§57-65

Week #05 Mathematics (8,9) Unit #03

Logarithm of a Real Number

If $a^x = y \Rightarrow \log_a y = x$, $a>0$, $y>0$, $a \neq 1$

$\Rightarrow a^x = y$

$\Rightarrow \log_a y = x \Leftrightarrow a^x = y$ (Equivalent equations)

Logarithm form ↓ Exponential form ↓

Note: $a^0 = 1 \Rightarrow \log_a 1 = 0$ and $a^1 = a \Rightarrow \log_a a = 1$

Characteristic and Mantissa of log of a number:

For example: $10^3 = 1000 \Leftrightarrow \log 1000 = 3$

$10^2 = 100 \Leftrightarrow \log 100 = 2$

$10^1 = 10 \Leftrightarrow \log 10 = 1$

$10^{-3} = 0.001 \Leftrightarrow \log 0.001 = -3$

Logarithm of any number consists of two parts

- an integral part ≥ 1 = +ve and < 1 = -ve characteristic
- a decimal part always +ve mantissa

Example: > 1 (ch)

$9.96 = 9.96 \times 10^0 = 1$

$1662.4 = 1.6624 \times 10^3 = 3$

< 1

example (ch)

$0.02 = 2 \times 10^{-2} = -2 = \bar{2}$

$0.00345 = 3.45 \times 10^{-3} = -3 = \bar{3}$

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Pg # 66-69

Week # 05 Unit # 03 (Sir Akhlaq)

Common and Natural logarithm

→ decadic logarithm, $\log x \Rightarrow \log_{10} x$

→ 1614, $\log_e x \Rightarrow \ln x$

Laws of logarithm:

(i) $\log_a(mn) = \log_a m + \log_a n$

Proof: Let $\log_a m = x$ and $\log_a n = y$

$\Rightarrow a^x = m$ and $a^y = n$ (writing in exponential form)

$\therefore a^x \times a^y = mn$

i.e $a^{x+y} = mn$

or $\log_a(mn) = x + y$

$\Rightarrow \log_a(mn) = \log_a m + \log_a n$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$ home work

(iv) $\log_a n = \log_b n \times \log_a b$ or $\frac{\log_b n}{\log_b a}$

Note: $\log_a a = 1$

Proof:

Let $\log_b n = x \Rightarrow b^x = n$

Taking log w.r.t base a, we have

$\Rightarrow \log_a n = \log_a b^x = x \log_a b = \log_b n \log_a b$

$\Rightarrow \log_a n = \log_b n \log_a b$, replace $n = a$

$\Rightarrow \log_b a \times \log_a b = \log_a a = 1$ or $\log_a b = \frac{1}{\log_b a}$

$\therefore \log_a n = \frac{\log_b n}{\log_b a}$

(iii) $\log_a m^n = n \log_a n$ ⇒ home work

50

Pg #70

Ex #3.3

Q1: (iii) $\log \frac{21 \times 5}{8}$

Sol: $\log 21 + \log 5 - \log 8$ { (i), (ii), (iv) and (vi) home work }

(v) $\log \frac{(22)^{1/3}}{5^3}$

Sol: $\log (22)^{1/3} - \log 5^3 \Rightarrow \frac{1}{3}\log 22 - 3\log 5$ Ans.

Q.2:

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$

Sol: $\log x - \log x^2 + \log (x+1)^3 - \log(x^2-1)$

$= \log \frac{x(x+1)^3}{x^2(x^2-1)} = \log \frac{(x+1)^3}{x(x+1)(x-1)}$

$= \log \frac{(x+1)^2}{x(x-1)}$ Ans.

Q.3: home work

Q.4: (ii) $\log_5 3 \times \log_3 25$ { (i) home work }

Sol: $\frac{\log_3 25}{\log_3 5} = \frac{\log 5^2}{\log 5} = 2$ Ans.

Q.5: $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$

(iii) $\log \sqrt{3\frac{1}{3}} = \log \left(\frac{10}{3}\right)^{1/2}$

$= \frac{1}{2}\log \left(\frac{10}{3}\right) \Rightarrow \frac{1}{2}\log\left(\frac{2 \times 5}{3}\right)$

$= \frac{1}{2}[\log 2 + \log 5 - \log 3] = \frac{1}{2}[0.3010 + 0.6990 - 0.3010]$

$= \frac{1}{2}(0.5229) = 0.2615$ Ans. { (ii), (i), (iv) and (v) home work }

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Week #05 Mathematics (9.1) Unit #03

Pg # 65 (Sir AKHtar)

$= 5521 + 9 \approx 5530$

$\therefore$ Ch $\left(\frac{1}{2}\right)$

$= 0.05530$ Ans.

(i) {Home Work}

Q.4 (ii) $\log_a 6 = 0.5$

Sol $\Rightarrow a^{0.5} = 6$ (Writing in the exponential form)

$\Rightarrow a^{5/10} = 6 \Rightarrow a^{1/2} = 6 \Rightarrow a = 36$ (Squaring)

$\Rightarrow \boxed{a = 36}$ ✓

(iii) $10^{\frac{P}{2}} = 46$

Sol $\Rightarrow \log 10^{\frac{P}{2}} = \log 46$ (Taking log) {(i) and (ii) Home work}

$\Rightarrow \frac{P}{2} \log 10 = 1.6628$ (using log table)

$\boxed{P = 1.6021}$ $(\log 10 = 1)$

Q.5: (ii) $\log 512$ To the base $2\sqrt{2}$

Sol Let $x = \log_{2\sqrt{2}}(512)$

$(2\sqrt{2})^x = 512$ (Writing in exponential form)

(i) Home work

$(2 \times 2^{\frac{1}{2}})^x = 512$

$(2^{\frac{3}{2}})^x = 512 \Rightarrow 2^{\frac{3x}{2}} = 2^9 \Rightarrow \frac{3x}{2} = 9$ ✓

$\Rightarrow \boxed{x = 6}$ ✓

Q.6: (iii) $\log_{64} 8 = \left(\frac{x}{2}\right)$

Sol $(64)^{\frac{x}{2}} = 8$ (Writing in exponential form)

$(8^2)^{\frac{x}{2}} = 8 \Rightarrow 8^x = 8 \Rightarrow \boxed{x = 1}$ ✓

Home work # (i), (ii), (iv) and (v)

by TSU in Grammar School G-9/3, Islamabad

Week # 05 Mathematics (8,9) Analogy

∴ Pg # 64-65 (Six Answers)

Ex 3.2

Q1: (i) 232.92

Sol log 232.92

(i) integral part has three (3) digits.

∴ ch = 3-1 = 2

(ii) To find mantissa, we round off 232.92 to 232.9

Log Table										Mean Differences			
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
23				3655															17

⇒ log 232.92 = 3655 + 17 = 2.3672

(iii) 0.000532

Sol ∴ ch = 4 {(ii) and (iv) have work}

Mantissa = 5051

⇒ log(0.000532) = 4.5051 Ans.

Q.2: (i) log 3.109, (ii) log 31.09, (iii) log 0.0031091, (iv) log 3109

If log 3.109 = 0.4926

⇒ log 3.109 = 0.4926 {(ii), (iii) and (iv) have work}

Q3: (i) 1.7427

Sol Let the number = x

log x = 1.7427

Now ch = 1

Anti-log Table

										Mean Differences			
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
74																			6

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Week # 05 Mathematics (8,9) Unit # 03

(Sir Akhtar)

Review Exercise 3

Pg # 72-73

Q1: (i) If $a^x = n$, Then ....

(a) $a = \log_x n$ (b) $x = \log_n a$ (c) $x = \log_a n$ (d) $a = \log_n x$

Sol: $\log_a n = x$ OR $x = \log_a n$.

(ii) ——— (x) home work

Q2:

(iv) If $x = \log y$, then y is called the Antilog of x.

(i), (ii), (iii) and (v) home work.

Q3: (iv) $\log_{64} x = -\frac{2}{3}$

Sol:

$x = (64)^{-\frac{2}{3}}$ (Writing into exponential form)

$= (4^3)^{-\frac{2}{3}}$

$x = 4^{-2} \Rightarrow x = \frac{1}{16}$ Ans.

(i), (ii) and (iii) home work.

Q4: (iv)

$\log(x) = \bar{1}.6238$

Sol:

Anti-log $(\log x)$ = Anti-log $(\bar{1}.6238)$ {Taking Anti-log}

$x = 0.4206$

{ $\because ch = \bar{1}$

Mantissa = 4198 + 8

= 4206

Pg #72

Unit #03 Mathematics (B.S.) X-Sir AKHTAR

Anti-log(log x) = Anti. log(ī.2834) (Taking Anti-log)

$x = .6229$ ✓

(viii) $\frac{(438)^3\sqrt{0.056}}{(388)^4}$

Sol: Let $x = \frac{(438)^3\sqrt{0.056}}{(388)^4}$

$x = \frac{(438)^3 \times (0.056)^{1/2}}{(388)^4}$

$\log x = \log\left(\frac{(438)^3 \times (0.056)^{1/2}}{(388)^4}\right)$ (Taking log on b/s.)

$\log x = 3\log 438 + \frac{1}{2}\log(.056) - 4\log 388$

$= 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$

$= 7.9245 + \bar{1}.3741 - 10.3552$

$= 7.9245 - 1 + .3741 - 10.3552$

$= (-3).0566$ ✓ $= -3.0566$ $(-1+1)$

$\log x = \bar{4}.9434$ ✓

Anti-log(log x) = Anti-log($\bar{4}$.9434) (Taking Anti-log)

$x = 0.0008778 \Rightarrow \boxed{x = 0.0008778}$

Home Work # (i), (ii), (iv), (v) & (vii)

Q.2: C.P

$C = PV^n$

Putting values of P, V, n.

$C = 80(3.1)^{5/4}$

(Sir Akhtar)

Unit # 03 Mathematics (IX) Week # 05

Ex 3.4 Pg # 72

$\Rightarrow \log C = \log 80 + \frac{5}{4}\log 3.1$ (Taking log)

$= 1.9031 + \frac{5}{4}(0.4970) = 1.9031 + 0.6143$

$\log C = 2.5174$ ✓

Anti-log (log C) = Anti-log (2.5174) (Taking Anti-log)

$\Rightarrow$ $C = 3292$ ✓

Q3, Q4 are home work

Q.5: Sol. $V = \frac{1}{3}\pi r^2 h$ ✓

Putting values of h, r, π.

$V = \frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times 4.2$ ✓

$\log V = \log\left(\frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times 4.2\right)$ (Taking log on b/s.)

$= \log 22 + 2\log 2.5 + \log 4.2 - \log 3 - \log 7$

$= 1.3424 + 2(0.3979) + 0.6232 - 0.4771 - 0.8451$

$= 1.3424 + 0.7958 + .6232 - 0.4771 - 0.8451$

$\Rightarrow \log V = 1.4392$ ✓

Anti-log (log V) = Anti-log 1.4392 (Taking Anti-log)

$V = 27.49$ ✓

55

Pg# 73-74 (Sir Akhtar)

Week # 05 Mathematics (e.g) Unit # 03

Q.5: If $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$

Then find value of (ii) $\log\left(\frac{16}{15}\right)$

Sol:

$$= \log\frac{2^4}{3\times 5} = 4\log 2 - \log 3 - \log 5$$

$$= 4(0.3010) - 0.4771 - 0.6990$$

$$= 1.2040 - 0.4771 - 0.6990 = \boxed{0.0279}$$ Ans.

(i) and (iii) home work.

Q6: (iii) $\dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Sol:

Let $x = \dfrac{(8.97)^3 \times (3.95)^2}{(15.37)^{1/3}}$

$\Rightarrow \log x = 3\log(8.97) + 2\log(3.95) - \frac{1}{3}\log(15.37)$ { Taking log

$= 2.8584 + 1.1932 - 3956$

$\Rightarrow \log(x) = 3.656$

$\Rightarrow$ Anti-log(log x) = Anti log (3.656) (Taking Anti-log)

$\boxed{x = 4529}$ (i) and (iii) home work.

Test # Ex 3.3 and 3.4 on Monday (20-4-2020)

Thanks!!!