Series: CBSE / SGP/ 2021-01 (Solved) **Code No. 041**

Roll No. ☐☐☐☐☐☐☐

- Please check that this question paper contains **4** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII
Sample Guess Paper 1 (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

Same as CBSE Sample Question Paper.

PART A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. Write the smallest equivalence relation R on Set A = {1, 2, 3}.

Or

Give an example to show that the relation R in the set of natural numbers, defined by $R = \{(x, y), x, y \in \mathbf{N}, x \le y^2\}$ is not transitive.

2. Write the number of all one-one functions from the set A = {a, b, c} to itself.

3. Let A = {1, 2, 3, 4}. Let R be the equivalence relation on A × A defined by (a, b) R(c, d) if $a + d = b + c$. Find the equivalence class [(1, 3)].

Or

Prove that the function $f: \mathrm{N} \to \mathrm{N}$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

4. If $A = \begin{bmatrix} 4 & 6 \\ 7 & 5 \end{bmatrix}$, then what is A. ($adj$.A)?

5. For what value of k, the matrix $\begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$ is skew symmetric?

Or

If $\begin{vmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\beta \end{vmatrix} = \frac{1}{2}$, where α, β are acute angles, then write the value of α + β.

6. If A is a square matrix of order 3 such that $|adj\ A| = 225$, find $|A'|$.

7. If $\int_0^1 (3x^2 + 2x + k)\,dx = 0$, write the value of k.

Or

Evaluate : $\int \cot x\,(\text{cosec}\, x - 1)\, e^x\, dx$

8. Write the value of : $\int \left(\dfrac{dy}{dx}\right)^3 dx$

9. If m and n are the order and degree, respectively of the differential equation $y\left(\dfrac{dy}{dx}\right)^3 + x^3\left(\dfrac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of $m + n$.

Or

Find the integral factor of differential equation : $\sec x \dfrac{dy}{dx} - y = \sin x$.

10. If $|\vec{a}| = a$, then find the value of $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$.

11. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles 30° and 120° respectively, then write the value of $|\vec{a}+\vec{b}|$.

12. If $\vec{a}$ and $\vec{b}$ are two vector of magnitude 3 and $\dfrac{2}{3}$ respectively such that $\vec{a}\times\vec{b}$ is a unit vector, write the angle between $\vec{a}$ and $\vec{b}$.

13. Find the distance of the point (a, b, c) from x-axis.

14. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane.

15. A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively, are $\dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}$ and $\dfrac{2}{3}$. What is the probability that at most one of them will solve the problem?

16. If $P(A) = \dfrac{2}{5}$, $P(B) = \dfrac{1}{3}$, $P(A \cap B) = \dfrac{1}{5}$, then find $P(\overline{A}/\overline{B})$.

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub parts from each question (17 – 21) and (22 – 26). Each question carries 1 mark.

17. **Case Study—**Factories that used to make perfumes,T-shirts and cars now making supplies to fight against the Corona virus. Manufactures, fashion designers, and 3D printing companies are now making face masks, PPE kits, gloves, ventilators and hand sanitizers.

A factory has three machines I, II and III which produce 30%, 50%, 20% respectively of the total items of the same variety. Out of these 2%, 5% and 3% respectively are found to be defective.

Answer the following questions :

(*i*) Probability of production by the three machines is:

(*a*) $\dfrac{3}{10}, \dfrac{5}{10}, \dfrac{2}{10}$ (*b*) $\dfrac{3}{100}, \dfrac{5}{100}, \dfrac{2}{100}$ (*c*) $\dfrac{30}{10}, \dfrac{50}{10}, \dfrac{20}{10}$ (*d*) $\dfrac{0.3}{100}, \dfrac{0.5}{100}, \dfrac{0.2}{100}$

(*ii*) Probability of production of three machines if an item is picked and found to be defective is:

(*a*) $\dfrac{2}{10}, \dfrac{5}{10}, \dfrac{3}{10}$ (*b*) $\dfrac{0.2}{100}, \dfrac{0.5}{100}, \dfrac{0.3}{100}$ (*c*) $\dfrac{2}{100}, \dfrac{5}{100}, \dfrac{3}{100}$ (*d*) $\dfrac{20}{10}, \dfrac{50}{10}, \dfrac{30}{10}$

(*iii*) Probability that the defective item is produced by machine III is:

(*a*) $\dfrac{3}{37}$ (*b*) $\dfrac{6}{37}$ (*c*) $\dfrac{2}{37}$ (*d*) $\dfrac{10}{37}$

(*iv*) Which of the following would satisfy the condition that Machine I and II are independent events?

(*a*) $P(I \cap II) = P(I) . P(II)$ (*b*) $P(I \cup II) = P(I) . P(II)$
(*c*) $P(I' \cap II) = P(I) . P(II)$ (*d*) None of the above

(*v*) Suppose $P(II) = \frac{2}{6}$, $P(I) = \frac{4}{6}$, $P(I \cap II) = \frac{2}{6}$, find $P(I \mid II)$.

(*a*) 1 (*b*) 0 (*c*) $\frac{2}{6}$ (*d*) $\frac{4}{6}$

18. Case Study—The following image shows Big Bazaar in which seller sells x items at a price of ₹ $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is ₹ $\left(\frac{x}{5} + 500\right)$.

Answer the following questions:

(*i*) Profit function in the form of x is represented as:

(*a*) $\frac{x^2}{100} + \frac{24x}{5} - 500$ (*b*) $\frac{x^2}{100} - \frac{24x}{5} + 500$ (*c*) $\frac{24x}{5} + \frac{x^2}{100} + 500$ (*d*) $\frac{24x}{5} - \frac{x^2}{100} - 500$

(*ii*) If the owner wants to maximize the profit how many items should be required to sell by him?

(*a*) 120 items (*b*) 240 items (*c*) 200 items (*d*) 100 items

(*iii*) If the total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

(*a*) 208 (*b*) 280 (*c*) 802 (*d*) 820

(*iv*) Find the intervals in which the function f is given by $f(x) = x^2 - 4x + 6$ is strictly increasing.

(*a*) $(-2, \infty)$ (*b*) $(-\infty, -2)$ (*c*) $(2, \infty)$ (*d*) $(-\infty, 2)$

(*v*) Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

(*a*) 674 (*b*) 764 (*c*) 476 (*d*) 746

PART B

Section III

19. Simplify: $\cot^{-1} \frac{1}{\sqrt{x^2 - 1}}$ for $x < -1$.

20. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

Or

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$. Hence find A^{-1}.

21. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$, then show that $A^2 - 4A + 7I = O$. Using this result calculate A^3 also.

22. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

23. Evaluate : $\int \frac{x^3 + x + 1}{x^2 - 1} dx$.

Or

Evaluate : $\int e^x \frac{(1 - \sin x)}{(1 - \cos x)} dx$.

24. Compute, using integration, the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$

25. Find the particular solution of the differential equation :
$(x - \sin y)\, dy + (\tan y)\, dx = 0$, given that $y = 0$ when $x = 0$

26. Find a unit vector perpendicular to the plane of triangle ABC where the vertices are A(3, –1, 2), B (1, –1, –3) and C (4, –3, 1).

27. Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also find their point of intersection.

28. There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them. Write the probability distribution for the selected persons who are non-violent.

Or

A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Show that the relation R in the set $A = \{x : x \in Z, 0 \le x \le 12\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

30. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

31. Show that the function $g(x) = |x-2|, x \in R$, is continuous but not differentiable at $x = 2$.

Or

Differentiate $\log(x^{\sin x} + \cot^2 x)$ with respect to x.

32. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub-intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing.

33. Evaluate: $\int_{-1}^{1} \frac{x+|x|+1}{x^2+2|x|+1} dx$.

34. Using integration find the area of the region $\left[(x, y) : x^2 + y^2 \le 1 \le x + \frac{y}{2}, x, y \in R\right]$.

Or

Find the area of the region $\{(x, y) : y \ge x^2 \text{ and } y = |x|\}$.

35. Find the particular solution of the differential equation $\cos x\, dy = \sin x(\cos x - 2y)\, dx$, given that $y = 0$ when $x = \frac{\pi}{3}$.

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Using matrices, solve the following system of equations:

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$, $\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$, $x \ne 0, y \ne 0, z \ne 0$.

Or

If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -2 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations:

$3x + 4y + 7z = 14$, $2x - y + 3z = 4$, $x + 2y - 3z = 0$.

37. Find the equation of the plane through the points A(1, 1, 0), B(1, 2, 1), and C(–2, 2, –1) and hence find the distance between the plane and the line $\frac{x-6}{3} = \frac{y-3}{-1} = \frac{z+2}{1}$.

Or

Find the vector and Cartesian equations of the plane containing the two lines

$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$

38. Solve the LLP graphically and find the maximum profit.

Maximize Profit, $Z = 200x + 20y$

Subject to the constraints: $3x + y \le 600$; $x + y \le 300$; $x - y \le 100$; $x, y \ge 0$.

Or

Solve the LLP graphically.

Minimize, $Z = 2x + 3y$

Subject to the constraints: $2x + 3y \ge 6$; $x - y \ge 0$; $2x + y \le 8$; $x, y \ge 0$.
