

General Instruction:

1. This question paper contains two parts **A and B**. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections- **I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections- **III, IV and V**.
2. Section **III** comprises of 10 questions of 2 marks each.
3. Section **IV** comprises of 7 questions of 3 marks each.
4. Section **V** comprises of 3 questions of 5 marks each.

PART - A
SECTION-I

Questions 1 to 16 carry 1 mark each.

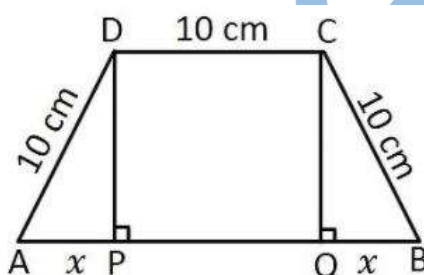
1. Check whether $f(x) = \tan x$ is one-one or not on \mathbb{R} .
2. Find the domain of $f(x) = \sin^{-1}(-x^2)$.
3. If A is a matrix of order 3×3 such that $|A| = 4$, find $|A^{-1}|$.
4. Find the value of $\cot\left(\frac{\pi}{3} - 2 \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$
5. If for the matrix A , $A^3 = I$, then find the value of A^{-1} .
6. If $2A + B + X = 0$, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$, then find the value of matrix X .
7. Find the equation of the tangent to the curve $y = x^2 + 4x + 1$ at the point where $x = 3$.
8. If $y = 7x^3 - 4x^2$, find $\frac{dy}{dx}$.
9. Evaluate: $\int \frac{dx}{x + x \log x}$

10. If A and B are events such that $P(A) = 0.3$, $P(B) = 0.6$ and $P(B/A) = 0.5$, find $P(A/B)$.
11. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement then find the probability that both drawn balls are black.
12. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, then find the value of $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$.
13. Find the value of λ if the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{j} + \lambda\hat{k}$ are perpendicular.
14. Write the cartesian equation of the plane whose vector equation is $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 7$.
15. Write the equation of line passing through the points $(-1, 2, 1)$ and $(3, 1, 4)$.
16. If $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = 144$ and $|\vec{a}| = 4$, then find the value of $|\vec{b}|$.

SECTION-II

Case study-based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. A building has front gate has the figure as shown below:
It is in the shape of trapezium whose three sides other than base in 10 m. Height of the gate is 'h' m.



On the basis of above figure answer the following questions:

- (i) The relation between a and h is
 (a) $x^2 + h^2 = 10$ (b) $x^2 + h^2 = 100$ (c) $h^2 - x^2 = 10$ (d) $x^2 - h^2 = 10$
- (ii) The area of gate A expressed as a function of x is
 (a) $(10+x)\sqrt{100+x^2}$ (b) $(10-x)\sqrt{100+x^2}$
 (c) $(10+x)\sqrt{100-x^2}$ (d) $(10-x)\sqrt{100-x^2}$
- (iii) The value of x when A is maximum is
 (a) 5 m (b) 10 m (c) 15 m (d) 20 m
- (iv) The value of h when A is maximum is
 (a) $5\sqrt{2}$ m (b) $5\sqrt{3}$ m (c) $10\sqrt{2}$ m (d) $10\sqrt{3}$ m
- (v) Maximum value of A is (in m^2) is
 (a) $\frac{75\sqrt{3}}{2}$ (b) $75\sqrt{3}$ (c) $\frac{75\sqrt{3}}{4}$ (d) 75

18. The probability distribution functions which shows the number of hours (X) a student study during lockdown period in a day, is given by (where $C > 0$)

X	0	1	2
P(X)	$3C^3$	$4C - 10C^2$	$5C - 1$

On the basis of above information answer the following questions:

- (i) The correct equation for C is
 (a) $3C^3 - 10C^2 + C - 2 = 0$ (b) $3C^3 + 10C^2 + C - 2 = 0$
 (c) $3C^3 - 10C^2 + 9C - 2 = 0$ (d) None of these
- (ii) The value of C is
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$
- (iii) $P(X < 2) =$
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$
- (iv) $P(X = 1) =$
 (a) $\frac{2}{9}$ (b) $\frac{1}{9}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$
- (v) $P(X \geq 0) =$
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 1 (d) 0

PART B
SECTION – III

Questions 19 to 28 carry 2 marks each.

19. An urn contains 5 white and 8 white black balls. Two successive drawing of three balls at a time are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and second draw gives 3 black balls.

20. Find X and Y, if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

21. Examine the continuity of the function: $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & x \neq 4 \\ 0, & x = 4 \end{cases}$ at $x = 4$.

22. If $y = \frac{1}{\sqrt{a^2 - x^2}}$, then find $\frac{dy}{dx}$.

23. Find the value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - 2 \sec^{-1}\left(2 \tan \frac{\pi}{6}\right)$.

24. Given $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then find $|\vec{a} \times \vec{b}|$.

25. If $x = 10(t - \sin t)$, $y = 12(1 - \cos t)$, find $\frac{dy}{dx}$.

26. Find the equation of the tangent to the curve $y = -5x^2 + 6x + 7$ at $(1/2, 35/4)$.

27. Evaluate: $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

28. Two vectors \vec{a} and \vec{b} , prove that the vector $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is orthogonal to the vector $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$.

SECTION – IV

Questions 29 to 35 carry 3 marks each.

29. Let the function $f: R^+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Prove that f is bijective.

30. Find the particular solution of differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$ for $x = 1$ and $y = 1$.

31. Evaluate: $\int_0^{\pi/2} \log(\sin x) dx$

32. Find the volume of the largest cylinder that can be inscribed in sphere of radius 'r'.

33. Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$.

34. Evaluate: $\int \frac{x^2}{x \sin x + \cos x} dx$

35. Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point $(1, 4)$.

SECTION – V

Questions 36 to 38 carry 5 marks each.

36. Solve the LPP graphically maximize, $Z = 1500(7x + 6y)$ subject to constraints;
 $x + y \leq 50$; $2x + y \leq 80$, $x, y \geq 0$.

37. Find the equation of plane determined by points $A(3, -1, 2)$, $B(5, 2, 4)$, $C(-1, -1, 6)$ and hence find the distance between plane and point $P(6, 5, 9)$.

38. Evaluate the product AB , where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Hence solve the system of linear equations $x - y = 3$, $2x + 3y + 4z = 17$ and $y + 2z = 7$.