Series: CBSE / SGP/ 2021-03 (Solved)

Code No. **041**

Roll No. | | | | | | | |

- Please check that this question paper contains **4** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Guess Paper 3 (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

Same as CBSE Sample Question Paper.

PART A

Section I

All questions are compulsory. In case of internal choices attempt anyone.

1. Let A = {1, 2, 3}, B = {4, 5, 6} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not.

Or

If R = $\{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R.

2. What are the possible number of relations on A = {1, 2, 3}.

3. Using the principal values, evaluate: $\tan^{-1} 1 + \sin^{-1}\left(-\frac{1}{2}\right)$.

Or

Using principal values evaluate: $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\frac{\sin 2\pi}{3}\right)$.

4. If A is a matrix of order 2 × 3 and B is a matrix of order 3 × 5, what is the order of matrix (AB)′ or T?

5. If A is a square matrix of order 3 such that $|adj\ A| = 64$, find $|A'|$.

Or

If A is a square matrix satisfying $A^2 = I$, then what is the inverse of A?

6. If $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$, Write the cofactor of a_{32} (the element of third row and second column).

7. Find $\int \frac{3 + 3\cos x}{x + \sin x}\, dx$.

Or

Find $\int(\cos^2 2x - \sin^2 2x)dx$

8. Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dy}{dx} = 1.$

9. Write the order and degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$.

Or

Find the integral factor for $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0.$

10. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

11. If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{a}| = 22$, then find $|\vec{b}|$.

12. Give an example of vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$.

13. Find the acute angle which the line with direction cosines $\frac{1}{\sqrt{3}}, \frac{1}{6}, n$ makes with positive direction of z-axis.

14. Find the direction cosines of the line: $\frac{x-1}{2} = -y = \frac{z+1}{2}$.

15. If A and B are two independent events, prove that A′ and B are also independent.

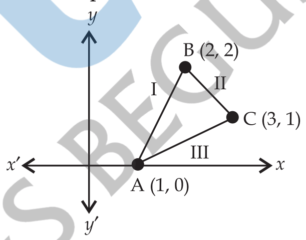
16. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red.

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question (17 – 21) and (22 – 26). Each question carries 1 mark.

17. Case Study—*Triangular Sandwich*

Top view of Sandwich



Answer the following the questions:

(*i*) Write the equation for line I in the form of x.

(*a*) $y = 2x + 1$ (*b*) $y = 2(x - 1)$ (*c*) $y = 2x + 2$ (*d*) $y = 3x + 1$

(*ii*) Write the equation for line II in the form of x.

(*a*) $y = 5 - x$ (*b*) $y = 3 - x$ (*c*) $y = 4 - x$ (*d*) $y = -x - 4$

(*iii*) Write the equation for line III in the form of x.

(*a*) $y = \frac{x+1}{3}$ (*b*) $y = x - 2$ (*c*) $y = \frac{x+1}{2}$ (*d*) $y = \frac{x-1}{2}$

(*iv*) Using integration, find the area of ΔABC.

(*a*) $\frac{3}{2}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{2}{3}$

(*v*) The relation between the lines are :

(*a*) parallel (*b*) perpendicular (*c*) Intersecting (*d*) None of the above

18. Case Study—A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The germination rate of three types of seeds are 45%, 60% and 35%.

Answer the following questions:

(*i*) The probability of random chosen seed to germinate:

(*a*) .69 (*b*) .39 (*c*) .49 (*d*) .59

(*ii*) The probability that seed will not germinate, given that seed is of type A_3

(*a*) $\frac{15}{100}$ (*b*) $\frac{65}{100}$ (*c*) $\frac{75}{100}$ (*d*) $\frac{55}{100}$

(*iii*) The probability that the seed is of type A_2 given that randomly chosen seed does not germinate.

(*a*) $\frac{22}{51}$ (*b*) $\frac{16}{51}$ (*c*) $\frac{20}{51}$ (*d*) $\frac{10}{51}$

(*iv*) Calculate the probability that it is of type A_1, given that randomly chosen seed does not germinate

(*a*) $\frac{51}{22}$ (*b*) $\frac{22}{51}$ (*c*) $\frac{16}{51}$ (*d*) $\frac{1}{51}$

(*v*) The probability that it will not germinate given that seed of type A_1:

(*a*) $\frac{55}{100}$ (*b*) $\frac{65}{100}$ (*c*) $\frac{35}{100}$ (*d*) $\frac{45}{100}$

PART B

Section III

19. Prove the following : $\cos [\tan^{-1} \{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$.

20. Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

Or

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = 5A + kI$.

21. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

22. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(*a*) strictly increasing (*b*) strictly decreasing

23. Evaluate : $\int \frac{1+\sin 2x}{1+\cos 2x} \cdot e^{2x}\, dx$.

Or

Evaluate: $\int \frac{\sin\phi}{\sqrt{\sin^2\phi + 2\cos\phi + 3}}\, d\phi$.

24. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

25. Find the shortest distance between the lines :

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \text{ and } \vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}).$$

26. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

27. Find the equation of the line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ passes through the point (1, 1, 1).

28. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the plan and variance of the distribution.

Or

A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Check whether the relation R in the set R of real numbers, defined by R = $\{(a, b) : 1 + ab > 0\}$, is reflexive, symmetric or transitive.

30. If the following function is differentiable at $x = 2$, then find the values of a and b, $f(x) = \begin{cases} x^2, & \text{if } x \le 2 \\ ax + b, & \text{if } x > 2 \end{cases}$

31. Let $y = (\log x)^x + x^{x \cos x}$, then find $\frac{dy}{dx}$.

Or

If $x = a \sin pt$, $y = b \cos pt$, then find $\frac{d^2y}{dx^2}$ at $t = 0$.

32. Find the equation of the normal to the curve $2y = x^2$, which passes through the point (2, 1).

33. Evaluate the following : $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

34. Compute, using integration, the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$

Or

Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

35. Solve the following differential equation, $(1 + y + x^2y)dx + (x + x^3)dy = 0$, where $y = 0$ when $x = 1$.

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$x + 3z = 9, -x + 2y - 2z = 4, \quad 2x - 3y + 4z = -3.$

Or

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations :

$x - y + 2z = 1; \quad 2y - 3z = 1; \quad 3x - 2y + 4z = 2.$

37. Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$.

Also, show that the plane contains the line $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \lambda(\hat{i} - 2\hat{j} - 5\hat{k})$

Or

Show that the lines $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$ are coplanar. Also, find the equation of the plane containing these lines.

38. Solve the following graphically and also find the maximum profit.

Maximum Profit, Z = $8000x + 12000y$

Subject to the constraints: $9x + 12y \le 180$; $3x + 4y \le 60$; $x \ge 0, y \ge 0$.

Or

Solve the following graphically and also find the maximum profit.

Maximum Profit, Z = $6x + 3y$

Subject to the constraints: $4x + y \ge 80$; $x + 5y \ge 0$; $3x + 2y \le 150$; $x \ge 0, y \ge 0$.