Series: CBSE / SGP/ 2021-04 (Solved)

Code No. **041**

Roll No. | | | | | | | |

- Please check that this question paper contains **5** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Guess Paper 4 (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

Same as CBSE Sample Question Paper.

PART A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. State the reason why the Relation $R = \{(a, b) : a \leq b^2\}$ on the set R of real numbers is not reflexive.

Or

State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

2. Find the derivative of $f(e^{\tan x})$ w.r.t to x at $x = 0$. It is given that $f'(1) = 5$.

3. Find the Projection (vector) of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$.

Or

For vector $\vec{a}$, if $|\vec{a}| = a$, then write the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$

4. If A is a square matrix of order 3 and $|2A| = k|A|$, then find the value of k.

5. Prove that the diagonal elements of a skew symmetric matrix are all zeroes.

Or

If $A = [a_{ij}]$ is a matrix of order 2 × 2, such that $|A| = -15$ and c_{ij} represents the cofactor of a_{ij}, then find $a_{21}c_{21} + a_{22}c_{22}$.

6. If A and B are invertible matrices of order 3, $|A| = 2$ and $|AB|^{-1} = \frac{-1}{6}$. Find $|B|$.

7. Find: $\int \frac{e^x(x-3)}{(x-1)^3} dx$

Or

Write the value of : $\int \frac{x + \cos 6x}{3x^2 + \sin 6x} dx$

8. Evaluate : $\int_e^{e^2} \frac{dx}{x \log x}$

9. Find the general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$.

Or

Find the integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$.

10. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

11. Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

12. Find a unit vector parallel to the sum of the vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 5\hat{k}$.

13. If the equations of a line AB are $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$, find the direction cosines of a line parallel to AB.

14. What is the distance (in units) between the two planes $3x + 5y + 7z = 3$ and $9x + 15y + 21z = 9$?

(*a*) 0 (*b*) 3 (*c*) $\frac{6}{\sqrt{83}}$ (*d*) 6

15. If P(A) = 0.6, P(B) = 0.5 and P(A | B) = 0.3, then find P(A ∪ B).

16. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question (17–21) and (22 – 26). Each question carries 1 mark.

17. **Case Study—**An insurance company insured 2000 scooterdrivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively.

One of the insured persons meets with an accident.

Answer the following questions:

(*i*) **Write the probability the events to insure scooter, car and truck driver respectively.**

(*a*) $\frac{2}{3}, \frac{1}{2}, \frac{4}{6}$ (*b*) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ (*c*) $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ (*d*) $\frac{2}{6}, \frac{2}{3}, \frac{4}{2}$

(*ii*) **What are the probability of an accident involving a scooter, a car and a truck?**

(*a*) $\frac{1}{100}, \frac{3}{100}, \frac{15}{100}$ (*b*) $\frac{2}{100}, \frac{6}{100}, \frac{3}{100}$ (*c*) $\frac{3}{100}, \frac{9}{100}, \frac{45}{100}$ (*d*) $\frac{6}{10}, \frac{30}{10}, \frac{7}{10}$

(*iii*) **What is the probability that the insured person who met with an accident, is a scooter driver?**

(*a*) $\frac{5}{52}$ (*b*) $\frac{4}{52}$ (*c*) $\frac{2}{52}$ (*d*) $\frac{1}{52}$

(*iv*) **Given that the events A and B are such that P(A) = $\frac{1}{2}$, P(A ∪ B) = $\frac{3}{5}$ and P(B) = p. Find p if they are mutually exclusive.**

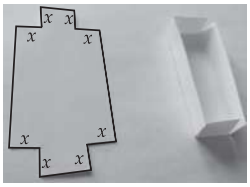
(*a*) $\frac{3}{10}$ (*b*) $\frac{2}{10}$ (*c*) $\frac{1}{10}$ (*d*) $\frac{1}{5}$

(*v*) **In addition to the point (*iv*), find p if they are mutually independent.**

(*a*) $\frac{1}{5}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{4}{5}$

18. **Case Study—*Rectangular Box***

This figure shows the Rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off squares from each corner and folding up the flaps.



Answer the following questions :

(*i*) Find the change in volume of the box w.r.t. the side to be cut *i.e.*, *x*.

(*a*) $x^2 + 23x - 90$ (*b*) $12(x^2 - 23x + 90)$

(*c*) $x^2 - 23x + 90$ (*d*) $6x^2 - 138x + 540$

(*ii*) What should be the side of the square to be cut off so that volume of the box is maximum?

(*a*) 5 (*b*) 12 (*c*) 18 (*d*) 24

(*iii*) What is the double derivative *i.e.*, $\frac{d^2V}{dx^2}$ at the value of *x*?

(*a*) –156 (*b*) –165 (*c*) –110 (*d*) 156

(*iv*) Write the dimensions of the Rectangular Box?

(*a*) 45, 24, 5 (*b*) 40, 19, 5 (*c*) 24, 45, 15 (*d*) 35, 14, 5

(*v*) What is the maximum value of the Box?

(*a*) 5240 (*b*) 4250 (*c*) 2450 (*d*) 2540

PART B

Section III

19. If $4\sin^{-1}x + \cos^{-1}x = \pi$, then find the value of *x*.

20. Two farmers X and Y cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale in rupees of these varieties of rice by both the farmers in the months of September and October are given by the following matrices A and B.

September sales in rupees (Basmati, Permal, Naura) **October sales in rupees** (Basmati, Permal, Naura)

$$A = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix}\begin{matrix} X \\ Y \end{matrix} \qquad B = \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}\begin{matrix} X \\ Y \end{matrix}$$

Find :

(*i*) What were the combined sales in September and October for each farmer in each variety?

(*ii*) If both farmers decided to donate 2% of the gross rupees sales in October, for the welfare of their workers, compute the total amount paid by each farmer for the welfare of the workers.

Or

To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel-sheets made by them using recycled paper at the rate of ₹ 20, ₹ 15 and ₹ 10 per unit respectively. School A sold 25 paper bags, 10 scrap-books and 30 pastel-sheets. School B sold 20 paper bags, 15 scrap-books and 30 pastel-sheets. While school C sold 25 paper bags, 18 scrap-books and 35 pastel-sheets. Using matrices, find the total amount raised by each school.

21. If $y = x^{\sin x} + \sin(x^x)$, find $\frac{dy}{dx}$

22. Find the equation(s) of the tangent(s) to the curve $y = (x^3 - 1)(x - 2)$ at the points where the curve intersects the *x*-axis.

23. Find $\int \left(\frac{1-x}{1+x^2}\right)^2 e^x\, dx$.

Or

Find: $\int \frac{(x^2 + \sin^2 x)\sec^2 x}{1+x^2}\, dx$.

24. Find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$ by using integration.

25. Solve the following differential equation $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$

26. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b}\times\vec{c})$.

27. Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the *z*-axis.

28. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of number of aces.

Or

A family has two children. What is the probability that both the children are boys, given that at least one of them is a boy?

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Let A = {1, 2, 3, ..., 9} and R be the relation in A × A defined by (a, b) R (c, d) if $a + d = b + c$ for $a, b, c, d \in$ A.
Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)].

30. Discuss the differentiability of the function $f(x) = \begin{cases} 2x - 1, & x < \frac{1}{2} \\ 3 - 6x, & x \geq \frac{1}{2} \end{cases}$ at $x = \frac{1}{2}$.

31. For what value of k is the following function continuous at $x = \frac{-\pi}{6}$?

$$f(x) = \begin{cases} \dfrac{\sqrt{3}\sin x + \cos x}{x + \frac{\pi}{6}}, & x \neq -\frac{\pi}{6} \\ k, & x = -\frac{\pi}{6} \end{cases}$$

Or

If $y = \log\left(\sqrt{x} + \frac{1}{x}\right)^2$, then prove that $x(x + 1)^2 y_2 + (x + 1)^2 y_1 = 2$.

32. Find the intervals in which the function $f(x) = -3\log(1 + x) + 4\log(2 + x) - \frac{4}{2 + x}$ is strictly increasing or strictly decreasing.

33. Find $\int \frac{\sec x}{1 + \operatorname{cosec} x}\,dx$.

34. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

Or

Using integration find the area of the region bounded by the triangle whose vertices are (1, 3), (2, 5) and (3, 4).

35. Find the general solution of the differential equation: $\frac{dx}{dy} = \frac{y\tan y - x\tan y - xy}{y\tan y}$

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. If A = $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations: $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$.

Or

If A = $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB.

Use this to solve the following system of equations :

$x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$

37. Find the distance of the point $3\hat{i} - 2\hat{j} + \hat{k}$ from the plane $3x + y - z + 2 = 0$ measured parallel to the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also, find the foot of the perpendicular from the given point upon the given plane.

Or

Find the equation of the plane through the point (4, −3, 2) and perpendicular to the line of intersection of the planes $x - y + 2z - 3 = 0$ and $2x - y - 3z = 0$. Find the point of intersection of the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ and the plane obtained above.

38. Solve the following graphically and also find the maximum profit.

Maximum Profit, $Z = 7.8x + 7.1y$

Subject to the constraints: $\frac{x}{4} + \frac{y}{3} \le 90$; $\frac{x}{8} + \frac{y}{3} \le 80$; $x \le 200$; $x \ge 0, y \ge 0$.

Or

Solve the following graphically and also find the maximum profit.

Maximum Profit, $Z = 50x + 60y$

Subject to the constraints: $20x + 10y \le 180$; $10x + 20y \le 120$; $10x + 30y \le 150$; $x \ge 0, y \ge 0$.