Series : CBSE / SGP/ 2021-04 (Unsolved) Code No. 041

Roll No. | | | | | | | |

- Please check that this question paper contains **5** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **38** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Guess Paper 4 (Unsolved)

Time Allowed : 3 hours *Maximum Marks : 80*

General Instructions:
Same as in Sample Guess Paper 1 (Unsolved).

PART A
SECTION I

All questions are compulsory. In case of internal choices attempt anyone.

1. Consider Set A = {1, 2, 3} and the relation R = {(1, 2)}, then it is a transitive relation. State true or false.

Or

State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

2. If $f, g : R \to R$ are two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x \ \forall \ x \in R$. Then, find *fog* and *gof*.

3. If R is a relation on the set N, defined by $\{(x, y) : 2x - y = 10\}$, then R is _____.

Or

If A = {1, 2, 5, 8} and B = {1, 2, 3, 4, 5, 6, 7, 8} then, what is the number of one-one functions from A to B?

4. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$, then A (*adj* A) is equal to _____.

5. If $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ and $Q = PP^T$, then the value of the determinant of Q is _____?

Or

If $A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$ and *adj* (A + B) $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find the value of x and y.

6. $\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x} dx$

Or

Find: $\int \sqrt{1 - \sin 2x}\, dx, \frac{\pi}{4} < x < \frac{\pi}{2}$.

7. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$.

8. $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx$

9. Find the order of $\left(1 + 5\frac{dy}{dx}\right)^{3/2} = 10\frac{d^3y}{dx^3}$.

Or

Find integrating factor of differential equation:

$$\frac{dy}{dx} - \frac{3x^2y}{1+x^3} = \frac{\sin^2 x}{1+x}.$$

10. If $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is _____?

11. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\vec{x} \cdot \vec{y} = 0$ then find the value of $|\vec{x} + \vec{y}|$.

12. Find the projection of $\vec{a} = 3\hat{i} - \hat{j} + 5\hat{k}$ on $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$.

13. If the plane $3x + y + 2z + 6 = 0$ is parallel to the line $\frac{3x-1}{2b} = 3 - y = \frac{z-1}{a}$, then value of $3a + 3b$ is _____.

14. Find the point at which the line joining (1, 1, 2) and (3, –2, 1) meets the plane $3x + 2y + z = 6$.

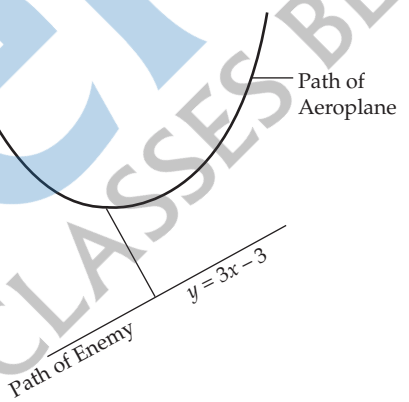
15. If A and B are two independent events, prove that $\overline{A}$ and B are also independent.

16. If $P(E) = \frac{1}{2}$ and $P(F) = \frac{1}{5}$. Find $P(\overline{E \cup F})$ if E and F are independent events.

SECTION II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. **Case Study—** An aeroplane is flying in the path of the curve $y = x^2 + 7x + 2$ and an enemy is following the path of straight line $y = 3x - 3$.



Answer the following Questions:

(*i*) The distance between aeroplane and an enemy at point (*x*, *y*) is D is equal to

(*a*) $D = \frac{(x_1 + 2)^2 + 1}{\sqrt{10}}$

(*b*) $D = \frac{(x_1 - 2)^2 + 1}{\sqrt{10}}$

(*c*) $\frac{(x_1 + 1)^2 + 1}{\sqrt{10}}$

(*d*) $\frac{(x_1 - 1)^2 + 1}{\sqrt{10}}$

(ii) The minimum distance between enemy and the aeroplane is

(a) $\frac{1}{\sqrt{20}}$ **(b)** $\frac{1}{\sqrt{10}}$ **(c)** $\frac{1}{\sqrt{30}}$ **(d)** $\frac{1}{\sqrt{40}}$

(iii) The equation of tangent to the slope of path of curve $y = x^2 + 7x + 2$ at $x = 0$ is

(a) $y = 2x - 7$ **(b)** $y = -2x - 7$ **(c)** $y = 2x$ **(d)** $y = 7x + 2$

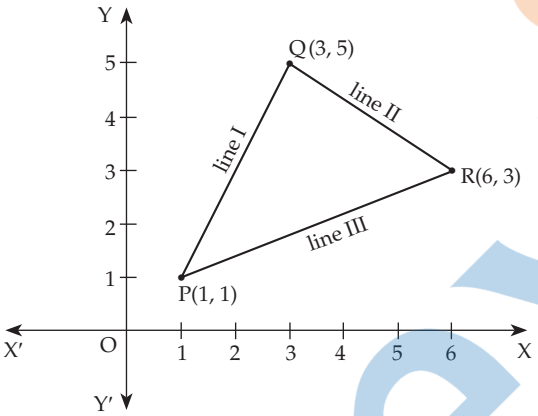
(iv) The y coordinate of point on line $y = 3x - 3$ which is nearest to the aeroplane is

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

(v) The integration of path of aeroplane $y = x^2 + 7x + 2$ is

(a) $\frac{x^3}{3} + 7x + 2$ **(b)** $\frac{x^3}{3} + \frac{7x^2}{2} + 2x + k$ **(c)** $\frac{x^3}{3} + \frac{7x^2}{2} + 3x + k$ **(d)** $\frac{x^3}{3} + 7x + \frac{2x^2}{3} + k$

18. Case Study— Top view of Pizza's triangular slice



Answer the following questions:

(i) Write the equation for the line I in the form of x:

(a) $y = 2x + 1$ **(b)** $y = 2x$ **(c)** $y = 2x - 1$ **(d)** $y = 3x + 1$

(ii) Write the equation for line II in the form of x:

(a) $y = \frac{18 - 2x}{3}$ **(b)** $y = \frac{18 + 5x}{3}$ **(c)** $y = 18 - x$ **(d)** $y = 18 + 8x$

(iii) Write the equation for line III in the form of x:

(a) $y = \frac{3x + 5}{2}$ **(b)** $y = \frac{2x + 5}{3}$ **(c)** $y = \frac{2x + 3}{5}$ **(d)** $y = \frac{2x - 3}{5}$

(iv) Using integration, find the area of ΔPQR.

(a) 4 **(b)** 5 **(c)** 1 **(d)** 3

(v) Area of the region bounded by the curve $x = 2y + 3$, the y-axis and between $y = -1$ and $y = 1$ is

(a) 4 **(b)** $\frac{4}{3}$ **(c)** 6 **(d)** 8

PART B

SECTION III

All questions are compulsory. In case of internal choices attempt anyone.

19. $\sin^{-1}\left[\frac{\sqrt{1+x} + \sqrt{1-x}}{2}\right]$, find the simple form.

20. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, find the value of $(A - 2I)(A - 3I)$.

Or

If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the value of $A^2 - 5A$.

21. Find the value of k, so that the function

$$f(x) = \begin{cases} \dfrac{(1-\cos 4x)}{8x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} \text{ is continuous at } x = 0.$$

22. Find the equation of the normal to the curve $x^2 = 4y$ which passess through point (–2, 4).

23. $\int \dfrac{dx}{1-3\sin x}$

Or

$\int \dfrac{x+1+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}\,dx$

24. Find the area bounded by $y^2 = 8x$ and $x^2 = 8y$.

25. Find the general solution of differential equation $\dfrac{dy}{dx} = e^y\,(e^x + e^{-x} + 2x)$.

26. If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

27. Find the distance between the line $\dfrac{x-5}{3} = \dfrac{y-4}{z} = \dfrac{z-8}{1}$ and the plane determined by the points. A(2, –2, 1), B(4, 1, 3) and C(–2, –2, 5).

28. A couple has 2 children. Find the probability that both are boys, if it is known

(*a*) One of them is boy **(*b*)** The older child is boy

Or

Find P(A ∪ B), if 2 P(A) = P(B) = $\dfrac{5}{13}$ and P(A | B) = $\dfrac{2}{5}$.

SECTION IV

All questions are compulsory. In case of internal choices attempt anyone.

29. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(*a*, *b*) : |*a* – *b*| is divisible by 2} is equivalence relation.

30. If $x^y = e^{x-y}$ then prove that

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

31. If $\sin(x+y) = \log(x+y)$, then find the value of $\dfrac{dy}{dx}$.

Or

Find the derivative of $\sin^3 x$ w.r.t. $\cos^3 x$.

32. Find the interval in which function $f(x) = \cos 2x$ is increasing and decreasing.

33. Prove that $\int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x})\,dx = \dfrac{\pi}{\sqrt{2}}$

34. Find the area bounded by the curve $y = \log x$, x-axis and the ordinates $x = 1$ and $x = 2$.

Or

Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|$.

35. Solve the Differential Equation

$$(x+1)\frac{dy}{dx} = 2e^{-y} - 1,\ y(0) = 0.$$

SECTION V

All questions are compulsory. In case of internal choices attempt anyone.

36. Find the image of the point (1, 2, 3) in the plane $x + 2y + 4z = 38$.

Or

Find the equation of plane containing the lines

$\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

$\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$

Also find the distance of this plane from the point (2, 2, 2).

37. Solve for x, y, z:

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$$

$$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

Or

Using matrix solve the following:

$x + y = 5$

$y + z = 3$

$z + x = 4$

38. Solve the LPP graphically:

Maximize $Z = 15x + 10y$

Subject to constraints:

$2x + 1y \leq 40;\ 2x + 3y \leq 80;\ x, y \geq 0$

Or

Solve the LPP graphically:

Minimize $Z = 7x + 4y$

Subject to constraints:

$3x + 2y \leq 12;\ 3x + y \leq 9;\ x, y \geq 0$