MODERN LANGUAGE SCHOOL & COLLEGE

15-6-20 MATHEMATICS 10^{th} Pg#1

Unit # 2: Theory of Quadratic Equations

Ex: 2.1

Q1) Find the discriminant of the following quadratic equations.

i) $2x^2 + 3x - 1 = 0$

$a = 2 \quad b = 3 \quad c = -1$

$$
\begin{aligned}
\text{Disc.} &= b^2 - 4ac \\
&= (3)^2 - 4(2)(-1) \\
&= 9 + 8 \\
&= 17 \quad \text{Ans.}
\end{aligned}
$$

* Do the remaining parts by your self.

Q2. Find the nature of roots and verify the result

i) $x^2 - 23x + 120 = 0$

$a = 1 \quad b = -23 \quad c = 120$

$$
\begin{aligned}
\text{disc.} &= b^2 - 4ac \\
&= (-23)^2 - 4(1)(120) \\
&= 529 - 480 \\
&= 49
\end{aligned}
$$

As the disc. is positive and perfect square so the roots are rational (real) and unequal.

Verification:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-23) \pm \sqrt{49}}{2(1)}$$

$$= \frac{23 \pm 7}{2}$$

$$x = \frac{23 + 7}{2} \quad ; \quad x = \frac{23 - 7}{2}$$

$$\boxed{x = 15} \quad ; \quad \boxed{x = 8}$$

* Solve the remaing parts by your self.

Q3: F

K

sol.

a =

Disc.

OR

3k

3k

3K

Ex: 2.1

Pg # 2

Q2 Find the value of k, the expression $k^2x^2 + 2(k+1)x + 4$ is a perfect square.

Sol:

$$k^2x^2 + 2(k+1)x + 4 = 0$$

$a = k^2$ $\quad b = 2(k+1)$ $\quad c = 4$

$$\text{Disc.} = b^2 - 4ac$$
$$= [2(k+1)]^2 - 4(k^2)(4)$$

$$= 4(k+1)^2 - 16k^2$$
$$= 4(k^2 + 1 + 2k) - 16k^2$$
$$= 4k^2 + 4 + 8k - 16k^2$$
$$0 = -12k^2 + 8k + 4$$
$$0 = -4(3k^2 - 2k - 1)$$
$$0 = 3k^2 - 2k - 1$$

OR

$$3k^2 - 2k - 1 = 0$$
$$3k^2 - 3k + k - 1 = 0$$
$$3k(k-1) + 1(k-1) = 0$$
$$(3k+1)(k-1) = 0$$

$3k + 1 = 0$; $k - 1 = 0$

$\boxed{k = -\frac{1}{3}}$; $\boxed{k = 1}$

Ex: 2.1 Pg# 3

Q4. (i) $(2k-1)x^2 + 3kx + 3 = 0$

Sol $a = (2k-1)$ $b = 3k$ $c = 3$

Disc. $= 0$

$b^2 - 4ac = 0$

$(3k)^2 - 4(2k-1)(3) = 0$

$9k^2 - 12(2k-1) = 0$

$9k^2 - 24k + 12 = 0$

$3(3k^2 - 8k + 4) = 0$

$3k^2 - 8k + 4 = 0$

$3k^2 - 6k - 2k + 4 = 0$

$3k(k-2) - 2(k-2) = 0$

$(k-2)(3k-2) = 0$

$k - 2 = 0$; $3k - 2 = 0$

$\boxed{k = 2}$; $\boxed{3k = 2} \Rightarrow k = \frac{2}{3}$

* Solve part (ii) & (iii) by your self.

Q5. Show that the equation $x^2 + (mx+c)^2 = a^2$ has equal roots if $c^2 = a^2(1+m^2)$

Sol

$x^2 + (mx+c)^2 = a^2$

$x^2 + m^2x^2 + c^2 + 2mcx = a^2$

$(1+m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$

$a = 1 + m^2$ $b = 2mc$ $c = c^2 - a^2$

Disc. $= 0$

$b^2 - 4ac = 0$

$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$

$4m^2c^2 - (4 + 4m^2)(c^2 - a^2) = 0$

$4m^2c^2 - (4c^2 - 4a^2 + 4m^2c^2 - 4m^2a^2) = 0$

$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$0m^2c^2 + 4m^2a^2 + 4a^2 = 4c^2$

Dividing both sides by 4

$m^2a^2 + a^2 = c^2$

Ex 2.1

Pg # 4

$c^2 = m^2 a^2 + a^2$

$c^2 = a^2 (m^2 + 1)$

$c^2 = a^2 (1 + m^2)$

Proved.

* Solve Q6 and Q7 by your self.

Q8. Show that the roots of the following equations are rational.

i) $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$

$a' = a(b-c)$ $\quad b' = b(c-a)$ $\quad c' = c(a-b)$

Disc. $= b'^2 - 4a'c'$

$b'^2 - 4a'c' = [b(c-a)]^2 - 4a(b-c)c(a-b)$

$= b^2(c-a)^2 - 4(ab-ac)(ac-bc)$

$= b^2(c^2+a^2-2ac) - 4(a^2bc - ab^2c - a^2c^2 + abc^2)$

$= b^2c^2 + b^2a^2 - 2ab^2c - 4a^2bc + 4ab^2c + 4a^2c^2 - 4abc^2$

$= a^2b^2 + b^2c^2 + 4a^2c^2 - 4a^2bc + 2ab^2c - 4abc^2$

$= a^2b^2 + b^2c^2 + 4a^2c^2 + 2ab^2c^2 - 4abc^2 - 4a^2bc$

By comparing

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$

$(ab)^2 + (bc)^2 + (-2ac)^2 + 2(ab)(bc) + 2(bc)(-2ac) + 2(ab)(-2ac)$

$= (ab + bc - 2ac)^2$

As the disc. is greater than zero and complete square therefore roots are rational.

* Solve part ii) by your self.

Ex. 2.1

Q1. For all values of k, Prove that the roots of equation Pg# 5

$$x^2 - 2\left(k + \frac{1}{k}\right)x + 4 = 0$$ are real

$k \neq 0$

Sol

$a = 1$, $b = -2\left(k + \frac{1}{k}\right)$, $c = 4$

Disc. $= b^2 - 4ac$

$= 4\left(k + \frac{1}{k}\right)^2 - 4(1)(4)$

$= 4\left(k^2 + \frac{1}{k^2} + 2\right) - 16$

$= 4k^2 + \frac{4}{k^2} + 8 - 16$

$= 4k^2 + \frac{4}{k^2} - 8$

$= 4\left(k^2 + \frac{1}{k^2} - 2\right)$

$= 4\left(k - \frac{1}{k}\right)^2$

Disc. $= \left[2\left(k - \frac{1}{k}\right)\right]^2$

As Disc. is greater than zero and a perfect square so roots are real.

Ex: 2.1 Pg # 6

Q10. Show that the roots of equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are real

Sol $a = (b-c)$ $b = (c-a)$ $c = (a-b)$

Disc. $= b^2 - 4ac$

$= (c-a)^2 - 4(b-c)(a-b)$

$= c^2 + a^2 - 2ac - 4(ab - b^2 - ac + bc)$

$= a^2 + c^2 - 2ac - 4ab + 4b^2 + 4ac - 4bc$

$= a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac$

Comparing with

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = (a+b+c)^2$

$(a)^2 + (-2b)^2 + c^2 + 2(a)(-2b) + 2(-2b)(c) + 2ac$

Disc. $= (a - 2b + c)^2$

As Disc. is greater than zero and a perfect square so roots are real.