35 (10) (Sir Akhtar)

MIIM Grammar School G-11/3, Islamabad.

03-04-2020 Mathematics (8, 9) Unit # 02

Pg # 47-49

Complex Number:

$$x^2+1=0 \Rightarrow x^2=(-1)$$

$\Rightarrow \sqrt{x^2} = \sqrt{-1}$ → Imaginary unit

$= x = \pm\sqrt{-1} \Rightarrow \sqrt{-1}, -\sqrt{-1}$ etc

here $\sqrt{-1} = i \Rightarrow i, -i \Rightarrow S.S = \{(i, -i)\}$

↳ Swiss Mathematician Leonard Euler (1707-1783)

$\sqrt{-1}, \sqrt{-5}$ etc. ⇒ Pure imaginary numbers

e.g: $i^{10} = (i^2)^5 = (-1)^5 = -1$.

Def:

$$z = a + bi, \quad i = \sqrt{-1}$$

Set:

$$C = \{z \mid z = a+bi, \text{ where } a, b \in R \text{ and } i = \sqrt{-1}\}$$

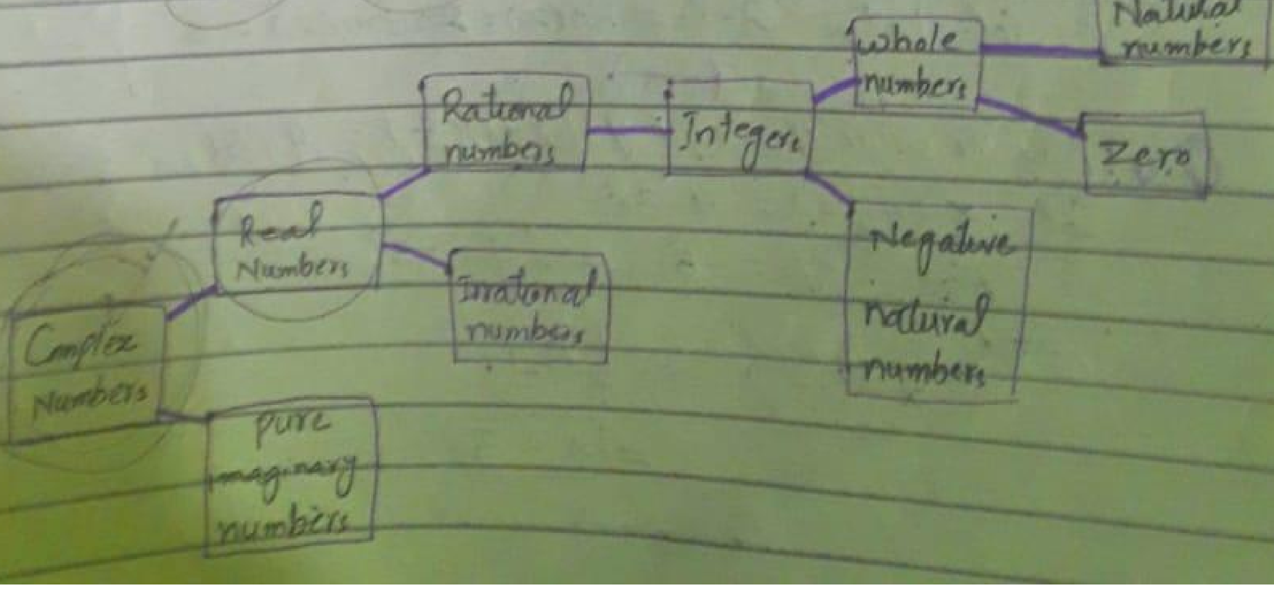
$$a = Re(z) \text{ and } b = Im(z)$$

Note: (i) $a \in R$ is a real number.

$a + 0.i$, $b = 0$

(ii) If $a = 0$, then $a + bi \Rightarrow bi$ → Purely imaginary number.

(iii) If $a = 0$ and $b = 0 \Rightarrow$ complex number: $0 + 0.i$



Pg # 49-52

Unit # 02

Week # 04 Ex. 2.6:

Q1: (i) $\sqrt{-3}\sqrt{-3} = 3$ False

$= \sqrt{3}i \times \sqrt{3}i$

$= \sqrt{3\times 3}\, i\cdot i = 3\times i^2 \quad \because i^2 = -1$

$= 3(-1)$

$= -3$

(vii) Product of a complex number and its conjugate is always a non-negative real number. True

Example:

$(2-3i)(2+3i)$

$= 2(2+3i) - 3i(2+3i) = 4+6i-6i-9i^2$

$= 4-9i^2 = 4+9 = 13 \quad (\because i^2 = -1)$

It is a non-negative real number.

(ii), (iii), (iv), (v), (vi) → Home Work

Q2: (i) $(2+3i)+(7-2i)$ Standard form $(a+bi)$

Solution: $= 2+3i+7-2i$

$= 2+7+3i-2i = 9+i$

(ii) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

Solution: $= 2(-1)+6i^2\cdot i+3(i^2)^8-6(i^2)^9\cdot i+4(i^2)^{12}\cdot i$

$= -2+6(-1)i+3(-1)^8-6(-1)^9 i+4(-1)^{12}\cdot i \quad \because i^2=-1$

$= -2-6i+3+6i+4i$

$= -2+3-6i+6i+4i$

$= 1+4i$

{ (ii) and (iii) → Home Work }

Q3: (ii) $(2-\sqrt{-4})(3-\sqrt{-4})$

$\sqrt{-4} = \sqrt{-1}\sqrt{4}$

$= \sqrt{4}\,i = 2i$

Sol: $(2-2i)(3-2i)$

$= 2(3-2i)-2i(3-2i)$

$= 6-4i-6i+4i^2 = 6-10i-4$

$= 2-10i$

MIIM Grammar School G-11/3, Islamabad

[illegible] (6-4-20) Mathematics (8-9) (Unit # 02)

Pg # 49-52

Basic operations on Complex Numbers:

① Addition Let

$z_1 = a+ib,\ z_2 = c+id,\ a,b,c,d \in R$

then sum $z_1+z_2 = (a+bi)+(c+di)$

$= (a+c)+(b+d)i$

② Multiplication: Let

$z_1 = a+ib,\ z_2 = c+id,\ a,b,c,d \in R$

then Product $z_1z_2 = (a+ib)(c+id)$

$= (ac-bd)+(ad+bc)i$

③ Subtraction Let

$z_1 = a+ib,\ z_2 = c+id,\ a,b,c,d \in R$

then subtraction $z_1-z_2 = (a-c)+(b-d)i$

④ Division Let

$z_1 = a+ib,\ z_2 = c+id, \quad a,b,c,d \in R$

then division

$$\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{a+ib}{c+id} \times \frac{c-id}{c-id}$$

$$= \frac{ac+bci-adi-bdi^2}{c^2-(di)^2}$$

$$= \frac{ac+bci-adi+bd}{c^2+d^2} \quad (\because i^2=-1)$$

$$= \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$$= \boxed{\frac{ac+bd}{c^2+d^2} + \left(\frac{bc-ad}{c^2+d^2}\right)i}$$

Pg # 49-52

Ex 2.6

$$= \frac{2+2i-3i-3i^2}{(4+9)(1+1)} = \frac{2-i-3(-1)}{(13)(2)}$$

$$= \frac{2+3-i}{26} = \frac{5-i}{26} = \frac{5}{26} - \frac{1}{26}i \text{ Ans.}$$

Q.4: (v) $\left(\frac{1+i}{1-i}\right)^2$

Sol: $\left[\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right]^2$

$$= \left[\frac{(1+i)^2}{1-i^2}\right]^2 = \left[\frac{1+2i+i^2}{1-(-1)}\right]^2 = \left[\frac{1+2i-1}{1+1}\right]^2$$

$$= \left[\frac{2i}{2}\right]^2 = i^2 = -1 \text{ Ans.}$$

Home Work # (i), (ii), (iv)

Q5: Calculate (a) $\bar{z}$ (b) $z+\bar{z}$ (c) $z-\bar{z}$ (d) $z\bar{z}$

(iv) $z = \frac{4-3i}{2+4i}$

Sol: $z = \frac{4-3i}{2+4i} \times \frac{(2-4i)}{2-4i}$

$$= \frac{(4-3i)(2-4i)}{(2)^2-(4i)^2}$$

$$= \frac{4(2-4i)-3i(2-4i)}{4-16(-1)}$$

$$= \frac{8-22i+12(-1)}{4+16} = -4-22i$$

Pg # 52-53

Sol $z = 2+3i \Rightarrow \bar{z} = 2-3i$

$w = 5-4i \Rightarrow \bar{w} = 5+4i$

Now $z+w = 2+3i + 5-4i$

$= 2+5+3i-4i = 7-i$

$\overline{z+w} = \overline{7-i} = 7+i$ —— (i)

$\bar{z}+\bar{w} = (2-3i)+(5+4i)$

$= 7+i$ —— (ii)

$\Rightarrow \overline{z+w} = \bar{z}+\bar{w}$ from (i) and (ii)

(iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$ where $w \neq 0$

Sol $\frac{z}{w} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i}$

$= \frac{(2+3i)(5+4i)}{(5)^2-(4i)^2} = \frac{2(5+4i)+3i(5+4i)}{25-16i^2}$

$= \frac{10+8i+15i+12i^2}{41}$

$= \frac{10+23i-12}{41} = \frac{-2+23i}{41}$

$= \frac{-2}{41} + \frac{23}{41}i$

$\overline{\left(\frac{z}{w}\right)} = \frac{-2}{41} - \frac{23}{41}i$ —— (i)

Now

$\frac{\bar{z}}{\bar{w}} = \frac{2-3i}{5+4i} = \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i}$

$= \frac{2(5-4i)-3i(5-4i)}{(5)^2-(4i)^2}$

144 MIIM Grammar school G-11/3, Islamabad

Week#04 => Mathematics (8,9) Unit#02

Pg#53-54

Review Exercise 2

Q1: (M.C.Qs)

(i) $(27x^{-1})^{-2/3}$ ......

Sol: $= \sqrt[3]{(27x^{-1})^{-2}} = \sqrt[3]{\frac{x^2}{(27)^2}} = \sqrt[3]{\frac{x^2}{729}}$

$\sqrt[3]{\frac{x^2}{9^3}} = \frac{\sqrt[3]{x^2}}{9}$ Ans.

$$\begin{array}{r|l} 9 & 729 \\ 9 & 81 \\ 9 & 9 \\ & 1 \end{array}$$

Remaining all parts home work

Q2: True or False

(i) Division is not an associative operation True

(vi) Subtraction is a Commutative operation False

Remaining parts are home work

Q3: home work (i) — (iv)

Q4: $\sqrt{\frac{(216)^{2/3} \times (25)^{1/2}}{(.04)^{-3/2}}} = \sqrt{\frac{(6^3)^{2/3} \times 5^{2 \times 1/2}}{\left(\frac{4}{100}\right)^{-3/2}}}$

$= \sqrt{\frac{6^2 \times 5}{\left(\frac{1}{25}\right)^{3/2}}} = \sqrt{\frac{6^2 \times 5^3}{(5^3)^{3/2}}} = \sqrt{\frac{6^2}{5^2}} = \frac{6}{5}$ tr

Q5: home work

Q6: Simplify $\left(\frac{a^{2l}}{a^{l+m}}\right)\left(\frac{a^{2m}}{a^{m+n}}\right)\left(\frac{a^{2n}}{a^{n+l}}\right)$

Sol: $= \left(a^{2l-l-m}\right)\left(a^{2m-m-n}\right)\left(a^{2n-n-l}\right)$

43 MIIM Grammar School G-11/3, Islamabad

(Sir Akhilar)

Pg # 52-53

Ex 2.6

$$= \frac{10-8i-15i+12i^2}{25-16(-1)}$$

$$= \frac{10-23i-12}{41} = \frac{-2}{41} - \frac{23}{41}i \quad \text{---(ii)}$$

hence $\left(\overline{\frac{z}{w}}\right) = \frac{\bar{z}}{\bar{w}}$ (from i and ii).

Home work # (ii) and (iii),

Q6 (v) $\frac{1}{2}(z+\bar{z})$ is the real part of z.

Sol

$$\frac{1}{2}(z+\bar{z})$$

$$= \frac{1}{2}\left[2+3i+2-3i\right] = \frac{1}{2}[4]$$

$= 2$ (2 is a real part of z)

(vi) is a home work

Q7: (iii) $(3+4i)^2 - 2(x-yi) = x+yi$

Sol $9+24i+16i^2-2x+2yi = x+yi$

$9+24i+16(-1)-2x+2yi = x+yi$

$-7-2x+(24+2y)i = x+yi$

$\Rightarrow -7-2x = x \quad , \quad 24+2y = y$

$-7 = x+2x \quad , \quad 24 = y-2y$

$\boxed{\frac{-7}{3} = x} \quad , \quad \boxed{-24 = y}$

Home work # (i) and (ii)

$= \frac{-4}{20} - \frac{22}{20}i = \frac{-1}{5} - \frac{22}{20}i = \frac{-1}{5} - \frac{11}{10}i$

$\Rightarrow \boxed{z = \frac{-1}{5} - \frac{11}{10}i}$

(a) $\bar{z} = \frac{-1}{5} + \frac{11}{10}i$

(b) $z + \bar{z} = \left(\frac{-1}{5} - \frac{11}{10}i\right) + \left(\frac{-1}{5} + \frac{11}{10}i\right)$

$= \frac{-1}{5} - \frac{1}{5} - \frac{11}{10}i + \frac{11}{10}i$

$= -\frac{2}{5}$

(c) $z - \bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right) - \left(-\frac{1}{5} + \frac{11}{10}i\right)$

$= \left(\frac{-1}{5} + \frac{1}{5} - \frac{11}{10}i - \frac{11}{10}i\right)$

$= \frac{-22}{10}i = -\frac{11}{5}i$

(d) $z\bar{z} = \left(\frac{-1}{5} - \frac{11}{10}i\right)\left(\frac{-1}{5} + \frac{11}{10}i\right)$

$= \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2 = \frac{1}{25} - \frac{121}{100}i^2$

$= \frac{1}{25} + \frac{121}{100} = \frac{4+125}{100} = \frac{125}{100} = \frac{5}{4}$

Home work # (i), (ii), and (iii)

Q6: If $z = 2+3i$ and $\omega = 5-4i$, show that

(i) $\overline{z+\omega} = \bar{z} + \bar{\omega}$

39 IMIIM Grammar School G-11/3, Islamabad

Pg # 49-52

Week #06 Ex 2.6 Unit #02

Q3:(iv) $(2-3i)(\overline{3-2i})$

Sol: $= (2-3i)(3+2i)$ (Taking conjugate)

$= 2(3+2i) - 3i(3+2i)$

$= 6+4i-9i-6i^2$

$= 6+4i-9i-6(-1)$

$= 6+4i-9i+6$

$= 12-5i$

{(i) and (iii) Home Work}

Q4:(iii) $\dfrac{2+3i}{4-i}$

Sol: $\dfrac{2+3i}{4-i} \times \dfrac{4+i}{4+i} = \dfrac{(2+3i)(4+i)}{4^2-i^2}$

$= \dfrac{2(4+i)+3i(4+i)}{16-(-1)} = \dfrac{8+2i+12i+3i^2}{16+1}$

$= \dfrac{8+14i+3(-1)}{17} = \dfrac{8+14i-3}{17}$

$= \dfrac{5+14i}{17} = \dfrac{5}{17} + \dfrac{14}{17}i$

Q4:(vi) $\dfrac{1}{(2+3i)(1-i)}$

Sol: $= \dfrac{1}{2+3i} \times \dfrac{1}{1-i}$

$= \dfrac{1}{2+3i} \times \dfrac{2-3i}{2-3i} \times \dfrac{1}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{2-3i}{(2+3i)(2-3i)} \times \dfrac{1+i}{(1-i)(1+i)}$

$= \dfrac{(2-3i)(1+i)}{(4-9i^2)(1-i^2)} = \dfrac{2(1+i)-3i(1+i)}{(4-9(-1))(1-(-1))}$

Pg # 47 - 49.

Conjugate of a Complex Number:

$$z = a + ib \Rightarrow \bar{z} = a - ib$$

Note (i) $\bar{\bar{z}} = z$

(ii) $z = a = a + 0i \Rightarrow \bar{z} = \overline{a+0i} = a - 0i = a$

Equality: $\forall a, b, c, d \in R$

$\Rightarrow a + bi = c + di \Leftrightarrow a = c$ and $b = d$

Note Properties of real numbers R are also valid for the set of complex numbers

Ex 2.5

Q1: (iv) $(-i)^8 = ((-i)^2)^4 = (-1)^4 = 1$

(vi) $i^{27} = (i^2)^{13} \times i = (-1)^{13} \times i = -i$

Home work # (i), (ii), (iii) and (v)

Q2: (i) $2 + 3i$

Sol Let $z = 2 + 3i \Rightarrow \bar{z} = 2 - 3i$

(iv) $-3 + 4i$

Sol Let $z = -3 + 4i \Rightarrow \bar{z} = -3 - 4i$

Home work # (ii), (iii), (v) and (vi)

Q3: (i) $1 + i$

Sol Let $z = 1 + i \Rightarrow Re(z) = 1$; $Im(z) = 1$

(vi) $2 + 0i$

Sol Let $z = 2 + 0i \Rightarrow Re(z) = 2$; $Im(z) = 0$

Homework (ii), (iii), (iv) and (v).

Q4: Find the value of x and y if $x + iy + 1 = 4 - 3i$

Sol $x + iy + 1 = 4 - 3i$

Here $iy = -3i \Rightarrow \boxed{y = -3}$

and

$x + 1 = 4 \Rightarrow x = 4 - 1$

$\Rightarrow \boxed{x = 3}$

45

(Sir Akhtar)

Pg # 53-54-55-56.

$$= a^{l-m} \times a^{m-n} \times a^{n-l}$$

$$= a^{l-\cancel{m}+\cancel{m}-\cancel{n}+\cancel{n}-l} = a^0 = 1$$

Q7: home Work