Roll No. □□□□□□□

- Please check that this question paper contains **4** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **38** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Guess Paper 5 (Unsolved)

Time Allowed: 3 hours *Maximum Marks: 80*

General Instructions:

Same as in Sample Guess Paper 1 (Unsolved).

PART A
SECTION I

All questions are compulsory. In case of internal choices attempt anyone.

1. Let A = {0, 1, 2, 3} and defined a relation R as follows:

R = {(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)}.

Is R reflexive, symmetric and transitive?

Or

If $f : R \to R$ is defined by $f(x) = (x^2 - 3x + 2)$. Find $fof\,(x)$.

2. Find the range of function $f(x) = x^2 + 2x + 3$.

3. Let A = {1, 2, 3, …n} and B = {a, b}. Then find the number of surjection from A into B.

Or

If the Set A contains 5 elements and Set B contains 6 elements, then the number of one to one and onto mappings from A to B is __________.

4. Solve the following matrix equation

$$[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$$

5. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

Or

If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew-symmetric find the value of a and b.

6. If A is a square matrix of order 3 such that $|adj\ A| = 64$, then find $|A|$.

7. Solve: $\int \frac{x-5}{(x-3)^2} e^x dx$

Or

Solve: $\int \sin x \log \cos x \, dx$

8. Find the area of curve $xy = 4$ bounded by the lines $x = 1$, $x = 3$ and x-axis.

9. Solve the differential equation $\sin^2 x \frac{dy}{dx} = 1$

Or

For what value of n the following differential equation $\frac{dy}{dx} = \frac{x^3 - y^n}{x^2 y + xy^2}$ is homogeneous differential equation.

10. If $\vec{a}$ and $\vec{b}$ are unit vectors, find the angle between $\vec{a}$ and $\vec{b}$, given that $\sqrt{3}\,\vec{a} - \vec{b}$ is a unit vector.

11. If $|\vec{a}| = 1$, $|\vec{b}| = 4$, $\vec{a} \cdot \vec{b} = 2$ and $\vec{c} = 2\vec{a} \times \vec{b} - 3\vec{b}$ then find the angle between $\vec{b}$ and $\vec{c}$.

12. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

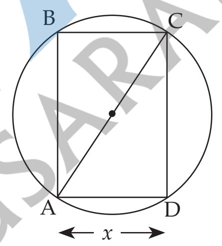
13. If a line makes angles 90°, 60° and θ with x, y and z axes respectively, where θ is acute, then find θ.

14. Find the intercepts cut of by the plane $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$ on three axes.
15. Two dice are rolled, if it is known that sum of numbers on the dice were more than 6, then find the probability of getting a sum 8.
16. Two unbiased dice are rolled. If it is known that sum of number obtained is 8 or greater, what is the probability that one number is 6.

SECTION II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. **Case Study—** A circular land is allotted by the government for residence of government officers. The government officers wanted to convert this land into a park. The officials municipal corporation said that they will plant grass in the rectangular area ABCD. Let the length of this area is x and width is y and diameter of circular land is 20 m.



(*i*) The relation between x and y is
(*a*) $x^2 + y^2 = 100$ (*b*) $x^2 + y^2 = 400$ (*c*) $x^2 + y^2 = 200$ (*d*) $x^2 + y = 300$

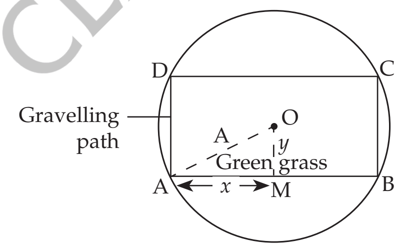
(*ii*) The perimeter of rectangular part ABCD
(*a*) $x + y$ (*b*) $2x + 2y$ (*c*) $x + 2y$ (*d*) $2x + y$

(*iii*) Maximum area of rectangle where grass is to be planted is
(*a*) 100 m² (*b*) 200 m² (*c*) 50 m² (*d*) 500 m²

(*iv*) The length of dimensions of rectangle of maximum area
(*a*) $2\sqrt{50}, 2\sqrt{50}$ (*b*) $2\sqrt{20}, 2\sqrt{20}$ (*c*) $2\sqrt{10}, 2\sqrt{10}$ (*d*) $\sqrt{50}, \sqrt{50}$

(*v*) If the area of rectangle will be maximum then it will become
(*a*) Square (*b*) Trapezium (*c*) Rhombus (*d*) Can not say

18. **Case Study—**An architect designs a garden in society. The garden is in the shape of a rectangle inscribed in a circle of radius 10 m as shown in given figure.



Based on the given information answer the following:

(*i*) $2x$ and $2y$ represents the length and breadth of the rectangular part, then the relation between the variables is
(*a*) $x^2 - y^2 = 10$ (*b*) $x^2 + y^2 = 10$ (*c*) $x^2 - y^2 = 100$ (*d*) $x^2 + y^2 = 100$

(*ii*) **The area of the green grass A expressed as a function of x is**

(*a*) $2x\sqrt{100-x^2}$ (*b*) $4x\sqrt{100-x^2}$ (*c*) $2x\sqrt{100+x^2}$ (*d*) $4x\sqrt{100+x^2}$

(*iii*) **The maximum value of area A is**

(*a*) 100 m^2 (*b*) 200 m^2 (*c*) 400 m^2 (*d*) 1600 m^2

(*iv*) **The value of length of rectangle, if A is maximum is**

(*a*) $10\sqrt{2}$ m (*b*) $20\sqrt{2}$ m (*c*) 20 m (*d*) $5\sqrt{2}$ m

(*v*) **The area of gravelling path is**

(*a*) $100(\pi+2)$ m^2 (*b*) $100(\pi-2)$ m^2 (*c*) $200(\pi+2)$ m^2 (*d*) $200(\pi-2)$ m^2

PART B

SECTION III

All questions are compulsory. In case of internal choices attempt anyone.

19. Find the domain of $\cos^{-1}(2x-1)$.

20. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then find α satisfying $0 < \alpha < \frac{\pi}{2}$ where $A + A^T = \sqrt{2}\,I_2$ where A^T is transpose of A.

Or

If A is 3 × 3 invertible matrix, then what will be the value of k, if $\det(A^{-1}) = (\det A)^k$.

21. Find $\frac{dy}{dx}$ at $x = 1, y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = k$.

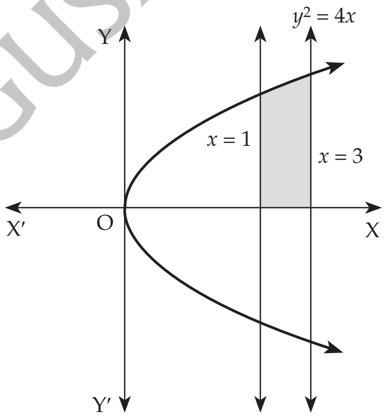
22. Find the point on the curve $9y^2 = x^3$ where the normal to the curve makes equal intercepts with the axes.

23. Evaluate: $\int \frac{xe^{2x}}{(1+2x)^2}\,dx$

Or

Evaluate: $\int_{\pi/4}^{\pi/2} \cos 2x \log \sin x\,dx$

24. The given parabola $y^2 = 4x$, find the area of shaded region.



25. Find the integrating factor of differential equation $\tan^2 x \frac{dy}{dx} + y\tan x = \sec x$.

26. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vector, then find the value of $|2\vec{a} + \vec{b} + \vec{c}|$.

27. Find the equation of plane passing through the points (–1, 2, 0), (2, 2, –1) and parallel to line whose direction ratios are (1, 2, –1).

28. A chairman is biased so that he selects his relatives for a job 4 times as likely as others. If there are 2 posts for a job. Find the probability distribution for selection of a person other than their relatives.

Or

A and B throw a pair of dice alternatively. A wins the game, if he gets a total of 7 and B wins the game, if he gets total of 10. If A starts the game, then find the probability that B wins.

SECTION IV

All questions are compulsory. In case of internal choices attempt anyone.

29. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on **R**.

30. Differentiate w.r.t. x: $(\sin x)^x + \sin^{-1}\sqrt{x}$.

31. Determine $f(0)$, so that function $f(x)$ defined by $f(x) = \dfrac{(4^x - 1)^3}{\sin\frac{x}{4}\log\left(1+\frac{x^2}{3}\right)}$ becomes continuous at $x = 0$.

Or

If $y = b\tan^{-1}\left(\frac{x}{a} + \tan^{-1}\frac{y}{x}\right)$, find $\dfrac{dy}{dx}$.

32. Find an angle θ, $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine.

33. Solve: $\int\left(\log(\log x) + \frac{1}{(\log x)^2}\right)dx$

Or

$\int \frac{4}{(x-2)(x^2+4)}\,dx.$

34. Often it is taken that a truthful person commands more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

35. Solve that the differential equation $\dfrac{dy}{dx} = \dfrac{y^2}{xy - x^2}$ is homogeneous and also solve it.

SECTION V

All questions are compulsory. In case of internal choices attempt anyone.

36. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve

$x - y + z = 4,\ x - 2y - 2z = 9$ and $2x + y + 3z = 1$.

Or

Solve by matrix method

$4x - 3y + 2z = 0$
$3x + 4y - 3z = 17$
$x + y - 2z = 6$

37. Find the equation of the plane passing through the line of intersection of planes

$2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and parallel to the line $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-5}{4}$.

Or

Find the coordinates of the foot of perpendicular from the point (–1, 3, –6) to the plane $2x + y - 2z + 5 = 0$. Also find the equation and length of the perpendicular.

38. Solve the problem graphically

Maximize $Z = 100x + 600y$

Subject to constraints:

$x + y \le 200;\ x \ge 20;\ y - 4x \ge 0;\ x, y \ge 0$

Or

The probability of solving A, B, C solving the problem are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously. Find the probability that

(*i*) Exactly one of them will solve it.

(*ii*) Atleast one of them will solve it.