Series: CBSE / SGP/ 2021-05 (Solved)

Code No. **041**

Roll No.

- Please check that this question paper contains **4** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Guess Paper 5 (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

Same as CBSE Sample Question Paper.

PART A

Section I

All questions are compulsory. In case of internal choices attempt anyone.

1. Find the number of all onto functions from the set {1, 2, 3, ..., 10} to itself.

Or

Let **R** be the relation in the set Z of integers given by R = {(a, b) : 2 divides $a - b$}. Show that the relation R transitive? Write the equivalence class [0].

2. A = {1, 2}. How many one-one functions from A to A possible? Also write them.

3. Write total number of functions from set A to set B, where set A = {1, 2, 3, 4}, set B = {a, b, c}.

Or

If X and Y are two sets having 2 and 3 elements respectively then find the number of functions from X to Y.

4. If for any 2 × 2 square matrix A, A(adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

5. Given A = $\begin{pmatrix} 4 & 2 & 5 \\ 2 & 0 & 3 \\ -1 & 1 & 0 \end{pmatrix}$, write the value of det. $(2AA^{-1})$.

Or

If $\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2\begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix}$ = I, where I is a 2 × 2 Unit matrix, find $(x - y)$.

6. If 3A – B = $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A.

7. Evaluate: $\int_{-2}^{2} (x^3 + 1)$.

Or

Find $\int x\, e^{(1+x^2)} dx$.

8. Write the angle between the vectors $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

9. Find the general solution of the differential equation $\frac{dy}{dx} + 2y = e^{3x}$.

Or

Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.

10. If $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$, then the values of p and q are?

11. The magnitude of projection of $(2\hat{i} - \hat{j} + \hat{k})$ on $(\hat{i} - 2\hat{j} + 2\hat{k})$ is ....

12. Vector of magnitude 5 units and in the direction opposite to $(2\hat{i} + 3\hat{j} - 6\hat{k})$ is ....

13. Find the direction cosines of the vector joining the points A(1, 2, –3) and B(–1, –2, 1) directed from B to A.

14. Find the vector equation for the line which passes through the point (1, 2, 3) and is parallel to the line $\frac{x-1}{-2} = \frac{1-y}{3} = \frac{3-z}{-4}$.

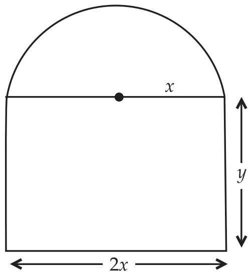
15. If A and B are two events such that P(A) = 0.2, P(B) = 0.4 and P(A ∪ B) = 0.5, then what is the value of P(A | B)?

16. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. What is the probability that they are of the different colours?

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question (17–21) and (22 – 26). Each question carries 1 mark.

17. **Case Study—**An architect designs a House in which a window for a study room in designed in the form of a rectangle above which there is a semi-circle, so that maximum sunlight can enter into the room. The perimeter of the window is P and the length and breadth of the Rectangular portion of window is given by $2x$ and y respectively.



Based on the above information answer the following questions:

(*i*) What should be the relation between the variables.

(*a*) $x + \pi y = P$ (*b*) $2y + x(4 + \pi)$ (*c*) $P = 2(2y + x)$ (*d*) $P = 2(y + 2x)$

(*ii*) The area of the rectangular region of window expressed as a function of x is

(*a*) $x(P - x(4 + \pi))$ (*b*) $x(P + (4 - \pi)x)$ (*c*) $P + x\pi + 2x$ (*d*) $P - x(4 + \pi)$

(*iii*) The maximum value of Area of Rectangular region.

(*a*) $\frac{P^2}{4(16+\pi)}$ (*b*) $\frac{P}{(4+\pi^2)}$ (*c*) $\frac{P^2}{4(4+\pi)}$ (*d*) $\frac{P}{2(2+\pi)}$

(*iv*) The owner of the house wants to maximize the area of the whole window including the semi-circle head. For this to happen the value of x should be

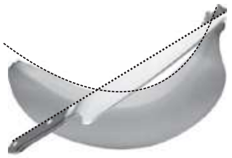
(*a*) $\frac{2(4+\pi)}{(P+\pi)}$ (*b*) $\frac{-2(P+\pi)}{4+\pi}$ (*c*) 0 (*d*) $\frac{(P+\pi)}{2(4+\pi)}$

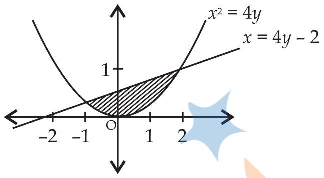
(*v*) Maximum area of entire window is:

(*a*) $P\left(\frac{P+\pi}{4+\pi}\right)$ (*b*) $2x + \frac{P+\pi}{4+\pi}$

(*c*) $\frac{4+\pi}{6+\pi} + x\pi$ (*d*) None of the above

18. Case Study—*This image shows a banana with is overheaded by a knife.*





Answer the following questions:

(*i*) Present the equation of the line in term of *x*.

(*a*) $y = 4(x - 2)$ (*b*) $2y = 4(x + 2)$ (*c*) $y = \frac{x-2}{4}$ (*d*) $y = \frac{x+2}{4}$

(*ii*) Find the intersection points of line and parabola.

(*a*) $(-1, 1)\ (2, 1)$ (*b*) $\left(1, \frac{-1}{4}\right)(2, -1)$ (*c*) $\left(-1, \frac{1}{4}\right)\ (2, 1)$ (*d*) $(1, 1)\ (2, 2)$

(*iii*) Find the area of the line.

(*a*) $\frac{13}{2}$ sq. unit (*b*) $\frac{15}{8}$ sq. unit (*c*) $\frac{4}{3}$ sq. unit (*d*) 2 sq. unit

(*iv*) Find the Area of parabola.

(*a*) $\frac{1}{2}$ sq. unit (*b*) $\frac{2}{3}$ sq. unit (*c*) $\frac{3}{2}$ sq. unit (*d*) $\frac{3}{4}$ sq. unit

(*v*) Find the shaded region of the figure.

(*a*) $\frac{9}{8}$ sq. unit (*b*) $\frac{9}{4}$ sq. unit (*c*) $\frac{8}{9}$ sq. unit (*d*) $\frac{4}{3}$ sq. unit

PART B

Section III

19. Express $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$; where $-\frac{\pi}{4} < x < \frac{\pi}{4}$, in the simplest form.

20. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method.

Or, Show that all the diagonal elements of a skew symmetric matrix are zero.

21. If $y = \log(1 + 2t^2 + t^4)$, $x = \tan^{-1} t$, find $\frac{d^2y}{dx^2}$.

22. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into subintervals in which the function $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing or strictly decreasing.

23. Evaluate : $\int \frac{x+2}{\sqrt{x^2 + 5x + 6}}\, dx$ ***Or,*** Evaluate : $\int_1^4 \{|x-1| + |x-2| + |x-4|\}\, dx$.

24. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = -6$ to $x = 0$.

25. Solve the differential equation: $x\frac{dy}{dx} + y - x + xy \cot x = 0, x \neq 0$

26. For three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ if $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \times \vec{c} = \vec{b}$, then prove that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, $|\vec{b}| = |\vec{c}|$ and $|\vec{a}| = 1$.

27. Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

28. Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X.

Or, There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head. What is probability that it was the two headed coin?

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Consider $f : \mathbf{R} - \mathbf{R} - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective.

30. If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.

31. Find 'a' and 'b', if the function given by $f(x) = \begin{cases} ax^2 + b, \text{ if } x < 1 \\ 2x + 1, \text{ if } x \geq 1 \end{cases}$ is differentiable at $x = 1$.

Or, Determine the values of 'a' and 'b' such that the following function is continuous at $x = 0$:

$$f(x) = \begin{cases} \frac{x + \sin x}{\sin(a+1)x}, & \text{if } -\pi < x < 0 \\ 2, & \text{if } x = 0 \\ 2\frac{e^{\sin bx} - 1}{bx}, & \text{if } x > 0 \end{cases}$$

32. Find the equation of the normal to the curve $2y = x^2$, which passes through the point (2, 1).

33. Evaluate : $\int_{-1}^{1} |x \cos \pi x| \, dx$.

34. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 1, x + y \geq 1, x \geq 0, y \geq 0\}$.

Or, Using integration find the area of the following region : $\left\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\right\}$

35. Solve the following differential equation : $xy \log\left(\frac{y}{x}\right) dx + \left(y^2 - x^2 \log\left(\frac{y}{x}\right)\right) dy = 0$

Section V

All questions are compulsory. In case of internal choices attempt anyone.

36. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations:

$3x + 3y + 2z = 1; \quad x + 2y = 4; \quad 2x - 3y - z = 5$

Or, Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

37. Find the distance of point $-2\hat{i} + 3\hat{j} - 4\hat{k}$ from the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ measured parallel to the plane: $x - y + 2z - 3 = 0$.

Or, Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3).

38. Solve the following graphically and also find the maximum profit.

Maximum Profit, $Z = 24x + 18y$

Subject to the constraints: $2x + 3y \leq 10$; $3x + 2y \leq 10$; $x \geq 0, y \geq 0$.

Or, (*a*) **The corner points of the feasible region determined by the following system of linear inequalities:** $2x + y \leq 10$, $x + 3y \leq 15$, $x \geq 0$, $y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q > 0$. Find the condition of p and q so that the maximum of Z occurs at both (3, 4) and (0, 5).

(*b*) **Solve the following graphically and also find the maximum profit.**

Minimize and Maximize, $Z = 5x + 10y$

Subject to the constraints: $1x + 2y \leq 120$; $1x + 1y \geq 60$; $x - 2y \geq 8$; $x \geq 0, y \geq 0$.