Series: CBSE / SGP/ 2021-01 (Unsolved)

Code No. 041

Roll No. ☐☐☐☐☐☐☐

- Please check that this question paper contains **5** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **38** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII

Sample Guess Paper 1 (Unsolved)

Time Allowed: 3 hours *Maximum Marks: 80*

General Instructions:

1. *The questions paper contains two parts **A and B**. Each part is compulsory. Part A carries **24 marks** and Part B carries **56 marks**.*
   ***Part A** has objective Type Questions and **Part B** has Descriptive Type Questions.*
2. *Both Part A and Part B have internal choices.*

Part-A

1. *It consists of two sections – **I and II***
2. ***Section I** comprises of 16 very short answer type questions.*
3. ***Section II** contains 2 case-studies. Each cash-study comprises of 5 case based MCQs. An examinee is to attempt any 4 out of 5 MCQs.*

Part-B

1. *It consists of three sections **III, IV and V**.*
2. ***Section III** comprises of 10 questions of **2 marks** each.*
3. ***Section IV** comprises of 7 questions of **3 marks** each.*
4. ***Section V** comprises of 3 questions of **5 marks** each.*
5. *Internal choice is provided in 3 questions of Section III, 2 questions of Section IV and 3 questions of Section V. You have to attempt only one of the alternatives in all such questions.*

PART A

SECTION I

All questions are compulsory. In case of internal choices attempt anyone.

1. How many relations are possible in set A = $\{a, b, c\}$.

Or

If $f(x) = 27x^3$ and $g(x) = x^{\frac{1}{3}}$ then find $gof(x)$.

2. Show that the relation R in set A = {1, 2, 3} given by R = {(1, 2)(2, 1)} is symmetric but neither reflexive nor transitive.

3. Prove that $f : R \to R$ given by $f(x) = x^3 + 1$ is one-one function.

Or

Find the domain of function $f(x) = \sqrt{\cos x}$.

4. If $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$. Then find A^2.

5. Find the solution of determinant $\begin{vmatrix} 4-x & 4-x & x-4 \\ -6 & 3 & 1 \\ 3 & 6 & 3 \end{vmatrix} = 0.$

Or

Evaluate $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix}$.

6. If the matrix $\begin{bmatrix} 2 & 1 & -5 \\ 0 & 3 & k \\ 1 & 3 & 2 \end{bmatrix}$ is singular then find k.

7. Evaluate $\int_1^2 |x-2|\,dx$

Or

Evaluate $\int \frac{dx}{\sin^2 x \cos^2 x}$

8. If $y = \log \frac{1-x}{1+x}$ then find $\frac{dy}{dx}$.

9. Find the equation of tangent of the curve $\frac{x-1}{x-2}$ at $x = 10$.

Or

Find the particular solution of $e^{\frac{dy}{dx}} = x$ given that $y = 0$, when $x = 1$.

10. Find a unit vector parallel to the vector $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}$.

11. Find the direction of the line $\frac{2x+1}{5} = \frac{y-2}{2} = \frac{z-6}{-2} = \gamma$.

12. Find the equation of plane in vector from $3x + 2y + 7z = 0$.

13. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = 3\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 6\hat{i} + 4\hat{j} + 6\hat{k}$.

14. Find equation of plane having normal vector $\hat{i} + \hat{j} + \hat{k}$ and unit distance from origin.

15. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

16. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a} + \vec{b} + \vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

SECTION II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. **Case Study—** An advertisement firm is supplied with decorative wire of length 34 m and are asked to cut the wire into 2 pieces. One of the pieces is to made into a circular sign board and the other into a square. The idea is to keep the combined area of circle and square to be minimum for writting slogans.

(*i*) **If a wire is cut at x m from one end and made into a circular board with radius r. Then find the relation between r and x.**

(*a*) $\pi r^2 = x$ (*b*) $\pi r = x$ (*c*) $2\pi r = x$ (*d*) $r = 2x$

(*ii*) **Area enclosed by circular board is**

(*a*) $\frac{x}{4\pi}$ m^2 (*b*) x^2 m^2 (*c*) πx^2 m^2 (*d*) $\frac{x^2}{4\pi}$ m^2

(*iii*) **Area enclosed by square frame is**

(*a*) $\frac{1}{4}(x-34)$ m^2 (*b*) $\frac{1}{16}(34-x)^2$ m^2 (*c*) $(34-x)^2$ m^2 (*d*) $\frac{1}{8}(34-x)^2$ m^2

(*iv*) **For what value of x, the combined area is minimum:**

(*a*) $\frac{34}{4+\pi}m$ (*b*) $\frac{17}{4+\pi}m$ (*c*) $\frac{34\pi}{4+\pi}m$ (*d*) $\frac{34\pi}{4-\pi}m$

(*v*) **What is ratio of the area of a circle and the area of a square when combined area is minimum**

(*a*) 11 : 14 (*b*) 14 : 11 (*c*) 7 : 11 (*d*) 11 : 7

18. **Case Study—** A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective.

Answer the following questions:

(*i*) **Probability of items produced by three operators on job:**

(*a*) $\frac{5}{10}, \frac{3}{10}, \frac{2}{10}$ (*b*) $\frac{3}{10}, \frac{5}{10}, \frac{2}{10}$ (*c*) $\frac{2}{10}, \frac{5}{10}, \frac{3}{10}$ (*d*) none of the above

(*ii*) **Probability of production by three operators if an item is chosen and found to be defective:**

(*a*) $\frac{5}{10}, \frac{7}{10}, \frac{1}{10}$ (*b*) $\frac{5}{100}, \frac{7}{100}, \frac{1}{100}$ (*c*) $\frac{1}{100}, \frac{5}{100}, \frac{7}{100}$ (*d*) $\frac{0.5}{100}, \frac{0.7}{100}, \frac{0.1}{100}$

(*iii*) **Probability that the defective item is produced by operator A:**

(*a*) $\frac{8}{37}$ (*b*) $\frac{5}{34}$ (*c*) $\frac{6}{34}$ (*d*) $\frac{7}{34}$

(*iv*) **If A and B one two events and A ≠ ϕ and B ≠ ϕ, then**

(*a*) $P(A|B) = P(A).P(B)$ (*b*) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

(*c*) $P(A|B).P(B|A) = 1$ (*d*) $P(A|B) = P(A)|P(B)$

(*v*) **If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B)$ $\frac{3}{5}$, then $P(B|A) + P(A|B)$ equals**

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{5}{12}$ (*d*) $\frac{7}{12}$

PART B

SECTION III

All questions are compulsory. In case of internal choices attempt anyone.

19. Find the principle value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

20. If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ then the value of matrix A.

Or

If A = $\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is identity matrix then find x.

21. Find the relationship between a and b so that function f defined by

$$f(x) = \begin{cases} ax + 1 & \text{if } x \le 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$$

Or

Find the value of k for which the function $f(x) = \begin{cases} \dfrac{x^2 + 3x - 10}{x - 2}, & x \ne 2 \\ k\,, & x = 2 \end{cases}$ is continuous at $x = 2$.

22. Show that $f(x) = 9x^3 + 18x^2 + 9x + 10$ is strictly increasing on R.

23. $\int x\sqrt{x+2}\,dx$

Or

$\int_0^3 \dfrac{dx}{9 + x^2}$.

24. Find the area of region bounded by $y^2 = 8x$ and a line $x = 2$ in first quadrant.

25. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2 + x}{x + 3}$

26. If $|\vec{a}| = 5$, $|\vec{b}| = 6$, $|\vec{c}| = 9$ then find the angle between $\vec{a}$ and $\vec{b}$.

Or

Find the vector equation of the line passing through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

27. Find the distance of the plane $2x - 3y + 2z - 14 = 0$ from the origin.

28. Find the probability distribution of x the number of heads in simultaneously toss of two coins.

SECTION IV

All questions are compulsory. In case of internal choices attempt anyone.

29. Solve the differential equation $x\dfrac{dy}{dx} + y = x\cos x + \sin x$ given that $y\left(\dfrac{\pi}{2}\right) = 1$.

30. If $\tan^{-1}\left(\dfrac{y}{x}\right) = \log\sqrt{x^2 + y^2}$ prove that $\dfrac{dy}{dx} = \dfrac{x + y}{x - y}$.

31. If $y = \sin^{-1}\dfrac{6x - 4\sqrt{1 - 4x^2}}{5}$, then find $\dfrac{dy}{dx}$.

Or

If $x^y + y^x = a^b$ then find $\dfrac{dy}{dx}$.

32. Find the Maximum area of isosceles triangle inscribed in the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ with its vertex at one end of major axis.

33. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{3/2}}{(\sin x)^{3/2} + (\cos x)^{3/2}}\,dx$

34. Using integration find the area of the region $\{(x, y) : 9x^2 + y^2 \le 36$ and $3x + y \ge 6$.

Or

Find the area enclosed between the parabola $4y = 3x^2$ and straight line $3x - 2y + 12 = 0$.

35. Find the general solution of differential equation:

$x\cos y\,dy = (xe^x \log x + e^x)\,dx$.

SECTION V

All questions are compulsory. In case of internal choices attempt anyone.

36. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find AB^{-1}.

Or

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$; $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

37. Two system of rectangular axes have same origin. If a plane cuts them at distance a, b, c and a', b', c' respectively from the origin then prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$.

Or

Prove that the line through A(0, –1, –1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(–4, 4, 4).

38. Solve the LPP graphically:

Maximize $Z = 7.5x + 5y$

Subject to constraint:

$2x + y \le 60$
$x \le 20$
$2x + y \le 120$

Or

Solve the following problem graphically:

Minimize and Maximize $Z = 3x + 9y$

Subject to the constraints:

$x + 3y \le 60$
$x + y \ge 10$
$x \le y$
$x \le 0, y \le 0.$