MODERN LANGUAGE SCHOOL & COLLEGE

8 June 2020 Mathematics Class 10th

Unit # 1 Quadratic Equations

Pg#05 Quadratic Formula:

Derivation of quadratic formula by using completing square method.

Standard form of quadratic equation is

$$ax^2+bx+c=0$$

Divide each term of equation by a we get

$$\frac{ax^2}{a}+\frac{bx}{a}+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

Shift constant term to R.H.S

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

Add $\left(\frac{b}{2a}\right)^2$ on both sides

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

By using $a^2+2ab+b^2=(a+b)^2$

$$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

(2)

Taking square root on both sides

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$ is known as quadratic formula.

This formula is used to solve quadratic equations.

→ Write the given equation in standard form of quadratic equation

$$ax^2 + bx + c = 0$$

→ Write the value a, b, c from equation

$$5x^2 - 9x - 2 = 0$$

$$a = 5 \qquad b = -9 \qquad c = -2$$

→ Put the values of 'a', 'b' and 'c' in above equation and solve further.

Maths 10th

Ex: 1.2

1. Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

First of all write above equation in standard form of quadratic equation.

$$-x^2 - 7x + 2 = 0$$

$a = -1 \qquad b = -7 \qquad c = 2$

Put these values in quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-1)(2)}}{2(-1)}$$

$$= \frac{7 \pm \sqrt{49 + 8}}{-2}$$

$$= \frac{7 \pm \sqrt{57}}{-2}$$

$$x = \frac{-7 \pm \sqrt{57}}{2}$$

$$S.S = \left\{ \frac{-7 \pm \sqrt{57}}{2} \right\} \quad \text{Ans.}$$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}x^2 + x - 4\sqrt{3} = 0$

$a = \sqrt{3}$ $\quad b = 1$ $\quad c = -4\sqrt{3}$

Put the values in quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2-4(\sqrt{3})(-4\sqrt{3})}}{2\sqrt{3}}$$

$\sqrt{3}\sqrt{3} = (\sqrt{3})^2$
$= 3$

$$= \frac{-1 \pm \sqrt{1+16\times3}}{2\sqrt{3}}$$

$$= \frac{-1 \pm \sqrt{1+48}}{2\sqrt{3}}$$

$$= \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$= \frac{-1 \pm 7}{2\sqrt{3}}$$

$x = \frac{-1+7}{2\sqrt{3}}$, $x = \frac{-1-7}{2\sqrt{3}}$

$x = \frac{6}{2\sqrt{3}}$; $x = \frac{-8}{2\sqrt{3}}$

$x = \frac{3}{\sqrt{3}}$; $x = \frac{-4}{\sqrt{3}}$

$x = \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}}$; $x = -\frac{4}{\sqrt{3}}$

$x = \sqrt{3}$; $x = \frac{-4}{\sqrt{3}}$

S.S = $\{\sqrt{3}, -4/\sqrt{3}\}$

(viii) $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$

Taking L.C.M on L.H.S

$$\frac{2x(x+2)-(x-1)(4-x)}{2x(x-1)} = \frac{7}{3}$$

$$\frac{2x^2+4x-(4x-x^2-4+x)}{2x^2-2x} = \frac{7}{3}$$

$$\frac{2x^2+4x-5x+x^2+4}{2x^2-2x} = \frac{7}{3}$$

$$\frac{3x^2-x+4}{2x^2-2x} = \frac{7}{3}$$

$$3(3x^2-x+4) = 7(2x^2-2x)$$

$$9x^2-3x+12 = 14x^2-14x$$

$$9x^2-14x^2-3x+14x+12=0$$

$$-5x^2+11x+12=0$$

$a=-5 \qquad b=11 \qquad c=12$

Now put the values of a, b and c in quadratic formula and solve by yourself.

(ix) $\frac{a}{x-b} + \frac{b}{x-a} = 2$

$$\frac{a(x-a)+b(x-b)}{(x-a)(x-b)} = 2$$

$$\frac{ax-a^2+bx-b^2}{x^2-bx-ax+ab} = 2$$

$$ax+bx-a^2-b^2 = 2(x^2-bx-ax+ab)$$

$$ax+bx-a^2-b^2 = 2x^2-2bx-2ax+2ab$$

Pg 46

$$2x^2 - 2bx - 2ax + 2ab = ax + bx - a^2 - b^2$$

$$2x^2 - 2bx - bx - 2ax - ax + 2ab + a^2 + b^2 = 0$$

$$2x^2 - 3bx - 3ax + a^2 + b^2 + 2ab = 0$$

$$2x^2 - (3a+3b)x + (a+b)^2 = 0$$

$a = 2 \qquad b = -(3a+3b) \qquad c = (a+b)^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-(3a+3b)) \pm \sqrt{[-(3a+3b)]^2 - 4(2)(a+b)^2}}{2(2)}$$

$$= \frac{(3a+3b) \pm \sqrt{3^2(a+b)^2 - 8(a+b)^2}}{4}$$

$$= \frac{(3a+3b) \pm \sqrt{9(a+b)^2 - 8(a+b)^2}}{4}$$

$$= \frac{(3a+3b) \pm \sqrt{(a+b)^2}}{4}$$

$$x = \frac{3a+3b \pm (a+b)}{4}$$

$$x = \frac{3a+3b+a+b}{4} \quad ; \quad x = \frac{3a+3b-a-b}{4}$$

$$= \frac{4a+4b}{4} \quad ; \quad x = \frac{2a+2b}{4}$$

$$= \frac{4(a+b)}{4} \quad ; \quad x = \frac{2(a+b)}{4} = \frac{a+b}{2}$$

$$x = (a+b)$$

$$S.S = \left\{(a+b), \frac{a+b}{2}\right\}$$

(x) $-(l+m) - lx^2 + (2l+m)x = 0$

$-lx^2 + (2l+m)x - (l+m) = 0$

Here

$a = -l \qquad b = (2l+m) \qquad c = -(l+m)$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(2l+m) \pm \sqrt{(2l+m)^2 - 4(-l)(-(l+m))}}{2(-l)}$$

$$= \frac{-2l-m \pm \sqrt{4l^2+m^2+4lm-4l^2-4lm}}{-2l}$$

$$= \frac{-2l-m \pm \sqrt{m^2}}{-2l}$$

$$= \frac{-2l-m \pm m}{-2l}$$

$$= \frac{-2l-m+m}{-2l} \quad ; \quad x = \frac{-2l-m-m}{-2l}$$

$$= \frac{-2l}{-2l} \quad ; \quad x = \frac{-2l-2m}{-2l}$$

$x = 1 \quad ; \quad x = \frac{-2(l+m)}{-2l}$

$$; \quad x = \frac{l+m}{l}$$

$$S.S = \left\{ 1 \ , \ \frac{l+m}{l} \right\}$$