + \lambda_1 \ln BM(t) + \lambda_2 \ln MV(t) + \lambda_3 Neg\,Y(t-1) + \lambda_4 Y(t-1)/B(t-1) \\ &+ \lambda_5\left[-AC(t-1)/B(t-1)\right] + \lambda_6\left[+AC(t-1)/B(t-1)\right] + \lambda_7 \Delta A(t)/A(t-1) \\ &+ \lambda_8 No\,D(t-1) + \lambda_9 D(t-1)/B(t-1) + \omega(t). \end{aligned} \qquad (5)$$

$$\begin{aligned} Y(t)/B(t) &= \alpha_0 + \alpha_1 \ln BM(t) + \alpha_2 \ln MV(t) + \alpha_3 Neg\,Y(t-1) + \alpha_4 Y(t-1)/B(t-1) \\ &+ \alpha_5\left[-AC(t-1)/B(t-1)\right] + \alpha_6\left[+AC(t-1)/B(t-1)\right] + \alpha_7 \Delta A(t)/A(t-1) \\ &+ \alpha_8 No\,D(t-1) + \alpha_9 D(t-1)/B(t-1) + \upsilon(t). \end{aligned} \qquad (6)$$

The time index *t* refers throughout to calendar year *t*. $\Delta A(t+1)/A(t)$ is the growth in total assets during the fiscal year that begins in calendar year *t* and $A(t)$ is the book value of assets at the beginning of that fiscal year. $Y(t)/B(t)$ is income before extraordinary items during the fiscal year that begins in calendar year *t* scaled by book equity at the beginning of that fiscal year. $\ln BM(t)$ is the log of the book to market ratio at the beginning of the fiscal year that begins in calendar year *t*. $\ln MV(t)$ is the log of the market value of equity at the beginning of the fiscal year that begins in calendar year *t*. $+AC(t)/B(t)$ is accruals in the fiscal year that begins in calendar year *t* relative to book equity at the beginning of that fiscal year for all firms with positive accruals and zero otherwise. $-AC(t)/B(t)$ is similarly defined. $D(t)/B(t)$ is the dividend yield over the fiscal year that begins in calendar year *t*. $Neg\,Y(t)$ is 1 if the firm experiences negative earnings in the fiscal year that begins in calendar year *t* and 0 otherwise. $No\,D(t)$ is 1 if the firm does not pay a dividend during the fiscal year that begins in calendar year *t* and 0 otherwise.

The first-stage regressions set out in (5) and (6) use the same set of explanatory variables as FF except that all variables are measured at the firm level. We also undertake first-stage regressions in which we add the percentage change in shares outstanding to the set of explanatory variables. We measure the change in shares outstanding as $\ln\left(n(t)/n(t-1)\right)$, where $n(t)$ is the split-adjusted shares outstanding at the beginning of the fiscal year beginning in calendar year $t$. FF choose to measure investment as growth in assets rather than growth in book equity. These two measures of investment will be highly correlated provided changes in book debt are small relative to changes in book equity. Our empirical work examines both potential measures of investment.

It might appear that the first-stage regressions are in fact identical whether the variables are measured at the firm level or the per-share level. But this is not the case. Consider the regression used to obtain estimates of expected profitability, regression (6). The lagged asset growth variable on the right-hand-side depends crucially on whether one considers the growth in assets or the growth in assets per share. The lagged growth in assets per share of $\frac{A(t)/n(t)-A(t-1)/n(t-1)}{A(t-1)/n(t-1)}$ will differ from $\Delta A(t)/A(t-1)$ whenever shares are either issued or repurchased during the fiscal year. Thus when estimated at a per-share level, regression (6) provides an alternate predictor of profitability to that produced when estimated at the firm level. Lagged asset growth is also an explanatory variable in the regression used to obtain estimates of expected investment, regression (5). The real problem with using per-shares measures occurs with the dependent variable in regression (5). The correctly specified valuation relation in (3) links discount rates to firm level expected asset growth, expected profitability and *BM*. A regression-based prediction of firm level expected asset growth requires that the dependent variable be firm level asset growth and not per-share level growth.

We use the time-series average of the coefficients estimated from the monthly first-stage regressions in conjunction with the explanatory variables of the first-stage regressions to form estimates of expected future investment and expected future profitability. We use these estimates in second-stage regressions in which we directly examine the following linearization suggested by the firm-level valuation formula:

$$r(i,t+1) = \gamma_0 + \gamma_1 \ln BM(t) + \gamma_2 \ln MV(t+1) + \gamma_3 \hat{E}\left(\Delta A(t+1)/A(t)\right) + \gamma_4 \hat{E}\left(Y(t)/B(t)\right) + \varepsilon(i,t+1), \quad (7)$$

where $r(i,t+1)$ denotes the return on a stock in the $i^{th}$ of the 12 months subsequent to the June of calendar year $t+1$. The accounting numbers that underlie the explanatory variables of the first-stage regressions used to determine expected investment during the fiscal year beginning prior to calendar year $t+1$, $\hat{E}\left(\Delta A(t+1)/A(t)\right)$, and expected profitability during the fiscal year beginning prior to calendar year $t+1$, $\hat{E}\left(Y(t)/B(t)\right)$, are all numbers for fiscal years ending in calendar year $t$. The explanatory variables $BM(t)$ and $MV(t)$ have slightly different meanings in the second-stage regression than they have in the first-stage regressions. In (7), $BM(t)$ is the ratio of the book value of equity at the beginning of the fiscal year that began during calendar year $t$ to the market value of equity at the end of calendar year $t$. And in (7), $MV(t+1)$ refers to the market value of equity at the end of June of calendar year $t+1$.

Our data are drawn from the CRSP and Compustat annual databases. The sample period is 1963 through 2009. Following FF, we exclude financial firms and very small firms with total assets of less than 25 million dollars or book equity of less than 12.5 million dollars. Firms are excluded from regressions used to predict investment and profitability and to explain cross-sectional differences in returns when the firm is missing any of the accounting and market information necessary to calculate the explanatory variables. We winsorize values of regression explanatory variables outside the 0.5% to 99.5% range, and for one-sided variables, either below the 0.5% value or above than 0.95% value as appropriate.[4] Our final sample includes 85,680 firm-years.

[Table 1 about here]

In Table 1 we present distributional data on selected firm level variables used in the first-stage regressions, in particular the inter-quartile break points and the spread between the $10^{th}$ and $90^{th}$ percentiles. We use this spread to demonstrate the importance of particular variables in the determination of the expected investment and expected profitability estimates provided by the first-stage regressions. The first two rows of Table 1 present the distributions of the dependent variables of the first-stage regressions and the remaining rows present the distributions of the independent variables used in the first-stage regressions. Rows three and four present the natural logs of *BM* and of the market value of equity at fiscal year ends. The final row presents distributional data on $\ln\left(n(t+1)/n(t)\right)$.

## 4. Firm level analysis

Our goal is to use a firm level analysis to investigate the correctly-specified valuation relation in (3). We first estimate expected investment and profitability at the firm level and then investigate the relation between stock returns, *BM* and firm level expected investment and profitability.

### *4.1. Firm level first-stage regressions*

Results of firm level first-stage regressions are presented in Table 2. Panels A and B report prediction models for investment and profitability in the coming fiscal year. Four variants of the prediction models are investigated. The four variants differ in the sample period examined and the explanatory variables used. The first variant is reported in the top row of panels A and B and is estimated over the 1963–2009 period and uses the same nine

[4] Fama and French (2006) trim firms with explanatory variable values outside the 0.5% to 99.5% range in regressions used to estimate expected profitability and expected investment and winsorize extreme explanatory variables values in regressions used to explain differences in average returns. Our reported results are materially unchanged by trimming in the investment and profitability predictor regressions and winsorizing in the stock returns egressions.

explanatory variables as the per-share first-stage regression of Table 2 of FF except that our measures are at the firm level.

The second variant of the first-stage regressions (reported in the second row) considers the 1963–2004 pre-global financial crises period examined in FF. The variant in the third row considers the full period and includes the percentage increase in shares outstanding as an additional explanatory variable. The variant reported in the fourth row differs from the third variant in that it uses the percentage increase in book equity as the measure of investment rather than the percentage increase in assets. In panel A the dependent variable is investment. In panel B the dependent variable is profitability.

[Table 2 about here]

Results for the first-stage firm level regressions have many similarities to those reported by FF at the per-share level. There is a significant negative relation between *BM* and expected investment, and also between *BM* and expected profitability. These findings are consistent with the view of low-*BM* firms as growth and glamour stocks (see, for example, Fama and French 1995). *BM* has the largest effect on future investment of all nine explanatory variables. Expected investment as proxied by percentage growth in book assets of a firm at the $10^{th}$ percentile of *BM* is expected to be higher than the asset growth of a firm at the $90^{th}$ percentile by fully 27.5%. The strong relation between *BM* and expected investment suggests that both variables are likely to capture similar economic forces and in our stock return regressions, an improvement in the predictive ability of one of these variables is accompanied by deterioration in the predictive ability of the second variable.

Controlling for the other explanatory variables, the size measure $\ln MV$ is negatively correlated with future investment but is not correlated with future profitability. The economic effect of size on investment is much less than of *BM*, as the difference between the expected investment of stocks at the $90^{th}$ and $10^{th}$ percentiles of size is less than 5%; i.e., less than one fifth the analogous effect associated with differences in *BM*. This small effect of size is in part due to the censoring of tiny firms.

There is a significant positive relation between current profitability and future profitability and between current profitability and future investment. The effect of current profitability on future profitability is the largest of all the variables—a firm at the 90th percentile of current profitability is expected to have future profitability that is higher by 22.3% relative to a firm at the 10th profitability percentile. The analogous effect of current profitability on expected investment is less than 5%. The binary variable *Neg Y* is significantly negatively related to both future profitability and future investment, while the binary variable *No D* has little economic effect on either expected investment or expected profitability. The accruals variables $\left[-AC/B\right]$ and $\left[+AC/B\right]$ are significant predictors of both future investment and future profitability. However, their economic significance is relatively slight. For example, the spread in expected investment between a firm in the 90th percentile and the 10th percentile of positive accruals is only 2%. There is a significant negative relation between current dividends and future investment and a significant positive relation between current dividends and future profitability. The dividend yield variable has a relatively large economic effect on expected investment (a difference of 6.2% between the 10th and 90th yield percentiles), but a smaller effect on expected profitability (a difference of 1.3%). Current investment measured as asset growth is significantly positively related to future investment and significantly negatively related to future profitability. The difference in expected investment between firms at the 10th and the 90th percentile of current asset growth is 5.3%, The spread in expected future profitability associated with that difference in current asset growth is 1.8%.

Turing to the second variant of the first-stage regressions we see that the coefficients estimated over the 1963–2004 period examined in FF are virtually identical to those estimated over the full 2003–2009 period; i.e., the inclusion of the global financial crises does not materially alter the prediction models. The third variant of the first-stage regressions includes the percent change in share outstanding as an explanatory variable, Previous research shows that firms that issue equity tend to increase their investment in the subsequent period (for example, Fama and French (2005); Lyandres et al. (2008)). Loughran and Ritter (1997) document that firms that issue equity suffer a decrease in profitability after the issuance. The first-stage regressions show that after controlling for the other explanatory variables, the percent change in shares outstanding is a significant positive predictor of future investment but not of future profitability. The difference in expected asset growth for firms at the 90th versus 10th percentile of net issuance is 2.7%.

The fourth variant measures investment by the percentage increase in book equity rather than by the percentage increase in assets. The major effect of this change is on the coefficient on current profitability as a predictor of future investment—the coefficient more than doubles, increasing from 0.15 to 0.42. Leverage will mean that the coefficient on current profitability as a predictor of future investment should be higher when that investment is measured by equity growth rather than asset growth. The coefficient on current investment as a predictor of future investment is smaller and less significant when equity growth rather than asset growth is used to measure investment and the coefficient on the binary variable *No D* changes sign from negative to significantly positive. Similarly, both coefficients of accruals change signs when equity growth rather than asset growth is used to measure investment. In contrast to these large changes in the coefficients of the expected investment first-stage regression, the coefficients of the various variables in the expected profitability regression hardly change.

Armed with the first-stage estimates of firm level expected profitability and expected investment we turn now to the link between future stock returns and firm level expected profitability, firm level expected investment, and current *BM*.

### *4.2. Firm level second-stage regressions*

We use the average coefficients from the first-stage regressions in conjunction with the first-stage explanatory variables to obtain estimates of firm level expected profitability and firm level expected investment and estimate the second-stage regression for each month from July 1963 to December 2009. The average coefficients and associated *t*-statistics are presented in Table 3. Just as Table 2 contains four variants of the first-stage regressions, Table 3 reports four sets of results for the second-stage regression. The different results correspond to the four different estimates of expected investment and expected profitability produced by the four variants of the first-stage regressions. The first row of Table 3 reports results based on the first rows of panels A and B of Table 2.

All the coefficients of the firm level second-stage regression are consistent with their expected signs: the coefficients of *BM* and expected profitability are positive and significant and most importantly, the coefficient of expected investment is negative and significant (−1.57; *t*-statistic = −2.17). This result is in sharp contrast with the FF result from an analysis of the flawed relation (4") which produces a positive and insignificant coefficient on expected investment measured at the per-share level.

Our sample period is five years longer than the one used in FF and these five years (2005–2009) include the global financial crises. Thus, it may be that the difference in results between our analysis and the FF results are not related to the differences between per-share and firm level analysis, but to the end years used in the samples. As noted the coefficients of the first-stage regressions hardly change as a result of the exclusion of the GFC period. Results of the second-stage regression confirm that the difference in the sample periods is not driving the difference in results between the FF analysis and our firm level analysis. The coefficients of the second-stage regression are virtually unchanged by the shortening of the sample period and importantly, the coefficient on expected investment remains negative and significant.

A large set of literature shows that stock issuance is followed by low returns.[5] Fama and French (2008a) argue that the apparent failure of the valuation formula in Fama and French (2006) is due to the fact that differences in *BM* across firms conflate changes in expected future cash flows with changes in the discount rate and argue that net issuance can serve as a variable that helps to separate the two effects. Their results show that post 1963 net issuance is negatively correlated with stock returns, even after controlling for *BM* and changes in book equity and in the market value of equity. We add to this literature by examining the effect of including net issuance as an additional explanatory variable in the first-stage regressions designed to predict future investment and profitability.

A comparison of the first and third rows of Table 3 shows that the coefficient on expected investment in the second-stage regression becomes more negative when net issuance is included as a predictor in the first-stage estimation of expected investment and

[5] For example, Loughran and Ritter (1995); Brav et al. (2000); Lyandres et al. (2008); Pontiff and Woodgate (2008); Fama and French (2008b).

expected profitability. The coefficient decreases from $-1.57$ (significant at the 5% level) to $-1.93$ (significant at the 1% level in a two-tailed test). The increase in the predictive power of expected investment is accompanied by a decrease in the predictive power of *BM*. The coefficient of *BM* falls from 0.21 to 0.16 and becomes significant at only the 10% level. This result is consistent with our previous observation that the predictive abilities of *BM* and expected investment are potentially driven by similar economic forces and an improvement in one can be at the expense of the other.

Finally, we note that the Miller-Modigliani valuation formula relates future stock returns to the expected growth in book equity. FF argue that asset growth, rather than equity growth, provides a better predictor of total investment, as it is a less noisy measure of total investment. While we agree with this statement, we believe it is still of interest to estimate the second-stage linearization of the valuation formula using the expected change in book equity rather than the expected change in total assets. The fourth row of Table 3 reports the result when growth in total assets is replaced by growth in book equity in both the first-stage and second-stage regressions.

Changing the measure of investment from asset growth to equity growth increases the coefficient on expected profitability from 1.48 to 2.25 and decreases the coefficient on expected investment from $-1.93$ to $-2.23$ but there is little change in the associated *t*-statistics. The change in the measure of investment leads to a large reduction in the coefficient on *BM*. The coefficient is reduced by almost a half and becomes statistically insignificant.

The global conclusion to draw from Table 3 is that the Miller-Modigliani valuation formula given in (3) is satisfied in the data. To see this express (3) in terms of perpetuity equivalent expected profitability, $E(Y/B)$, and perpetuity equivalent expected investment, $E(\Delta B/B)$: $\frac{V(t)}{B(t)} = \frac{E(Y) - E(\Delta B/B)}{\rho}$. Linearizing and adding size as a control variable gives the second-stage regression reported in Table 3. In all (4) variants of the second-stage regression, the valuation model predictions re expected profitability and expected investment are borne out: Expected profitability is significantly positively related to future stock returns, and expected investment is significantly negatively related to future returns. And also as predicted by the valuation model, *BM* is always positively related to returns, though the relation becomes insignificant once growth in shares outstanding is included as a right-hand-side variable in the first-stage regression-based predictions of future profitability and investment.

*4.3. Firm level second-stage regressions within BM quintiles*

In section 4.2 we estimated a linear approximation of the relation between expected returns and proxies for variables in the valuation relation (3). In this section we allow for both a non-linear effect of *BM* and for the empirical validity of the proxies for expected profitability and expected investment to vary between value and growth firms. To do so we divide the sample into quintiles based on *BM* and estimate within each quintile a variant of the second-stage regression:

$$r_j(i,t) = \theta_0 + \theta_1 \ln MV_j(t) + \theta_2 \hat{E}\left(\Delta A_j(t+1)/A_j(t)\right) + \theta_3 \hat{E}\left(Y_j(t)/B_j(t)\right) + u_j(i,t). \quad (8)$$

Quintile 1 is the set of stock with the smallest values of *BM*. This analysis serves as a robustness test for our results in Table 3. Expected profitability and expected investment are determined by the first-stage regression coefficients in row 1 of panels A and B of Table 2.

[Table 4 about here]

The first two columns of Table 4 present the equal-weighted average and median (in brackets) of expected investment and expected profitability in each of the *BM* quintiles. The first column confirms the strong negative relation between expected investment and *BM*. The average expected investment in the lowest BM quintile is more than twice as large as the average investment in the median quintile (27% expected growth in assets compared with 12%). Further, in the highest *BM* quintile, the average and median expected investment is negative. Additionally, we can report that less than 0.2 per cent of firms in the high-*BM* quintile have expected asset growth larger than the 27% average expected asset growth of firms in the low-BM quintile. Column 2 presents the average and median expected profitability across *BM* quintiles and shows that the negative relation between expected profitability and *BM* reported in panel B of Table 2 is monotonic across the quintiles.

The next four columns present the results of the regression in (8). The coefficient on expected investment is negative in four of the quintiles and significant in three of them. In the one case where it is positive, it is insignificant. Importantly, we find that the coefficient of expected investment is negative and significant among both high and low-*BM* firms. The coefficient of expected profitability is positive in all quintiles and significant in the two lowest *BM* quintiles. This finding is consistent with Novy-Marks (2012) who reports that profitability has a larger effect among low-*BM* firms than among high-*BM* firms. The important point is that the basic predictions of the valuation formula are largely satisfied within BM quintiles. Finally, the coefficient of size is negative in all portfolios, but significant only among value firms.

## 5. Conclusion

Fama and French (2006) observe that the Miller-Modigliani valuation formula implies a set of basic relations between four variables: future stock returns, current *BM*, firm level expected profitability, and firm level expected investment. However, their empirical work failed to find the predicted negative relation between expected investment and stock returns after controlling for the other two variables. This is because their tests examined per-share measures of expected investment and expected profitability and the valuation formula does not necessarily hold in per-share analysis. While investment can be proxied by either asset growth or equity growth, share issuance and repurchase will change the number of shares outstanding and per share growth can differ from firm level growth. In fact, they need not even have the same sign.

Our empirical work supports the Fama-French valuation insight. We find that the empirical predictions of the valuation formula are true in the data, once variables are measured at the firm level. As predicted by the valuation formula, we find a positive relation between *BM* and returns, a positive relation between expected profitability and returns, and crucially a negative relation between expected investment and returns. We further study the robustness of these results and show that the predictions of the valuation formula hold in all *BM* quintiles with but one exception, namely an insignificant positive coefficient on asset growth for firms in quintile four. In summary: Measuring investment at firm level rather than per-share level is the key to empirically understanding the simultaneous relation between expected returns, *BM*, expected profitability, and expected investment.

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Table 1

Quartile break points and the spread between the 10th and 90th percentiles for selected variables

Distributional information for the following variables: $Y(t)/B(t)$ is income before extraordinary items scaled by book equity; $\Delta A(t+1)/A(t)$ is the growth in total assets over a year; $\ln BM(t)$ is the natural log of the book to market ratio at the beginning of year *t*; $\ln MV(t)$ is the natural log of the market value of equity at the beginning of year *t*; $+AC(t)/B(t)$ is year *t* accruals relative to book equity for firms with positive accruals; $-AC(t)/B(t)$ is similarly defined; $D(t)/B(t)$ is the year *t* dividend yield; and $\ln(n(t+1)/n(t))$ is the natural log of the ratio of the split-adjusted shares outstanding at the beginning of year $t+1$ to the split-adjusted shares outstanding at the beginning of year *t*.

| | 25% | Median | 75% | 10% to 90% Spread |
|---|---|---|---|---|
| $Y(t)/B(t)$ | 0.031 | 0.096 | 0.149 | 0.348 |
| $\Delta A(t+1)/A(t)$ | 0.006 | 0.091 | 0.219 | 0.590 |
| $\ln BM(t)$ | −0.971 | -0.399 | 0.129 | 2.117 |
| $\ln MV(t)$ | 3.983 | 5.181 | 6.554 | 4.720 |
| $+AC(t)/B(t)$ | 0.000 | 0.019 | 0.088 | 0.192 |
| $-AC(t)/B(t)$ | −0.032 | 0.000 | 0.000 | 0.116 |
| $D(t)/B(t)$ | 0.000 | 0.006 | 0.038 | 0.063 |
| $\ln(n(t+1)/n(t))$ | 0.000 | 0.006 | 0.032 | 0.183 |

Table 2

Firm level first-stage regression predictors of investment and profitability

Panel A reports the average coefficients and associated *t*-statistics of sets of Fama-Macbeth regressions used to predict investment for the 1963–2009 period:

$$\Delta A(t+1)/A(t) = \lambda_0 + \lambda_1 \ln BM(t) + \lambda_2 \ln MV(t) + \lambda_3 Neg\, Y(t-1) + \lambda_4 Y(t-1)/B(t-1)$$
$$+ \lambda_5 \left[-AC(t-1)/B(t-1)\right] + \lambda_6 \left[+AC(t-1)/B(t-1)\right] + \lambda_7 \Delta A(t)/A(t-1)$$
$$+ \lambda_8 No\, D(t-1) + \lambda_9 D(t-1)/B(t-1) + \omega(t).$$

The dependent variable in the first three rows of panel A is investment during fiscal year $t$, $\Delta A(t+1)/A(t)$, measured as growth in assets from the beginning of fiscal year $t$ to the beginning of fiscal year $t+1$. The explanatory variables in the first row of Panel A are as described in Table 1 and are the same as those in Table 2 of Fama and French (2006). The two variables not described in Table 1 are dummy variables: $Neg\, Y(t)$ which is 1 if the firm experiences negative earnings in year *t*, and $No\, D(t)$ which is 1 if the firm does not pay a dividend that year. The first-stage regression results reported in the second row of Panel A are for the FF period 1963–2004. The regression in third row differs from that in the first in that it adds $\ln\left(n(t)/n(t-1)\right)$, the percentage increase in shares outstanding, as an additional explanatory variable. The fourth row differs from the third in that it uses the percentage increase in book equity as the measure of investment rather than the percentage increase in assets in determining both the dependent variable and the explanatory variable of lagged investment.

The first row of Panel B reports the average coefficients and associated *t*-statistics of sets of Fama-Macbeth regressions used to predict profitability for the 1963–2009 period:

$$Y(t)/B(t) = \alpha_0 + \alpha_1 \ln BM(t) + \alpha_2 \ln MV(t) + \alpha_3 Neg\, Y(t-1) + \alpha_4 Y(t-1)/B(t-1)$$
$$+ \alpha_5 \left[-AC(t-1)/B(t-1)\right] + \alpha_6 \left[+AC(t-1)/B(t-1)\right] + \alpha_7 \Delta A(t)/A(t-1)$$
$$+ \alpha_8 No\, D(t-1) + \alpha_9 D(t-1)/B(t-1) + \upsilon(t).$$

The second row contains the results for the FF period. The third adds the percentage increase in shares outstanding as an additional explanatory variable, and the fourth substitutes growth in book equity for growth in assets as an explanatory variable. * and ** denote that an average coefficient is significant at the 5% and 10% levels respectively.

Panel A: Firm level first-stage regression to predict investment

| $\ln BM(t)$ | $\ln MV(t)$ | $Neg\ Y(t-1)$ | $\frac{Y(t-1)}{B(t-1)}$ | $\frac{-AC(t-1)}{B(t-1)}$ | $\frac{+AC(t-1)}{B(t-1)}$ | $\Delta A(t)/A(t-1)$ $\Delta B(t)/B(t-1)$ | $No\ D(t-1)$ | $\frac{D(t-1)}{B(t-1)}$ | $\ln\left(\frac{n(t)}{n(t-1)}\right)$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1963–2009: Explanatory variables are firm level and otherwise match those in Table 2 of Fama and French (2006) (FF) | | | | | | | | | | |
| –0.13** | –0.01** | –0.07** | 0.13** | 0.07** | –0.10** | 0.09** | –0.00 | –0.99** | | 0.193 |
| 1963–2004: Explanatory variables are firm level and otherwise match those in Table 2 of FF | | | | | | | | | | |
| –0.13** | –0.01** | –0.07** | 0.14** | 0.05* | –0.10** | 0.09** | –0.01* | –1.05** | | 0.202 |
| 1963–2009: Firm level FF explanatory variables plus the % change in shares outstanding | | | | | | | | | | |
| –0.13** | –0.01** | –0.07** | 0.15** | 0.06** | –0.10** | 0.06** | –0.01* | –1.01** | 0.15** | 0.211 |
| 1963–2009: Firm level FF explanatory variables plus the % change in shares outstanding with investment proxied by growth in book equity | | | | | | | | | | |
| –0.15** | –0.01** | –0.05** | 0.42** | –0.09** | 0.03 | 0.02* | 0.02** | –1.11** | 0.15** | 0.213 |

Panel B: Firm level first-stage regression to predict profitability

| $\ln BM(t)$ | $\ln MV(t)$ | $Neg\ Y(t-1)$ | $\frac{Y(t-1)}{B(t-1)}$ | $\frac{-AC(t-1)}{B(t-1)}$ | $\frac{+AC(t-1)}{B(t-1)}$ | $\Delta A(t)/A(t-1)$ $\Delta B(t)/B(t-1)$ | $No\ D(t-1)$ | $\frac{D(t-1)}{B(t-1)}$ | $\ln\left(\frac{n(t)}{n(t-1)}\right)$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1963–2009: Explanatory variables are firm level and otherwise match those in Table 2 of Fama and French (2006) (FF) | | | | | | | | | | |
| –0.05** | 0.00 | –0.03** | 0.64** | –0.10** | –0.03* | –0.03** | –0.02** | 0.20** | | 0.453 |
| 1963–2004: Explanatory variables are firm level and otherwise match those in Table 2 of FF | | | | | | | | | | |
| –0.05** | –0.00 | –0.02* | 0.66** | –0.09** | –0.05** | –0.02** | –0.02** | 0.18** | | 0.449 |
| 1963–2009: Firm level FF explanatory variables plus the % change in shares outstanding | | | | | | | | | | |
| –0.05** | 0.00 | –0.03** | 0.64** | –0.10** | –0.03* | –0.03** | –0.02** | 0.20** | –0.01 | 0.454 |
| 1963–2009: Firm level FF explanatory variables plus the % change in shares outstanding with investment proxied by growth in book equity | | | | | | | | | | |
| –0.05** | 0.00 | –0.03** | 0.63** | –0.11** | –0.04* | –0.01 | –0.02** | 0.20** | –0.03* | 0.453 |

Table 3

The firm level relation between stock returns, *BM*, expected profitability, and expected investment

In the second-stage regression we examine directly the relation suggested by the firm level valuation formula. $r(i,t+1)$ denotes the return on a stock in the $i^{th}$ of the 12 months subsequent to the June of calendar year $t+1$. The coefficients obtained from the first-stage regressions are used in the estimation of expected profitability, $\hat{E}(Y(t)/B(t))$, in the fiscal year beginning in calendar year *t*, and expected investment, $\hat{E}(\Delta A(t+1)/A(t))$, in the fiscal year beginning in calendar year *t*. $BM(t)$ is the book value of equity at the beginning of the fiscal year beginning in calendar year *t* relative to the market value of equity at the end of calendar year *t*. $MV(t+1)$ is the market value of equity at the end of June of calendar year $t+1$. Numbers in parentheses are *t*-statistics.

$$r(i,t+1)=\gamma_0+\gamma_1 \ln BM(t)+\gamma_2 \ln MV(t+1)+\gamma_3 \hat{E}(\Delta A(t+1)/A(t))+\gamma_4 \hat{E}(Y(t)/B(t))+\varepsilon(i,t+1)$$

| *Intercept* | $\ln BM(t)$ | $\ln MV(t)$ | $\hat{E}(Y(t+1)/B(t+1))$ | $\hat{E}(\Delta A(t+2)/A(t+1))$ |
|---|---|---|---|---|
| July 1963–December 2009: Predicted profitability and investment reflect the first-stage regressions in row 1 of panels A and B of Table 2 | | | | |
| 1.67<br>(4.25) | 0.21<br>(2.29) | –0.07<br>(–1.77) | 1.47<br>(2.36) | –1.57<br>–2.12 |
| July 1963–December 2004: Predicted profitability and investment reflect the first-stage regressions in row 2 of panels A and B of Table 2 | | | | |
| 1.72<br>(4.13) | 0.22<br>(2.17) | –0.07<br>(–1.60) | 1.65<br>(2.37) | –1.64<br>(–2.09) |
| July 1963–December 2009: Predicted profitability and investment reflect the first-stage regressions in row 3 of panels A and B of Table 2 that include the % change in shares outstanding as an explanatory variable | | | | |
| 1.73<br>(4.41) | 0.16<br>(1.80) | –0.08<br>(–1.93) | 1.48<br>(2.39) | –1.93<br>(–2.79) |
| July 1963–December 2009: Predicted profitability and investment reflect the first-stage regressions in row 4 of panels A and B of Table 2 that not only include the % change in shares outstanding as an explanatory variable but also measure investment as growth in book equity rather than asset growth | | | | |
| 1.71<br>(4.70) | 0.09<br>(1.11) | –0.09<br>(–2.44) | 2.25<br>(2.52) | –2.23<br>(–2.52) |

Table 4

Firm level relation between stock returns, expected profitability, expected investment and size within *BM* quintiles

For each *BM* quintile we estimate the following regression:

$$r_j(i,t) = \theta_0 + \theta_1 \ln MV_j(t) + \theta_2 \hat{E}\left(\Delta A_j(t+1)/A_j(t)\right) + \theta_3 \hat{E}\left(Y_j(t)/B_j(t)\right) + u_j(i,t)$$

$r_j(i,t)$ denotes the return on stock *j* in the $i^{th}$ of the 12 months subsequent to the June of calendar year *t*. Expected profitability for the fiscal year beginning in calendar year *t*, $\hat{E}\left(Y(t)/B(t)\right)$, and expected investment in the fiscal year beginning in calendar year *t*, $\hat{E}\left(\Delta A(t+1)/A(t)\right)$, are determined by the coefficients reported in row 1 of panels A and B of Table 2. The sample period is July 1963 to December 2009. For the 12 monthly cross-sectional regressions run from July onwards for each calendar year *t*, stocks are allocated to *BM* quintiles formed on the basis of $BM(t-1)$ calculated as the natural log of book value of equity at the end of the fiscal year that ended during calendar year *t* – 1 relative to the market value of equity at the end of calendar year *t* – 1. The monthly cross-sectional regression is then run for each quintile with quintile 1 being the set of stock with the smallest values of *BM*. Column 2 and Column 3 present the average and median (in brackets) of expected investment and expected profitability in each of the *BM* quintiles. Columns 4 to 7 present the monthly averages of the intercepts and the coefficients of size, expected investment and expected profitability in the above series of cross-sectional regressions for each quintile. Numbers in parentheses are *t*-statistics.

| | Average investment and profitability within *BM* quintiles | | Average coefficient estimates for regressions performed using stocks in *BM* quintiles | | | |
|---|---|---|---|---|---|---|
| *BM* quintile | $\overline{\hat{E}\left(\Delta A_j(t+1)/A_j(t)\right)}$ | $\overline{\hat{E}\left(Y_j(t)/B_j(t)\right)}$ | *Intercept* | $\ln MV(t)$ | $\hat{E}\left(\Delta A_j(t+1)/A_j(t)\right)$ | $\hat{E}\left(Y_j(t)/B_j(t)\right)$ |
| 1 low *BM* | 0.27 (0.26) | 0.13 (0.18) | 1.15 (2.248) | −0.01 (−0.282) | −2.43 (−3.743) | 1.87 (3.342) |
| 2 | 0.18 (0.17) | 0.09 (0.13) | 1.58 (3.435) | −0.07 (−1.471) | −1.98 (−2.156) | 1.58 (3.436) |
| 3 | 0.12 (0.12) | 0.07 (0.10) | 1.51 (3.584) | −0.05 (−1.198) | −0.83 (−0.734) | 1.10 (1.214) |
| 4 | 0.07 (0.07) | 0.03 (0.07) | 1.58 (4.04) | −0.06 (−1.417) | 0.410 (0.329) | 0.76 (0.671) |
| 5 high *BM* | −0.01 (−0.00) | −0.03 (0.01) | 2.20 (5.824) | −0.16 (−3.237) | −2.23 (−2.296) | 0.83 (0.839) |