A Theory of Justices’ Retirement

Álvaro Bustos Pontificia Universidad Católica de Chile School of Management, and Tonja Jacobi Northwestern University School of Law

Send correspondence to: Tonja Jacobi, Northwestern University, School of Law, Chicago, IL, USA; E-mail: t-jacobi@law.northwestern.edu

This paper introduces a formal model of Supreme Court retirement, in which the justices, the President and the Senate are rational agents who aim to shift the median ideology of the Court as close as possible to their own ideologies. The model shows that the probability of retirement depends on a set of personal, contextual, and political variables. It provides a rigorous theory for the effect of extant variables, and identifies variables that have not previously been fully appreciated. In particular, it shows the impact of the ideologies of the non-retiring justices and whether the ideology of the retiring justice is moderate or extreme. This more complete explanation of strategic judicial retirements raises empirically testable predictions to differentiate among the disparate findings of the existing literature. (JEL: K10, K30, K40)

1. Introduction

In order to comprehensively model strategic judicial behavior, a full understanding of the endgame of judicial behavior—judicial retirement—is essential. Yet law and economics’ modeling of the decision process of justices facing retirement is limited. There is a substantial empirical literature on the topic, but it shows some inconsistent results, and would benefit from formalized theory. Yet there are only two pre-existing formal models. One concludes that political configurations are not highly relevant to the decision to retire (Bailey and Yoon, 2011)—a conclusion we dispute. The other takes the decision to retire as given (Bustos and Jacobi, 2014b). This paper

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provides a comprehensive formal model of judicial retirement, including a number of significant variables that have previously been overlooked. In particular, it shows that the ideologies of non-retiring justices and the relative moderation or extremity of the retiring justice both impact the decision to retire. In addition, it reveals that a number of non-trivial predictions follow from the interaction of the ideologies of the President and the Senate, and that simple proximity to the nominating agents does not always yield a greater probability of retirement.

The strategic judicial behavior literature expects that rational justices should tailor their retirements to political cycles, so as to coincide with presidential administrations of similar political leanings. Why, then, many Court observers have asked, has Justice Ginsburg not resigned during the Obama administration (e.g. Chemerinsky, 2014)? In this study, we show that, although the presidential alignment theory has merit, the motivations for strategic retirement extend well beyond the ideology of the President, or even the Senate (Moraski and Shipan, 1999). We show that there will be many situations in which even when the ideology of the potentially retiring justice is aligned with either the President or the Senate, the justice will nevertheless have a low probability of retiring.

The retirement decision centrally depends on the probability of “forced retirement”—the possibility of an exit being driven by exogenous factors, such as age, death, or ill health. We show that the choice of retirement depends on whether this probability is larger than a certain threshold, a ratio of the ideological distance between the current median of the Court and the median of the court if the justice retires in the first period, over the distance between the current median and the expected median of the Court if the justice retires in the next period. From this relationship, numerous conclusions follow, many of them testable.

First, we can fine-tune the predictions of when variation in the ideologies of the President and the Senate impact the decision to retire. Our model shows that from two potentially retiring justices, the one with an ideology farther away from the ideology of the President may in fact have a higher probability of retirement. The reason is that the nomination game played by the President and the Senate to fill the vacancy may be different for the two

1. At least in the modern context of retirement benefits (see Vining et al., 2006).
retiring justices, and ergo define different expected Court medians. In addition, when the nominating agents are moderately positioned, justices will retire more often when the President is close to the current Court median, or the Senate is closer to the midpoint between the current Court median and the default Court median. Whereas when the nominating agents are more extreme, marginal changes in their ideologies do not affect the probability of retirement.

Second, our model predicts systematic differences in the retirement decisions made by extreme and moderate justices. Conservative justices retire more (less) than moderate justices when the ideology of the Senate and the Court both are conservative (liberal) enough. The reason for this prediction is that when the Senate and the Court are conservative (liberal) enough, the nomination games always define medians of the Court that are more conservative (liberal) if the conservative justice, rather than the liberal justice, retires. The surprising part of this result is that it holds for all possible ideologies of the President.

Finally, our predictions also allow for clarification of the effect of the ideologies of the non-retiring justices on the retirement decisions of their colleagues. More specifically, retirement is more likely when the ideologies of all the justices are closer to each other. Also, conservative (liberal) justices are more likely to retire than moderate justices when the ideology of the median of the Court is liberal (conservative) enough. Both predictions originate from the fact that, after retirement, the President and the Senate are more likely to set a new median of the Court that is closer to the current median when the Court is concentrated or the ideologies of the retiring justice and the median are opposed.

Part 2 reviews the existing literature. Part 3 sets up the model. Part 4 provides the solution. Part 5 sets out the results and draws testable hypotheses. Part 6 provides discussion of possible extensions and the effect of relaxing some of our assumptions, and Part 7 concludes.

2. Literature Review

Even early narrative biographical accounts of judicial retirement preceding any judicial strategy literature recognized the strategic attempts of justices to structure their retirement around the ideology of their expected
replacement (Schmidhauser, 1962, p. 127). Yet the largely empirical literature that followed has been conflicted on whether political variables are significant predictors of judicial retirement. At the Supreme Court level, Brenner (1999), Squire (1988), and Yoon (2006) found no evidence of a pattern of politicized departure of justices, but Hagle (1993) and King (1987) found political effects on departures from the Court. Yet others reported mixed results (e.g. Zorn and Van Winkle, 2000). Stolzenberg and Lindgren (2010, p. 291) argue that the mixed results were a product of methodological problems, and they comprehensively showed that political factors are highly significant, a result that has been echoed consistently in studies of the lower federal courts (Barrow and Zuk, 1990; Spriggs and Wahlbeck, 1995; Barrow et al., 1996; Nixon and Haskin, 2000; Choi et al., 2013).

The existing literature has not coalesced on an agreement of which other variables should be included in the analysis, or why. For instance, tests are often included for the significance of each year of a president’s term: one study found that retirements occur more during the beginning of each president’s term (Hagle, 1993), whereas others found evidence of more retirements occurring only at the beginning of the President’s second term, but found weak evidence on retirements being driven by political factors (Zorn and Van Winkle, 2000). Perhaps more consistent results, or a method of sorting through the different conclusions, could be achieved with a clear theory for including timing within an administration’s term. If timing constitutes an extension of the political effects thesis, then we need to query results that combine a significant timing effect while disputing a political effect.

Despite the centrality of the concept of relative ideological positioning of the political agents in models of judicial behavior, there is little consideration of the effect of whether a justice is positioned extremely or moderately when assessing justices’ retirement strategies. This may be because much of the literature arose before reliable cross-institutional measures of ideology existed (see Bailey, 2007). We find that a justice’s retirement propensity depends on a number of factors, most importantly their own relative ideological extremity. The only pre-existing formal model of the judicial decision to retire is Bailey and Yoon (2011). It focuses on the effect of retirements on the position of the median of the Court, and finds that political considerations are not highly relevant. However, we allow for variation in
the ideologies of the President and Senate beyond the discrete categories of Republican and Democrat, and we consider all of the possible relative arrays between the potential retiree, the Court median, the President and the Senate. When all of these factors are included, the effect of political alignment becomes clear.

Although a more comprehensive model may not fully resolve these disputes, the contradictory results arising from existing analyses indicates that there is benefit to be gained from developing greater theoretical scaffolding for this inquiry.

3. The Model

3.1. Players

Suppose that the Court has three Justices, \( \{ J_1, J_2, J_3 \} \), and operates for two periods \( t \in \{ 1, 2 \} \). We assume that justice \( J_r \) ("r" for retiring) is considering retirement at the beginning of the first period. The other justices do not retire ("r−" for not retiring in which \( r^- \in \{ 1, 2, 3 \} \backslash \{ r \} \))—in Section 6.1, we discuss the possibility of multiple potentially retiring justices. If \( J_r \) does not retire during the first period, then she faces probability \( p \) of involuntary retirement at the beginning of the second period due to exogenous reasons (e.g. death or health issues). If \( J_r \) does not retire due to exogenous factors, then she faces a second and final opportunity to freely choose retirement.

Whenever a justice dies or retires, the President, \( P \), and the Senate, \( S \), play a one-period game, as in Moraski and Shipan (1999, hereinafter “MS”), to fill the vacancy. That is, \( P \) proposes a candidate and \( S \) confirms or rejects the nominee. If \( S \) confirms, the nominee becomes the new justice; if \( S \) rejects, then the seat remains vacant and the Court keeps the default median, constituted by the mid-point of the two remaining justices. In order to incorporate the possibility of a change of a political cycle, in period 1 the President and the Senate are \( P_1 \) and \( S_1 \), respectively, and in period 2 they are \( P_2 \) and \( S_2 \).

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2. Bailey and Yoon also use a Moraski and Shipan model, but only consider the unconstrained game described below.

3. We do not consider the probability that \( J_r \) may have to retire for exogenous reasons at the beginning of the first period because in that case the Justice does not face a retirement decision in the first and/or second periods anymore.
3.2. Players’ Preferences and Payoffs

Justice $J_g$ has ideology $\alpha_g \in [0, 1]$. The closer $\alpha_g$ is to 1, the more conservative the justice is. In order to identify a liberal, a moderate, and a conservative justice, we consider that $0 < \alpha_1 < \alpha_2 < \alpha_3 < 1$. That is, $J_1$ is the liberal justice, $J_2$ is the moderate justice, and $J_3$ is the conservative justice. All justices’ ideologies are known. We denote the ideology of $P_t$ as $\alpha_{P1} \in [0, 1]$ and the ideology of the $S_t$ as $\alpha_{S1} \in [0, 1]$. While $\alpha_{P1}$ and $\alpha_{S1}$ are known, the values of $\alpha_{P2}$ and $\alpha_{S2}$ are distributed according to density functions $f(x)$ and $g(x)$, respectively, in which $F(\alpha) = \int_0^\alpha f(x) \, dx$ and $G(\alpha) = \int_0^\alpha g(x) \, dx$. We do not impose restrictions on the values of $\alpha_{P1}$ and $\alpha_{S1}$ or in their relative position. At this point, we introduce an important assumption; we address the effect of relaxing this assumption in Section 6.2.

Assumption 1 Ideologies of the President and Senate in the second political cycle are independent of the ideologies of the President and Senate in the first political cycle.

The only agents that make decisions are: $J_r$, $P_t$, and $S_t$. For all players, we assume that their goal is to minimize the distance between their own ideologies and the expected median of the Court. We consider that all the agents care about the long-term median of the Court. There are just three possible scenarios for the long-term median. First, if $J_r$ does not retire in period 1 or period 2, the median remains at $\alpha_2$. If $J_r$ retires in period 1, the median becomes $m_r$ and if she retires at period 2, the median becomes $M_r$. Both $m_r$ and $M_r$ will be estimated in the solution of the model. The discount factor is 1.

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4. The term “moderate” refers to the justice’s position relative to the rest of the Court, not to any exogenous political reference point.

5. That occurs as long as the justices have been with the Court long enough (see Bustos and Jacobi, 2014a).

6. That means that the President can be more liberal than the Senate, or vice versa.

7. Without this assumption, the probability distribution of the ideologies of $P_2$ and $S_2$ will be conditional on $P_1$ and $S_1$.

8. In our model, the game ends with a retirement. Hence the nomination game played by $P_t$ and $S_t$ considers a one period horizon. If that was not the case, then the nomination game a la Moraski and Shipan would be more complicated, as P and S would have to consider the possibility of future retirements.
3.3. Timing of Actions

Figure 1 summarizes the sequence of events and decisions described above.

3.4. Nomination Games

Because it will clarify the rest of the paper, here we explain the logic of the MS nomination games played by P and S when a Court vacancy occurs at the end of the first period. Without loss of generality, we assume that $J_3$ is the retiring justice and $\alpha_{P_1} < \alpha_1$.\(^9\)

*P and S have opposed ideologies* ("FC," for a fully constrained president): if $\alpha_{P_1} < (\alpha_1 + \alpha_2)/2 < \alpha_{S_1}$, the President nominates a new justice with ideology $(\alpha_1 + \alpha_2)/2$, who is confirmed by the Senate. Even if the President would like to nominate a new justice more liberal than $J_1$, so that $J_1$ becomes the new median, any justice with ideology farther left than $(\alpha_1 + \alpha_2)/2$ will be rejected by the Senate, since that is the default in case of no agreement.

*P and S have semi-opposed ideologies* ("SC" for semi-constrained): if $\alpha_{P_1} < (3\alpha_1 + \alpha_2)/4 < \alpha_{S_1} < (\alpha_1 + \alpha_2)/2$, the President nominates a new

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9. The analysis of all the other cases is analogous. In the proof of Proposition 1, we cover all of them.
justice with ideology $2\alpha_{S_1} - (\alpha_1 + \alpha_2)/2$, who is confirmed by the Senate. Again the President would like to appoint a new justice who is more liberal, but any nominee whose ideology lies at a distance from the Senate’s ideology larger than the distance of the Senate’s ideology from the default median will be rejected by the Senate. Then if we denote the ideology of the new nominee as $\alpha_N$, the Senate will reject any nominee where $\alpha_N - \alpha_{S_1} > \alpha_{S_1} - (\alpha_1 + \alpha_2)/2$. In other words, the most liberal new nominee that the Senate is willing to confirm is $\alpha_N = 2\alpha_{S_1} - (\alpha_1 + \alpha_2)/2$.

$P$ and $S$ have aligned ideologies (“UC” for unconstrained): If $\max\{\alpha_{P_1}, \alpha_{S_1}\} < (3\alpha_1 + \alpha_2)/4$, the President is free to nominate a new justice with ideology $\alpha_{P_1}$, who is confirmed by the Senate, because then the new median of the Court is $\alpha_1$. The Senate prefers $\alpha_1$ to $(\alpha_1 + \alpha_2)/2$.

4. Solution of the Model

In this section, we characterize $J_r$’s optimal retiring strategy. First, it is clear that she never retires during the second period. The reason is that regardless of whether she is a moderate or an extreme justice (liberal or conservative), if she retires, she leaves the Court with a median who is at best in the same position as if she does not retire, and in the most likely case she leaves the Court farther from her ideology.

The characterization of the retirement strategy at the beginning of the first period requires a distinction in the ideology of the retiring justice. We focus in the cases in which $J_r$ is the conservative or the moderate justice. The liberal case is analogous to the conservative case.

The retiring Justice is Conservative ($r = 3$):

A conservative justice prefers to retire at $t = 1$ if the following inequality holds:

$$\alpha_3 - m_3 < \alpha_3 - ((1 - p)\alpha_2 + pM_3).$$

10. If she is the moderate justice, then her ideology is the median of the Court, hence a new median of the Court can never be closer to her own ideology (the best that can happen is that the new median will be $(\alpha_1 + \alpha_3)/2 = \alpha_2$, but that is a particular case). If the new justice is conservative, then the new median of the Court will be a value within the interval $[\alpha_1, \alpha_2]$, which cannot be closer to $\alpha_3$ than $\alpha_2$. Finally, if the new justice is liberal, then the new median of the Court will be a value within the interval $[\alpha_2, \alpha_3]$, which cannot be closer to $\alpha_1$ than $\alpha_2$. 
The left-hand side of (1) tells us that if J retires in the first period, then the Court will be left with a median equal to \( m_3 \). As is shown in Figure 2, the values of \( m_3 \) (when \( r = 3 \) and \( r^− = 2 \) in the figure) depend on the ideologies of \( P_1 \) and \( S_1 \). For example, if both the President and the Senate are conservative enough (\( \alpha_{P_1} > \alpha_2 \) and \( \alpha_{S_1} > (\alpha_1 + 3\alpha_2)/4 \)), then \( m_3 = \alpha_2 \), but if both are liberal enough (\( \alpha_{P_1} < \alpha_1 \) and \( \alpha_{S_1} < \alpha_1 \)), then \( m_3 = \alpha_1 \).

In addition, the right-hand side of (1) tells us that in the second period the justice faces the possibility of a forced retirement (probability \( p \)). In that case, the median of the Court will be \( M_3 \); this is an expected value that is conditional on the expected ideologies of the President and the Senate in the second period as well. In the Appendix, we write \( M_3 \) explicitly; for now, we continue our analysis keeping in mind that \( M_3 \in [\alpha_1, \alpha_2] \). Given that we know that J3 does not freely retire in the second period, in case of no forced retirement (probability \( 1 - p \)), the median of the Court remains as \( \alpha_2 \).

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11. Applying the logic explained in Section 3.4.
With all these considerations in mind, we determine that a conservative justice retires at the beginning of the first period if and only if

\[ p > \tilde{p} = \frac{\alpha_2 - m_3}{\alpha_2 - M_3} \]

We discuss the intuition behind \( \tilde{p} \) after we state Proposition 1.

**The retiring Justice is Moderate (r = 2):**

Following the same steps as the conservative case, it follows that justice \( J_2 \) prefers to retire at \( t = 1 \) if the following inequality holds:

\[ |\alpha_2 - m_2| < |\alpha_2 - ((1 - p)\alpha_2 + pM_2)|. \]  \hspace{1cm} (2)

Figure 2 shows the possible values of \( m_2 \) when \( r = 2 \) and \( r^- = 3 \). As in (1), inequality (2) tells us that in the second period the justice faces the possibility of a forced retirement, in which case the median of the Court will be \( M_2 \). Also in Appendix, we write \( M_2 \) explicitly and show that \( M_2 \in [\alpha_1, \alpha_3] \). Hence it follows that a moderate justice retires at the beginning of the first period if and only if

\[ p > \tilde{p} = \frac{|\alpha_2 - m_2|}{|\alpha_2 - M_2|}. \]

Although it is still the case that the values of \( \tilde{p} \) can range from 0 to 1, the conditions in which \( \tilde{p} = 0 \) or \( \tilde{p} = 1 \) occur are very different in the scenario where the retiring justice is the conservative justice than when she is the moderate justice. We leave that discussion for the next section. Here, we summarize the properties of the solution in Proposition 1.

**Proposition 1 (Retirement Strategy)** If the retiring justice is \( J_r \), then she retires at the beginning of the first period if and only if her exogenous probability of retirement is larger than

\[ \tilde{p}_r = \frac{|\alpha_2 - m_r|}{|\alpha_2 - M_r|} \]

in which the value of \( m_2 \) and \( m_3 \) are conditional on \( P_1 \) and \( S_1 \) and are defined by Figure 2, and the values of \( M_2 \) and \( M_3 \) are conditional on the expected values of \( P_2 \) and \( S_2 \), and are calculated in Appendix.

**Proof.** For the proof, see Appendix. \( \square \)
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Figure 3. Value of $\bar{p}$ if J$_3$ Considers Retirement at $t = 1$.

Figures 3 and 4a and b summarize possible values of $\bar{p}$ in the plane $(\alpha_P, \alpha_S)$. Note that in the figures we treat any value of $\bar{p}$ larger than 1 as 1. Also note that if we do not know $p$ but we expect it to be uniformly distributed in $[0,1]$, then $1 - \bar{p}$ becomes the expected probability with which J$_r$ retires.

The intuition behind Proposition 1 is as follows. We know that if a forced retirement was not possible, J$_r$ would prefer not to retire. The reason is that $|\alpha_r - \alpha_2|$, which is the disutility that J$_r$ suffers from the ideology of the median of the Court being different from her own ideology, is always smaller than or equal to $|\alpha_r - m_r|$ or $|\alpha_r - M_r|$. These two last expressions correspond to the disutility associated with the ideological difference from the median of the Court that J$_r$ suffers when she retires in the current or next period, respectively. Hence, the decision to retire in the current period is a

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12. $|\alpha_r - m_r|$ and $|\alpha_r - M_r|$ provide a measure of how much the expected Court decisions change with a retirement. For example, these distances can be interpreted as the fraction of cases in which the Court makes a liberal instead of conservative decision.
Figure 4. (a) Value of $\bar{p}$ if $J_2$ Considers Retirement at $t = 1$ when $\frac{\alpha_1 + \alpha_3}{2} < \alpha_2$.
(b) Value of $\bar{p}$ if $J_2$ Considers Retirement at $t = 1$ when $\frac{\alpha_1 + \alpha_3}{2} > \alpha_2$. 
Table 1. Possible Expected Medians if Retirement happens in the First Period

<table>
<thead>
<tr>
<th>Nomination game</th>
<th>Values of α_P and α_S</th>
<th>Possible values of m_r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>α_P and α_S are both extremely small or extremely large*</td>
<td>α_1, α_2, α_3</td>
</tr>
<tr>
<td>Unconstrained</td>
<td>α_P is moderate**</td>
<td>α_P</td>
</tr>
<tr>
<td>Semi-constrained</td>
<td>α_S is moderate***</td>
<td>2α_S - (α_1 + α_2)/2, 2α_S - (α_1 + α_3)/2</td>
</tr>
<tr>
<td>Fully-constrained</td>
<td>α_P is extremely large and α_S is extremely small or vice versa****</td>
<td>(α_1 + α_2)/2, (α_1 + α_3)/2</td>
</tr>
</tbody>
</table>

In Figure 2:* areas in which m_r ∈ {α, α_r−}; ** areas in which m_r = α_p; *** areas in which m_r = 2α_S - (α_1 + α_2 - α_r)/2; **** areas in which m_r = (α_1 + α_2 - α_r)/2.

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decision motivated to avoid the potentially larger reduction in utility associated with a forced retirement in the next period.13

Note that \( \bar{P} \) does not directly depend on the ideology of the retiring justice because it is the median of the Court, and not the ideology of the retiring justice, that determines the decisions of the Court. Hence it is not safe to assert that the closer the ideology of the President is to the ideology of a justice, the more likely that this justice is to retire.

Our capacity to make predictions about \( \bar{P}_r \) based on \( M_r \) is limited by the assumptions we impose over \( f(x) \) and \( g(x) \). We can be more explicit about predictions about \( \bar{P}_r \) originating on \( m_r \). Table 1 summarizes the possible values of \( m_r \) conditional on the nomination game played by the President and the Senate.

It follows that the likelihood with which \( \bar{P}_r = 0 \) (or equivalently \( m_r = \alpha_2 \)) centrally depends on three elements: first, the ideologies of the President and the Senate; second, the ideology of the retiring justice; third, the ideologies of the other members of the Court. In the next section, we characterize these connections systematically.

(13) In other words, J_r either retires in the current period and suffers disutility equal to |α_2 - m_r|, or does not retire but faces the possibility of a forced retirement in the next period and suffers disutility equal to |α_2 - M_r|. If |α_2 - m_r| < |α_2 - M_r| and the probability of a forced retirement in the next period is large enough, then J_r retires immediately. But if |α_2 - m_r| > |α_2 - M_r|, there is no point in retiring now as the “harm” is smaller if retirement takes place in the second period.
5. Results

In this section, we present our main results. In Proposition 2 we formally prove them. At the end of the section we derive testable predictions.

Overview

Our main results originate from a common logic: by changing the ideologies of the agents of the model we might change the nomination game that P and S play to fill any vacancy in the Court. Because the properties of the expected median of the Court are different in UC, FC, and SC games, we find that the probabilities of retirement (and their properties) can be very different in scenarios in which only one of the ideologies of the agents is different. For example, in some nomination games, the expected median of the Court explicitly depends on the ideologies of P and S, while in the others that is not true. In addition, while in some nomination games the expected median of the Court moves to the right when the ideologies of the members of the Court go up, in the other games the effect is exactly the opposite.

Our first main result is that the probability of retirement does not change with the ideology of the President or the Senate when any of these agents have an extreme ideology, but the same probability might go up or down with changes in the ideologies of P or S when these ideologies are moderate. The reason is that in the first scenario, the expected median of the Court in the case of a retirement is a constant, independent of the ideology of P or S, but in the second scenario, it is a function of these ideologies.

The second main result is that conservative justices retire more (less) than moderate justices when the ideology of the Senate and the Court both are conservative (liberal) enough, regardless of the ideology of the President.14 Here we have to consider that when the ideology of the Senate is extreme, then only the UC or FC nomination games are possible. The relevance of that observation is that under these two nomination games, the expected median of the Court is more to the right when the moderate retires than when the conservative does, regardless of the ideology of the President. That is not the case under the SC nomination game; there the inequality

14. This is a striking result because ordinarily the literature assumes that retirement is more likely as a product of proximity in ideologies between the retiring justice and the President.
reverts and that is why the testable result does not hold when the Senate is moderate.\footnote{When the ideology of the Senate is moderate, the nomination game can be UC, FC, or SC.}

The third main result relates to the importance of the ideology of the other members of the Court. Here, we can abstract from the ideologies of the President and Senate, and notice that for all nomination games, the closer the ideologies of the members of the Court are to one another, the larger is the probability of retirement (both for the moderate and the conservative justice). That is because the more tightly clustered the judicial ideologies, the more likely that the expected median will be equal to the current median. However, whereas in the case of the moderate justice potentially retiring, both the liberal and the conservative justice have to be closer to the moderate justice to increase the retirement probability, in contrast in the case of the conservative justice potentially retiring, only the liberal has to be closer to the moderate. Hence, if the median of the Court is liberal enough, in expected value, the probability of retirement for the conservative justice has to be larger than the probability of retirement for the moderate.

5.1. Current Ideologies of the President and the Senate

Retirement decisions depend on the ideology of the President and the ideology of the Senate. It is not enough to consider only the ideology of one actor. We discuss that fact at three levels.

First, our model predicts that when the President and Senate both have extreme ideologies, the probability of judicial retirement does not vary with the ideology of P or S.\footnote{These are the four corner areas in Figures 3 and 4. Note that the ideologies do not need to be aligned. Proof of Proposition 2 provides a formal definition of extreme ideologies.} From Table 1, this occurs when P and S play a FC nomination game or a UC nomination game (P and S being extreme). Then the expected median of the Court, and ergo \(\bar{p}\), are constants independent of \(\alpha_P\) or \(\alpha_S\). However, when at least one of P or S has moderate ideology, the marginal effect of a change in their ideologies over the retirement probability is not zero.\footnote{The marginal effect can be positive or negative. These are the intermediate areas in Figures 3 and 4. A formal definition of moderate is in the proof of Proposition 2.} To see this, also from Table 1, the expected median depends
on $\alpha_P$ or $\alpha_S$ when P and S play a SC nomination game or a UC nomination game (P or S being moderates).

The intuition of this result is connected to the relative bargaining power that P and S have during the corresponding nomination game. When both agents have extreme and opposed ideologies, they negate each other and make the default median the new median of the Court, hence none of the agents impose its preferred ideology. But when the ideology of the Senate or the President is moderate enough, the moderate side can partially or totally impose its preferences upon the new median of the Court.18

Second, when P or S has a moderate ideology, the closer $\alpha_P$ is to $\alpha_2$ or the closer $\alpha_S$ is to $\alpha_2/2 + (\alpha_1 + \alpha_r)/4$, the more likely it will be that $J_r$ decides to retire (the smaller the value of $\bar{p}_r$). Figures 3 and 4 indicate these scenarios with the white areas. A natural question is: why is it not the case that retirement is more likely when $\alpha_S$ is closer to $\alpha_2/2 + (\alpha_1 + \alpha_r)/4$? The answer lies in the logic of the SC nomination game: in a SC nomination game, the President can counter-balance the capacity of the Senate to block the new nominee and set a median equal to the default median, which is $(\alpha_1 + \alpha_r)/2$. Hence the ideology of the Senate that induces the President to nominate a justice with ideology $\alpha_2$ is the one that mirrors $\alpha_2$ when the new median is $2\alpha_S - (\alpha_1 + \alpha_r)/2$, which is $\alpha_S = \alpha_2/2 + (\alpha_1 + \alpha_r)/4$. This implies that retirement with certainty for the moderate justice ($\bar{p}_2 = 0$) only occurs if the ideology of the Senate is more conservative than the current median when $\alpha_2 > (\alpha_1 + \alpha_3)/2$ (Figure 4a), but retirement with certainty only takes place if the ideology of the Senate is more liberal than the current median when $\alpha_2 < (\alpha_1 + \alpha_3)/2$ (Figure 4b).19

Third, when we fix the ideology of one of the agents (P or S), it follows that the variance of the probability of retirement increases the farther away the ideology of the other agent is from $(\alpha_1 + \alpha_r)/2$. For example, when $\alpha_P = (\alpha_1 + \alpha_2)/2$, the threshold probability for $J_3$ to retire in the current period is $(\alpha_2 - \alpha_1)/(2(\alpha_2 - M_2))$ for any possible ideology of the Senate,

18. For example: $m_3 = \alpha_P$ when $\alpha_S < (3\alpha_1 + \alpha_3)/4$ and $\alpha_P \in [\alpha_1, (\alpha_1 + \alpha_3)/2]$ (the Senate is extremely liberal and the President is moderately liberal). As the ideology of the President lies within $[\alpha_1, \alpha_2]$ and the Senate accepts any candidate more liberal than $(\alpha_1 + \alpha_3)/2$, the President can impose his own ideology to the Court.

19. It is enough to notice that $\alpha_2/2 + (\alpha_1 + \alpha_3)/4 > (\alpha_1 + \alpha_3)/2$ when $\alpha_2 > (\alpha_1 + \alpha_3)/2$ but the opposite is true when $\alpha_2 < (\alpha_1 + \alpha_3)/2$. 


but if $\alpha_P > \alpha_2$, then that same threshold for $J_2$ can take any value from 0 to $(\alpha_2 - \alpha_1)/(2(\alpha_2 - M_2))$, depending on the value of $\alpha_S$. This means that retirement becomes much more unpredictable when the ideologies of the President and the Senate are more extreme. Note that here the key value is not $\alpha_2$ but $(\alpha_1 + \alpha_r - \epsilon)/2$, because only this last ideology defines the same $m_r$ for all nomination games. This result further illustrates that retirement predictions depend on all of the agents, not only on $\alpha_r$ and/or $\alpha_P$. There are examples in which the same $\alpha_P$ but a different $\alpha_S$ generates different retirement probabilities, and a smaller $|\alpha_P - \alpha_r|$ does not imply that $J_r$ retires with a larger probability.\(^{20}\)

5.2. Ideology of the Retiring Justice

When we vary the ideology of the retiring justice, many of the results above still hold, but there are some important differences between the decisions made by a retiring justice with an extreme or a moderate ideology.\(^{21}\) In terms of similarities, an inspection of Figures 3 and 4 shows that in both cases $\bar{p}$ can take values that range from 0 to 1. Those values occur for different ideologies of the President and the Senate but any value in the interval $[0,1]$ is feasible, regardless of whether the retiring justice is moderate or extreme. In addition, whether we are dealing with an extreme or a moderate justice, in both cases the probability of retirement reduces the closer $m_r$ is to $M_r$. That is, the closer are the expected ideologies of the medians of the Court in the current and new political cycles, the more likely it is that the justice will decide not to retire (the closer $\bar{p}$ is to 1).

\(^{20}\) Consider a scenario 1 in which $(\alpha_P, \alpha_S) = (\alpha_2 - \epsilon/2, (\alpha_1 + \alpha_3)/2 - \mu)$ and a scenario 2 in which $(\alpha_P, \alpha_S) = (\alpha_2 - \epsilon, (\alpha_1 + 3\alpha_3)/4 + \mu)$. Also, the moderate justice is the retiring justice and $0 < \epsilon < (\alpha_2 - (\alpha_1 + \alpha_3)/2)/2$ together with $0 < \mu$. Then, even when in scenario 1 the ideology of the retiring justice is closer to the ideology of $P$ than in scenario 2, nevertheless in the first scenario $\bar{p} = (|\alpha_2 - (\alpha_1 + \alpha_3)| + |\alpha_2 - M_2|)/2|\alpha_2 - M_2|$, which is larger than $\bar{p} = \epsilon/|\alpha_2 - M_2| < |\alpha_2 - (\alpha_1 + \alpha_3)/2|/2|\alpha_2 - M_2|$ from the second scenario.

\(^{21}\) The terminology of extreme and moderate refers to a given Court in the sense that for example the moderate justice could be the conservative or the liberal justice in a different Court. Also the terminology of extreme and moderate must be understood in relative terms. For example, the moderate justice could have an extreme ideology (measured in absolute terms) but because another member of the Court is even more extreme, the first mentioned justice lies in the Court’s middle and ergo plays the role of the moderate justice.
Figure 5. Comparing Retirement Probabilities ($\bar{p}_2$ and $\bar{p}_3$).

The retirement decisions of a moderate and extreme justice also show important differences. Figure 5 identifies the regions in the plane ($\alpha_p$, $\alpha_S$) in which: the moderate justice always retires with a larger or equal probability than the extreme justice (region A); the conservative justice always retires with a larger or equal probability than the moderate justice (region B); and the probability could go either way depending on the comparison of $\alpha_2$ with $(M_2 + M_3)/2$ (regions C and D).

Region A: $\bar{p}_2 \leq \bar{p}_3$

Region B: $\bar{p}_3 \leq \bar{p}_2$

Region C: If $M_2 > \alpha_2$ and $\alpha_2 \leq \frac{M_2 + M_3}{2}$ then $\bar{p}_2 \leq \bar{p}_3$.
If $M_2 < \alpha_2$ or $\alpha_2 > \frac{M_2 + M_3}{2}$ then $\bar{p}_2 \geq \bar{p}_3$

Region D: If $M_2 > \alpha_2$ and $\alpha_2 \leq \frac{M_2 + M_3}{2}$ then $\bar{p}_3 \leq \bar{p}_2$.
If $M_2 < \alpha_2$ or $\alpha_2 > \frac{M_2 + M_3}{2}$ then $\bar{p}_3 \geq \bar{p}_3$
the current period when P and S are extreme liberals. But for the moderate justice, retirement in the first period is possible when P and S are extreme liberals. The moderate justice retires if he expects that the second period new median will be far enough from the current median.22

Region B: In contrast, the median of the Court becomes $\alpha_r$ if both P and S are extreme conservatives and a retirement takes place in the current period. For the conservative justice, a first period retirement always dominates a second period retirement because in this latter case, the expected median moves to the left of $\alpha_2$. Then, $J_3$ always retires in the current period. But for the moderate justice, given conservative nominating agents, retirement in the first period is worse than in the second period if the second period new median is close to the current median.23

Regions C and D: The comparison of probabilities critically depends on whether $\alpha_2 < (M_2 + M_3)/2$ holds. That inequality tells us that the retiring justice expects that the median of the Court will move to the right in case of a retirement.24 Then, in regions in which the appeal of retirement for the conservative justice is smaller than it is for the moderate justice ($\alpha_2 - m_3 > |\alpha_2 - m_2|$ which is the case for region C), the probability that the moderate justice retires is larger than the probability that the conservative does. But if $M_2 > \alpha_2$ and $(M_2 + M_3)/2 < \alpha_2$, we cannot conclude within region C whether $\bar{p}_2$ or $\bar{p}_3$ is larger. Predictions reverse when we are inside region D because there $\alpha_2 - m_3 < |\alpha_2 - m_2|$.25

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22. If $(\alpha_2 - \alpha_1)/|\alpha_2 - M_2| < 1$, then retirement in the first period takes place if $p$ is large enough. In other words, no retirement with certainty ($\bar{p} = 1$) is a likely outcome for extreme justices (a priori that probability is $\alpha_1 (3\alpha_1 + \alpha_2)/4$), whereas for moderate justices that outcome depends on $M_2$.

23. Hence, whereas for a conservative justice retirement with certainty ($\bar{p} = 0$) happens with a strictly positive probability (a priori that probability is $(1 - \alpha_2) (1 - (\alpha_1 + 3\alpha_2)/4)$, for a moderate justice that same decision, a priori, takes place with probability 0 (more specifically, it only occurs in two particular cases, which are: (1) when $\alpha_P = \alpha_2$ and $\alpha_S > \alpha_2$; (2) when $\alpha_P = (\alpha_2 + (\alpha_1 + \alpha_3)/2)/2$ and $\alpha_P > \alpha_S$—in which we are considering the case that $\alpha_2 > (\alpha_1 + \alpha_3)/2$).

24. Actually, the median always moves to the left when the conservative justice retires (because $M_3 \leq \alpha_2$) but the expected move to the right in the case that the moderate justice retires is so big that in expected value (for any retirement) the median moves to the right.

25. Once more the implications for a SC nomination game are different than the ones for a FC or UC game.
Implications from Figure 5. Despite this variability, we can draw an important implication from the analysis of regions A–D: with some minor conditions on M, regardless what the ideology of P is, we can predict whether the extreme or the moderate justice is more likely to retire, as long as the ideology of S is extreme enough.\textsuperscript{26} It may not seem surprising that the probability of retirement for the conservative justice is larger than the same for the moderate justice when the President is very conservative, but it is surprising to make that same prediction when the President is very liberal; all that is needed is that the Senate is conservative enough.

To better understand the previous result, we say that “retirement of justice $i$” stochastically dominates “retirement of justice $j$” over the domain $[0, \alpha]$ conditional on $\alpha_{a_2}$ when

$$E[\tilde{\rho}_i(\alpha_{a_1} = \alpha)|\alpha_{a_2}] < E[\tilde{\rho}_j(\alpha_{a_1} = \alpha)|\alpha_{a_2}],$$

in which $\{i, j\} \in \{2, 3\}; \{a_1, a_2\} \in \{P, S\}$ and $\alpha_P, \alpha_S$ are uniformly distributed in $[0,1]$. In the particular case that “retirement of justice $i$” stochastically dominates “retirement of justice $j$” for all values of $\alpha$, we just say that $J_i$ stochastically dominates $J_j$.

Then retirement of the conservative justice dominates retirement of the moderate justice when the ideology of the Senate is conservative enough, and in addition $M_3$ is small enough or $M_2$ is close enough to $\alpha_2$. To see that, note that in case of a retirement, only FC or UC nomination games can take place (SC nomination game takes place in region D) after retirement. The most important point is that under both the UC and FC nominations games, it is the case that the new median of the Court is closer to the current median if the conservative justice retires, instead of the moderate justice.\textsuperscript{27} It follows that unless the conservative justice expects a median of the Court much closer to the current median in the second period than the moderate justice does, the conservative justice is more likely to retire than the moderate justice.\textsuperscript{28}

\textsuperscript{26} By extreme enough we mean $\alpha_S \geq (\alpha_1 + 3\alpha_3)/4$ or $\alpha_S \leq (3\alpha_1 + \alpha_2)/4$.

\textsuperscript{27} In the case of a UC, we have that $m_3 \in \{\alpha_2, \alpha_P\}$ and $m_2 \in \{\alpha_3, \alpha_P\}$, which implies that $|m_3 - \alpha_2| \leq |m_2 - \alpha_2|$. And in the case of a FC, we have that $m_3 = (\alpha_1 + \alpha_2)/2 < (\alpha_1 + \alpha_3)/2 = m_2$.

\textsuperscript{28} Using analogous reasoning we find that retirement of $J_2$ stochastically dominates retirement of $J_3$ when the Senate is liberal enough. Again the key is that only the FC and UC nomination games take place.
Unlike the analysis for the Senate, we do not expect to find stochastic dominance conditional on $\alpha_P$ being extreme, because when we fix the ideology of P and cover all the possible ideologies of S, we always cross regions C and D. These two regions provide opposite predictions on whether $\bar{p}_2 < \bar{p}_3$ or $\bar{p}_3 < \bar{p}_2$, and so stochastic dominance is different for different values of $\alpha_S$.

However, we do find stochastic dominance relating to the ideology of the President in an intuitive scenario. We do find that, regardless what the ideology of S is, the moderate justice is more likely to retire than the conservative justice if the ideology of P is moderate within a certain range. The intuition of that result comes from the fact that, conditional on the ideology of the President being moderate enough, the new median of the Court will be closer to the current median if the moderate and not the conservative justice retires. This result might seem to reflect the common belief that retirement is more likely when the ideologies of the President and the ideology of the retiring justice are more aligned, but notice that the moderate justice is the current median of the Court, hence the effect that retirement is more likely arises because the ideologies of the President and the current median are more aligned.

5.3. Ideologies of the Non-retiring Justices

The more concentrated are the ideologies of the Court (the closer $\alpha_1$ and $\alpha_3$ are to $\alpha_2$), the more likely it is that retirement takes place. This can be seen in Table 1, since $m_r$ can be $\alpha_1, \alpha_2, \text{ or } \alpha_3$ under a UC nomination game and can be $(\alpha_1 + \alpha_3)/2$ or $(\alpha_1 + \alpha_2)/2$ under a FC nomination game. In both cases, the closer $\alpha_1$ and $\alpha_3$ are to $\alpha_2$, the closer $m_r$ is to $\alpha_2$ (we consider SC in the next paragraph). The intuition of this result is that the more concentrated the Court is, the smaller the set of possible ideologies for a new median of the Court is in case of a retirement (which is $[\alpha_1, \alpha_r - ]$). As the current median of the Court always belongs to the set of possible ideologies ($\alpha_2 \in [\alpha_1, \alpha_r - ]$), a smaller set implies a larger probability that the new median will be the current median.

29. By moderate within a certain range we mean $\alpha_P \in [(\alpha_1 + \alpha_2)/2, (\alpha_1 + \alpha_3)/2]$. Note from Figure 5 that in that range, we only find region C when we vary the value of $\alpha_S$.

30. See the details in the proof of ii.b in Proposition 2.
There is an important difference in the implications of this result for the extreme versus the moderate justice. For a conservative justice, a move-ment of $J_1$ closer to $J_2$ unequivocally implies a larger expected probability of retirement. But for the moderate justice, a movement of $J_1$ closer to $J_2$ implies a larger expected probability of retirement if and only if $J_1$ is not too close to $J_2$ (the same is true for $J_3$). The reason for this asymmetry is that whereas for a conservative justice the proximity of $J_1$ to $J_2$ implies that the SC nomination game tends to disappear, and ergo $\bar{p}$ becomes 0 for all ideologies of P and S, this is not so for a moderate justice. Even when $\alpha_1 = \alpha_2$, it is not the case that the SC nomination game disappears from the possible nomination games; for that to happen requires both $\alpha_1$ and $\alpha_3$ to be equal to $\alpha_2$. This discussion generates two important results if the ideologies of P and S are uniformly distributed and $M_2$ and $M_3$ are close enough (see the characterization of iii.b in Proposition 2).

First, if the ideology of the current median of the Court is liberal enough, regardless what the ideology of the conservative justice is, then a priori, we should expect a larger probability of retirement for the conservative justice than for the moderate justice. The intuitive reason is that while for the conservative justice the expected value of $m_3$, which belongs to $[\alpha_1, \alpha_2]$, gets closer to $\alpha_2$ when $\alpha_2$ gets closer to $\alpha_1$ (increases the probability of retirement), for the moderate justice the expected value of $m_2$, which belongs to $[\alpha_1, \alpha_3]$, gets further away from $\alpha_2$ when $\alpha_2$ gets closer to $\alpha_1$ (decreases the probability of retirement).

Second, if the ideology of the current median of the Court is conservative enough and the ideology of the conservative justice is not extremely
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conservative, then a priori, we should expect a larger probability of retirement for the moderate justice than for the conservative justice. Now $m_3$ gets further away from $\alpha_2$ when $\alpha_2$ gets closer to $\alpha_3$ (decreases the probability of retirement) and $m_2$ gets closer to $\alpha_2$ when $\alpha_2$ gets closer to $\alpha_3$ (increases the probability of retirement). However, $\alpha_3$ cannot be too big for this to be true.

In order to see why, suppose that $m_2$ and $m_3$ are uniformly distributed and also $\alpha_3 > 3\alpha_2 - 2\alpha_1$, then it follows that $m_2 - \alpha_2 = (\alpha_2 - \alpha_1)/2$ is smaller than $|m_3 - \alpha_2| = (\alpha_1 + \alpha_3)/2 - \alpha_2$, which implies that the conservative justice is more likely to retire than the moderate.36

5.4. Other Considerations

Our model also allows us to verify the role that the literature has shown that age plays in the retirement decision. With age it becomes more likely that the threshold imposed by parameter $\bar{p}$ will be satisfied and very likely, however, the effect is not linear because retirement decisions take place if and only if the perceived probability of exogenous retirement is large enough. Then in many cases justices will “die unintentionally”—that is, they will play with the odds of being able to voluntarily determine their own retirement, and many times they will lose that gamble (Stolzenberg and Lindgren, 2010).

An additional observation is provided by behavioral considerations. If for any reason a justice approximates37 her perceived probability of exogenous retirement as 0, then that justice never retires. This does not depend on her ideology, the ideology of the rest of the Court, or the current or the

36. Region B (which is a region in which conservative retirement is more likely than moderate retirement) tends to disappear when $\alpha_2$ goes up. However, if in addition $\alpha_3$ is large, then region C tends to be replaced by region D (which is a region in which conservative retirement is more likely in the case that $M_1 = M_2 = M$). If the second effect dominates the first, then retirement of the conservative is more likely than the moderate.

37. Tversky and Kahneman (1982) show that, when making decisions under uncertainty, human beings tend not to look for exact solutions in a maximization problem but instead they use heuristics—approximated solutions. They document at least three of these heuristics: representivity, availability, and anchorage. One of the most well-documented behaviors is that when faced with unpleasant events, such as accidents, illness, or death, people tend to underestimate the probability that such an event will happen to them, in many times acting as if that probability was 0.
expected ideologies of the President or the Senate. In other words, whenever justices, either because of age or overconfidence, believe that their forced probability of retirement is practically zero, they will always decide to stay in service, regardless what their true probability of retirement is or what the apparent political convenience of their retirement is.

The results found in Sections 5.1–5.3 are summarized in Proposition 2.

**Proposition 2 (Main results)**

i. *Current ideologies of P and S:*

a. If P and S have extreme ideologies, then \( \frac{\partial \bar{p}_r}{\partial \alpha_P} = \frac{\partial \bar{p}_r}{\partial \alpha_S} = 0 \) (marginal changes in the ideologies of P and S do not affect the probability of retirement).

b. If P or S have moderate ideologies then: (i) when P and S play a UC nomination game, then \( \frac{\partial \bar{p}_r(\alpha_P)}{\partial |\alpha_P - \alpha_S|} < 0 \) (the probability of retirement is smaller the closer the ideology of P is to the current median of the Court); (ii) when P and S play a SC nomination game then \( \frac{\partial \bar{p}_r(\alpha_P)}{\partial |\alpha_S - (\alpha_2 + (\alpha_1 + \alpha_r)/2)|} < 0 \) (the probability of retirement is smaller the closer the ideology of S is to the average of the current median of the Court and the default median of the Court).

c. \( \text{Var}(\bar{p}_r|\alpha_P = (\alpha_1 + \alpha_r)/2) = 0 \) and \( \text{Var}(\bar{p}_r|\alpha_P = (\alpha_1 + \alpha_r)/2) \geq 0 \) (the variance of the probability of retirement weakly increases with the distance between the ideology of P and the ideology of the default median of the Court).

ii. *Ideology of the retiring Justice:*

a. If \( \alpha_2 \) and \( \alpha_S \) are large (small) enough, then \( \bar{p}_3 \leq \bar{p}_2(\bar{p}_2 \leq \bar{p}_3) \) (if the Court and the Senate are conservative (liberal) enough,

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38. This result derives from the fact that in general justices see permanence on the Court as a dominant strategy over retirement. That logic becomes very clear when we analyze the decision of the justice in the second period, and that logic holds in the context of a model with more than two periods (see Section 6.4).
then the probability of retirement for the conservative justice is larger (smaller) than the same for the moderate justice).

b. If \( M_2 = M_3 \alpha \in [(\alpha_1 + \alpha_2)/2, (\alpha_1 + \alpha_3)/2] \), then \( \tilde{p}_2 \leq \tilde{p}_3 \), \( \forall \alpha_s \) (if the President is moderate within a certain range, then the expected probability of retirement for the moderate justice is larger than the same for the conservative justice).

iii. **Ideology of the non-retiring Justices:**

a. \( \frac{\partial \bar{p}_3}{\partial \alpha_1} \leq 0 \) (the closer that the ideology of the liberal justice is to the current median, the larger the probability that the conservative justice retires) and \( \frac{\partial \bar{p}_2}{\partial (|\alpha_1 - \alpha_2| + |\alpha_3 - \alpha_2|)} \leq 0 \) (the closer the ideologies of the liberal and the conservative justices are to the current median, the larger the probability that the moderate justice retires).

b. If \( M_2 = M_3 \), then \( \exists \alpha_2 \) and \( \alpha_2^{**} \) such that: (i) If \( \alpha_2 \leq \alpha_2^{**} \), then \( E[\tilde{p}_3] \leq E[\tilde{p}_2] \) for all Courts \((\alpha_1, \alpha_2, \alpha_3)\) (if the Court is liberal enough, then the expected probability of retirement for the moderate justice is larger than the same for the conservative); (ii) If \( \alpha_2 \geq \alpha_2^{**} \), then \( \exists \alpha_1^*(\alpha_2) \) and \( \alpha_3^*(\alpha_1, \alpha_2) \) such that: \( E[\tilde{p}_2] \leq E[\tilde{p}_3] \) for all Courts \((\alpha_1, \alpha_2, \alpha_3)\) in which \( \alpha_1 \in [\alpha_1^*, \alpha_2] \) and \( \alpha_3 \in [\alpha_2, \alpha_3^*](\alpha_1, \alpha_2, \alpha_3) \) (if the Court is conservative and the justices are clustered enough, then the expected probability of retirement for the conservative justice is larger than the same for the moderate justice).

**Proof.** For the proof, see Appendix.

Proposition 2 allows us to draw three testable predictions that are novel within the literature.

5.5. Testable Predictions

**TP1** (Ideologies of the Senate and the President. Proposition 2i.b): when the ideologies of the President and the Senate are not extreme, justices tend to retire more when: (i) the ideology of the President is closer to the current median of the Court, (ii) the ideology of the Senate is closer to the average of the current median of the Court and the default median of the Court.
TP2 (Ideologies of the retiring Justices. Proposition 2ii.a): if the Court and the Senate are conservative (liberal) enough, then conservative justices tend to retire more (less) than moderate justices, regardless of the ideology of the President.

TP3 (Ideologies of the non-retiring Justices. Proposition 2iii.b): if the ideology of the median of the Court is liberal, then conservative justices are more likely to retire than moderate justices. If the median of the Court is conservative, then moderate justices are more likely to retire than conservative justices if and only if the ideology of the latter is not too extreme.

6. Discussion of the Model

6.1. When More Than One Justice Retires

The model is flexible enough to consider retirement decisions in cases in which more than one justice is of retirement age. For example, consider that if in the second period the retiring justice does not retire, then there still exists a probability that the other original potentially retiring justice will retire. Under those conditions, our model predicts that the probability of retirement goes up with the probability of the other original justices’ forced retirement probabilities; but this occurs if and only if the ideology of \( J_r \) is closer to the expected median of the Court when she retires in this period than if she retires in the next period.\(^{39}\)

\(^{39}\) If \( J_2 \)’s unforced retirement probability is \( p' \) then (1) becomes

\[
\alpha_3 - m_3 < \alpha_3 - ((1 - p')(p'\alpha_2 + (1 - p')M_2) + pM_3).
\]

Then if we consider that \( M_2 > \max\{m_3, M_3\} \), the new value of \( \bar{p} \) is given by

\[
\frac{p'M_2 + (1 - p')\alpha_2 - m_3}{p'M_2 + (1 - p')\alpha_2 - M_3}
\]

which is decreasing in \( p' \) when \( M_3 < m_3 \) but is increasing in the same probability otherwise. Hence, if the conservative justice expects that the Court will be more conservative if she retires in the current period rather than in the next period, and she knows that the moderate justice might retire in the next period, then she will be more inclined to retire immediately. This is because if she waits for the next period, the Court median can move to the left, not only because of the possibility of her own retirement but because of the possible retirement of the moderate justice.
6.2. Correlation Between Political Cycles

The assumption that the political ideologies of the President and the Senate in subsequent cycles are independent may be too strong. There is plenty of evidence of persistence in the political affiliation of presidents over time and historical electoral patterns in the Senate show tendencies that change only periodically. Our results are still valid with a degree of correlation between ideologies in the different cycles. The novelty is that predictions are more specific.

Suppose that $m_3$ and $M_3$ can be written as $m_3 = \mu \alpha_1 + (1 - \mu) \alpha_2$ and $M_3 = \delta \mu \alpha_1 + (1 - \delta \mu) \alpha_2$, in which $\mu \in [0, 1]$ captures the degree that the new median in case of a first period retirement leans towards the left and $\delta \in [0, 1/\mu]$ captures the correlation between $m_3$ and $M_3$. Not only (1), and ergo Proposition 1, are still true here, but also $\bar{p}$ becomes simply $1/\delta$. Three points follow from that last condition.

First, the probability of retirement is bounded from above; that is, in all possible scenarios of correlation between the ideologies of the President and Senate in the two periods, retirement never takes place if the exogenous probability of retirement is smaller than $\mu$. Second, the more liberal the expected ideology of the Court is if the retiring justice retires in the first period, then the more likely that the retiring justice will decide to retire in the first period (as the range of possible values for $\bar{p}$ is $[\mu, 1]$ then a reduction in the value of $\mu$ makes retirement more likely). Third, the stronger the correlation is between the two political cycles, the more likely it is that the retiring justice will retire in the current period.

6.3. Considering a 9 Justice Court

Clearly, a three justice Court is a simplification. However, that simplification is harmless for our analysis because our predictions (Propositions 1 and 2) are the same for all justices that sit to the right of the median justice and are the same for all the justices that sit to the left of the median. The median justice is the only moderate in our terminology.

In order to see this, consider a nine member Court $\{\alpha_1, \alpha_2, \ldots, \alpha_8, \alpha_9\}$. Then, according to our model, justices $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ are liberal, justices
\{\alpha_6, \alpha_7, \alpha_8, \alpha_9\} are conservative, and justice \{\alpha_5\} is moderate. Then, consider that the retiring justice is \{\alpha_6\}, such that the justices that play the role of the liberal and moderate justices in our analysis are \{\alpha_4\} and \{\alpha_5\}, respectively.\footnote{These are the justices that P and S consider to calculate the default median of the Court.} If, instead, the retiring justice is \{\alpha_7\}, then still the liberal and the moderate are \{\alpha_4\} and \{\alpha_5\}—the same for the other two farther to the right. Hence, \( \tilde{p} \) is the same for all justices to the right of \{\alpha_5\} and our predictions hold. Note that in the case of the moderate justice, \{\alpha_6\} plays the role of the conservative justice.

6.4. Multi-Period Model

At first glance, it might seem that a major limitation of the model is that we consider that there are only two periods. However, the logic of Proposition 1 and all the results derived from Table 1 hold if we consider that the model has many or even infinite periods after the first one.\footnote{We do not rule out that all Proposition 2 also holds, but that would require a much more involved analysis.} The reason is that within a multi-period model the retirement condition at the current period (we can treat it as \( t = 1 \)) is given by

\[
p_r > \frac{\left| \alpha(\alpha_2, \alpha_r, M_r) - m_r \right|}{\left| \alpha(\alpha_2, \alpha_r, M_r) - M_r \right|} = \tilde{p}_r(1).
\]

In which

\[
\alpha(\alpha_2, \alpha_r, M_r) = \alpha_2 - \left( \alpha_2 \int_{m_r^*}^{\alpha_r} f_{m_r}(x) \, dx - \int_{m_r^*}^{\alpha_r} x f_{m_r}(x) \, dx \right) + \left( 1 - \int_{m_r^*}^{\alpha_r} f_{m_r}(x) \, dx \right) \delta E[V_1]
\]

where \( m_r^* \) is the cutoff median over which retirement takes place with certainty in the next period, \( f_{m_r}(x) \) is the distribution for the expected median which by itself will depend on \( f(x) \) and \( g(x) \). And \( E[V_1] = \sum_{i=0}^{\infty} \delta^i (1 - p_r)^i [\left| \alpha_r - M_r \right| d_i + (1 - d_i) \left| \alpha_r - \alpha_2 \right|] \) where \( d_i \) is 1 if there is retirement at \( t \) and 0 otherwise—is the expected discounted loss associated with the
distance of the median of the Court and the retiring justice in all the future periods (to infinity).\textsuperscript{42}

All these may look very complicated but if the discount factor $\delta$ is not too small, then $\bar{p}_r(t)$ is increasing in $t$ because $E[V_t]$ is decreasing in $t$. Hence (3) tells us that either $J_r$ never retires (voluntarily) or retires at $t = 1$. So once more our predictions on retirement centrally depend on how close $m_r$ gets to a certain ideology, that this time is not $\alpha_2$ but $\alpha(\alpha_2, \alpha_r, M_r)$.\textsuperscript{43}

7. Conclusions

Most scholars of judicial retirement have found that political factors affect the decision to retire, but this model includes numerous factors not previously fully appreciated by the literature. In particular, we show that the retirement decision varies with the extremeness or moderation of the ideology of the retiring justice—both relative to the rest of the Court and relative to the political actors. This creates quite different predictions than focusing exclusively on the nominating agents. For instance, for the current Court, given the new Republican Senate, we predict that the four of the five oldest justices who are extremists are now less likely to retire during the remainder of President Obama’s term, as extremists are less likely to retire when the President and Senate have extreme ideologies of the opposite type. This model offers a mechanism for developing a deeper understanding of the centrally important question of when and why judges make the decision to retire.

Appendix Mathematical Proofs

Calculating $M_3$. Using Figure 2 as a reference, we can split the calculation of $M_3$ into three expressions that represent the nomination

\textsuperscript{42} This is the solution of the recursive equation $E[V_t] = \delta E[V_{t+1}] + (1 - p_r)[(\alpha_r - M_r)(d_r + (1 - d_r)(\alpha_r - \alpha_2))]$.

\textsuperscript{43} Note that the driving force to delay retirement is that the retiring justice “loses less” if she stays longer on the Court, as in that case the discounted addition to the distance of her ideology from the median of the Court is smaller ($E[V_t]$ is decreasing in $t$). $E[V_t]$ decreases even more sharply if we consider that other justices can also retire; then the retiring justice would fear that the retirement of the other justices will distort the median even more.
games that P and S might play. That is, P and S play the FC game when $\alpha_P < (\alpha_1 + \alpha_3)/2$ and $\alpha_S > (\alpha_1 + \alpha_3)/2$ or $\alpha_P > (\alpha_1 + \alpha_3)/2$ and $\alpha_S < (\alpha_1 + \alpha_3)/2$. P and S play a UC game when $\alpha_P < (\alpha_1 + \alpha_3)/2$ and $\alpha_S < \max\{(3\alpha_1 + \alpha_3)/4, \alpha_P/2 + (\alpha_1 + \alpha_3)/4\}$, or $\alpha_P > (\alpha_1 + \alpha_3)/2$ and $\alpha_S > \min\{(\alpha_1 + 3\alpha_3)/4, \alpha_P/2 + (\alpha_1 + \alpha_3)/4\}$ and P and S play a SC game for the rest of the possible values of $(\alpha_P, \alpha_S)$. After adding all the parts we get that $M_3$ is equal to a weighted average of a Court median when P and S play a FC, UC, or SC game, respectively.

$$M_3 = \begin{cases} 
\left( 1 - F \left( \frac{\alpha_1 + \alpha_2}{2} \right) \right) G \left( \frac{\alpha_1 + \alpha_2}{2} \right) \frac{\alpha_1 + \alpha_2}{2} & \text{P and S play a FC game} \\
+ F \left( \frac{\alpha_1 + \alpha_2}{2} \right) \left( 1 - G \left( \frac{\alpha_1 + \alpha_2}{2} \right) \right) \frac{\alpha_1 + \alpha_2}{2} \quad & \text{P and S play an UC game} 
\end{cases}$$

$$G \left( \frac{3\alpha_1 + \alpha_2}{4} \right) \left( \alpha_1 F(\alpha_1) + \int_{\alpha_1}^{\frac{3\alpha_1 + \alpha_2}{4}} f(x) x \, dx \right)$$

$$+ \left( 1 - G \left( \frac{\alpha_1 + 3\alpha_2}{4} \right) \right) \left( \alpha_2 (1 - F(\alpha_2)) + \int_{\frac{\alpha_1 + 3\alpha_2}{4}}^{\alpha_2} f(x) x \, dx \right)$$

$$+ \int_{\alpha_1}^{\frac{3\alpha_1 + \alpha_2}{4}} f(x) \left( \int_{\frac{x + \alpha_1 + \alpha_2}{2}}^{\frac{3\alpha_1 + \alpha_2}{4}} g(y) \, dy \right) x \, dx$$

$$+ \int_{\frac{\alpha_1 + 3\alpha_2}{4}}^{\alpha_2} f(x) \left( \int_{\frac{x + \alpha_1 + \alpha_2}{2}}^{\frac{3\alpha_1 + \alpha_2}{4}} g(y) \, dy \right) x \, dx$$

$$+ \left( G \left( \frac{\alpha_1 + \alpha_2}{2} \right) - G \left( \frac{3\alpha_1 + \alpha_2}{4} \right) \right)$$

$$F(\alpha_1) \left( 2E \left[ \alpha_S \mid \alpha_S \in \left[ \frac{3\alpha_1 + \alpha_2}{4}, \frac{\alpha_1 + \alpha_2}{2} \right] \right] - \frac{\alpha_1 + \alpha_2}{2} \right)$$

$$+ \int_{\alpha_1}^{\frac{3\alpha_1 + \alpha_2}{4}} \left( \frac{G \left( \frac{\alpha_1 + \alpha_2}{2} \right) - G \left( \frac{x + \alpha_1 + \alpha_2}{4} \right)}{G \left( \frac{\alpha_1 + \alpha_2}{2} \right) - G \left( \frac{3\alpha_1 + \alpha_2}{4} \right)} \right)$$

$$\times \left( 2E \left[ \alpha_S \mid \alpha_S \in \left[ \frac{x}{2}, \frac{\alpha_1 + \alpha_2}{4} \right] \right] - \frac{\alpha_1 + \alpha_2}{2} \right) \right) f(x) \, dx$$
Calculating $M_2$. The expression for $M_2$ is the same as $M_3$ but now $\alpha_2$ has to be replaced by $\alpha_3$. In addition, in order to take into account that $\alpha_2$ can be larger or smaller than $(\alpha_1 + \alpha_3)/2$, $2E[\alpha_S|\bullet] - (\alpha_1 + \alpha_3)/2$ has to be written in absolute terms. That is as $|2E[\alpha_S|\bullet] - (\alpha_1 + \alpha_3)/2|$.

**Proof Proposition 1.** We need to verify that the values of $m_r$ are as given in Figure 2.

If $\alpha_p < (\alpha_1 + \alpha_{r-})/2 < \alpha_S$ or $\alpha_S < (\alpha_1 + \alpha_{r-})/2 < \alpha_p$, then $P_1$ and $S_1$ nominate and confirm a new justice with ideology $(\alpha_1 + \alpha_{r-})/2$ when $J_r$retires because $P_1$ and $S_1$ play a fully constrained nomination game.

On the other side, if $\alpha_p < \alpha_1$ and $\alpha_S < (3\alpha_1 + \alpha_{r-})/4$, then the president nominates a candidate with ideology $\alpha_p$ which is supported by the Senate and the new median of the Court becomes $\alpha_1$. Note that the Senate does not need to be to the left of $J_1$ for this UC nomination game to take place. It is enough that the ideology of the Senate is closer to $\alpha_1$ than to $(\alpha_1 + \alpha_{r-})/2$ which is equivalent to $\alpha_S < (3\alpha_1 + \alpha_{r-})/4$. Analogously a UC nomination game takes place when $\alpha_p > \alpha_{r-}$ and $\alpha_S > (\alpha_1 + 3\alpha_{r-})/4$ only in this case the new median of the Court becomes $\alpha_{r-}$.

The third and final scenario in which a UC nomination game takes place is when the ideology of the Senate is to the same side of the median of the Court as the President is and, in addition, the Senate is closer to the ideology of the President than the ideology of the default median. This happens when $\alpha_1 < \alpha_p < (\alpha_1 + \alpha_{r-})/2$ and $\alpha_S < \alpha_p/2 + (\alpha_1 + \alpha_{r-})/4$ or when $(\alpha_1 + \alpha_{r-})/2 < \alpha_p < \alpha_{r-}$ and $\alpha_S < \alpha_p/2 + (\alpha_1 + \alpha_{r-})/4$. 

$$
\left(1 - F(\alpha_2)\right) \left(2E[\alpha_S|\alpha_S \in \left[\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + 3\alpha_2}{4}\right] - \frac{\alpha_1 + \alpha_2}{2}\right) + \int_{\alpha_1 + \alpha_2}^{\alpha_2} \left\{ \frac{G\left(\frac{\alpha_1 + \alpha_2}{4}\right) - G\left(\frac{\alpha_1 + \alpha_2}{2}\right)}{G\left(\frac{\alpha_1 + 3\alpha_2}{4}\right) - G\left(\frac{\alpha_1 + \alpha_2}{2}\right)} \right\} f(x) \, dx
$$

$P$ and $S$ play a SC game
We know from Section 3.4 that the remaining set of scenarios in the universe of \((\alpha_p, \alpha_S)\) in which the nomination game is neither FC nor UC, the nomination game is SC. In those games, the new median of the Court becomes \(2\alpha_S - (\alpha_1 + \alpha_r)/2\). In our analysis, the set of scenarios is \(\alpha_p < (\alpha_1 + \alpha_r)/2\) and \(\max((3\alpha_1 + \alpha_r)/4, \alpha_p/2 + (\alpha_1 + \alpha_r)/4) < \alpha_S < (\alpha_1 + \alpha_r)/2\) or \(\alpha_p > (\alpha_1 + \alpha_r)/2\) and \(\min((3\alpha_1 + \alpha_r)/4, \alpha_p/2 + (\alpha_1 + \alpha_r)/4) > \alpha_S > (\alpha_1 + \alpha_r)/2\).

**Proof of Proposition 2.** (i) We start by defining the cases in which the President and the Senate have extreme or moderate ideologies.

President and Senate have extreme ideologies in the next four cases \(\equiv \{\alpha_{p_1} < \alpha_1 \& \alpha_{S_1} < (3\alpha_1 + \alpha_r)/4\} \text{ or } \{\alpha_{p_1} < (\alpha_1 + \alpha_r)/2 \& \alpha_{S_1} > (\alpha_1 + \alpha_r)/2\} \text{ or } \{\alpha_{p_1} > (\alpha_1 + \alpha_r)/2 \& \alpha_{S_1} < (\alpha_1 + \alpha_r)/2\} \text{ or } \{\alpha_{p_1} > \alpha_r \& \alpha_{S_1} > (\alpha_1 + 3\alpha_r)/4\}.

President or Senate has moderate ideology in the next four cases \(\equiv \{\alpha_{p_1} < \alpha_1 \& \alpha_{S_1} \in [(3\alpha_1 + \alpha_r)/4, (\alpha_1 + \alpha_r)/2]\} \text{ or } \{\alpha_{p_1} \in [\alpha_1, (\alpha_1 + \alpha_2)/2] \& \alpha_{S_1} \in [(\alpha_1 + \alpha_r)/4, (\alpha_1 + \alpha_r)/2]\} \text{ or } \{\alpha_{p_1} \in [(\alpha_1 + \alpha_r)/2, \alpha_r] \& \alpha_{S_1} > (\alpha_1 + \alpha_r)/2\} \text{ or } \{\alpha_{p_1} > \alpha_r \& \alpha_{S_1} \in [(\alpha_1 + \alpha_r)/2, (\alpha_1 + 3\alpha_r)/4]\}.

Then

a. By inspection of Figures 3 and 4, when P and S are extreme then \(m_r \in [\alpha_1, \alpha_2, (\alpha_1 + \alpha_2)/2, (\alpha_1 + \alpha_3)/2]\) which implies that \(\bar{p}_r\) does not depend on \(\alpha_p\) or \(\alpha_S\).

b. By inspection of Figures 3 and 4, when P and S are moderate and the nomination game is UC then \(\bar{p}_r = |\alpha_2 - \alpha_p|/|\alpha_2 - M_r|\) which implies that the closer is \(\alpha_p\) to \(\alpha_2\), the closer is \(\bar{p}_r\) to 0. Also when P and S are moderate and the nomination game is SC then \(\bar{p}_r = |\alpha_2 + (\alpha_1 + \alpha_r)/2 - 2\alpha_S|/|\alpha_2 - M_r|\) which implies that the closer is \(\alpha_S\) to \(\alpha_2/2 + (\alpha_1 + \alpha_r)/4\) the closer is \(\bar{p}_r\) to 0.

c. We do it only for the conservative justice and conditional on the ideology of the President. The steps are analogous for the moderate justice and conditional on the ideology of the Senate. We calculate the values of \(\var{\bar{p}_3|\alpha_p}\) for the next four intervals: \(\alpha_p \leq \alpha_1; \alpha_p \in [\alpha_1, (\alpha_1 + \alpha_2)/2]; \alpha_p \in [(\alpha_1 + \alpha_2)/2, \alpha_2]; \alpha_p > \alpha_2\). When \(\alpha_p \leq \alpha_1\) then

\[
\var{\bar{p}_3|\alpha_p} = \frac{(\alpha_2 - \alpha_1)^2 (128\alpha_1 + 64\alpha_2 - 3(5\alpha_1 + 3\alpha_2)^2)}{768(\alpha_2 - M_2)^2}
\]
which is a constant that does not depend on \( \alpha_p \). When \( \alpha_p \in [\alpha_1, (\alpha_1 + \alpha_2)/2 \) then

\[
\text{Var}[\tilde{\beta}_3 | \alpha_p] = \frac{(\alpha_1 + \alpha_2 - 2\alpha_p)^2(-27(\alpha_1 + \alpha_2)^2 - 12\alpha^2_p - 36\alpha_p(\alpha_1 + \alpha_2) + 64(\alpha_1 + \alpha_2 + \alpha_p))}{768(\alpha_2 M_2)^2}
\]

Note that \( \text{Var}[\tilde{\beta}_r | \alpha_p = (\alpha_1 + \alpha_2)/2 = 0 \). In addition, \( \partial \text{Var}[\tilde{\beta}_3 | \alpha] / \partial \alpha_p < 0 \).

When \( \alpha_p \in [(\alpha_1 + \alpha_2)/2, \alpha_2] \) then

\[
\text{Var}[\tilde{\beta}_3 | \alpha_p] = \frac{(\alpha_1 + \alpha_2 - 2\alpha_p)^2(-27(\alpha_1 + \alpha_2)^2 - 12\alpha^2_p - 36\alpha_p(\alpha_1 + \alpha_2) + 16(\alpha_1 + \alpha_2 + 2\alpha_p))}{768(\alpha_2 - M_2)^2}
\]

Once more \( \text{Var}[\tilde{\beta}_3 | \alpha_p = (\alpha_1 + \alpha_2)/2 \) but this time \( \partial \text{Var}[\tilde{\beta}_3 | \alpha_p] / \partial \alpha_p > 0 \).

Finally, when \( \alpha_p > \alpha_2 \) then

\[
\text{Var}[\tilde{\beta}_3 | \alpha_p] = \frac{(\alpha_2 - \alpha_1)^2(80\alpha_1 + 112\alpha_2 - 3(5\alpha_1 + 3\alpha_2)^2)}{768(\alpha_2 - M_2)^2}
\]

which is a constant that does not depend on \( \alpha_p \). Putting all these pieces together we conclude that \( \text{Var}(\tilde{\beta}_r | \alpha_p \) strictly increases with \( |\alpha_p - (\alpha_1 + \alpha_2 - 1)/2 \) as long as \( \alpha_p \in [\alpha_1, \alpha_2] \) but it is a constant otherwise.

ii)

a. If \( \alpha_2 < \text{Min}\{M_2, (M_2 + M_3)/2 \) then \( \tilde{\beta}_2 \leq \tilde{\beta}_3 \) in region C. Hence when \( \alpha_S < (3\alpha_1 + \alpha_2)/4 \) it is true that \( \tilde{\beta}_2 \leq \tilde{\beta}_3 \) for all values of \( \alpha_p, \alpha_S \) as in region A it is always the case that \( \tilde{\beta}_2 \leq \tilde{\beta}_3 \). On the other side, when \( \alpha_2 > M_2 + |\alpha_2 - M_2|((\alpha_2 - \alpha_1)/2(\alpha_2 - (\alpha_1 + \alpha_3)/2) \) then \( \tilde{\beta}_2 \geq \tilde{\beta}_3 \) in region C. Hence when \( \alpha_S > (\alpha_1 + 3\alpha_3)/4 \) it is true that \( \tilde{\beta}_3 \leq \tilde{\beta}_2 \) for all values of \( \alpha_p, \alpha_S \) as in region B it is always the case that \( \tilde{\beta}_3 \leq \tilde{\beta}_2 \).

b. Suppose that \( M_2 = M_3 = M < \alpha_2 \) and \( \alpha_p \in [(\alpha_1 + \alpha_2)/2, (\alpha_1 + \alpha_3)/2 \) then \( |\alpha_2 - m_2| \leq |\alpha_2 - m_3| \) for all possible values of \( \alpha_S \). We consider only the case in which \( (\alpha_1 + \alpha_3)/2 < \alpha_2 \). The analysis of the other case is analogous.

When \( \alpha_S < (\alpha_1 + \alpha_2)/2 \), then \( m_2 = \alpha_p \) and \( m_3 + (\alpha_1 + \alpha_2)/2 \), which implies that \( |\alpha_2 - m_2| \leq |\alpha_2 - m_3| \) because \( \alpha_p > (\alpha_1 + \alpha_2)/2 \).
When $\alpha_S \in [(\alpha_1 + \alpha_2)/2, \alpha_P/2 + (\alpha_1 + \alpha_2)/4]$, then $m_2 = \alpha_P m_3 = 2\alpha_S - (\alpha_1 + \alpha_2)/2$, which implies that $|\alpha_2 - m_2| \leq |\alpha_2 - m_3|$ because $\alpha_S < \alpha_P/2 + (\alpha_1 + \alpha_2)/4$.

When $\alpha_S \in [\alpha_P/2 + (\alpha_1 + \alpha_2)/4, \alpha_P/2 + (\alpha_1 + \alpha_3)/4$, then $m_2 = \alpha_P$ and $m_3 = \alpha_P$, which implies that $|\alpha_2 - m_2| = |\alpha_2 - m_3|$.

When $\alpha_S \in [\alpha_P/2 + (\alpha_1 + \alpha_3)/4, (\alpha_1 + \alpha_3)/2]$, then $m_2 = 2\alpha_S - (\alpha_1 + \alpha_3)/2$ and $m_3 = \alpha_P$, which implies that $|\alpha_2 - m_2| \leq |\alpha_2 - m_3|$ because $\alpha_S \geq \alpha_P/2 + (\alpha_1 + \alpha_3)/4$.

Finally, when $\alpha_S > (\alpha_1 + \alpha_3)/2$, then $m_2 = (\alpha_1 + \alpha_3)/2$ and $m_3 = \alpha_P$, which implies that $|\alpha_2 - m_2| \leq |\alpha_2 - m_3|$ because $\alpha_P \leq (\alpha_1 + \alpha_3)/2$.

Which ends the proof.

iii) It follows from inspection of Table 1. Alternatively, the same result follows after we note that if a priori the ideologies of P and S are uniformly distributed then

\[
E \left[ \tilde{p}_2 \mid \alpha_2 > \frac{\alpha_1 + \alpha_3}{2} \right] = \frac{9\alpha_1^2(\alpha_3 - \alpha_1) + (48 - 18\alpha_1 + (\alpha_3 + \alpha_2))}{48|\alpha_2 - M_2|} \left\{ \begin{array}{l}
9\alpha_1^2(\alpha_3 - \alpha_1) + (48 - 18\alpha_1 + (\alpha_3 + \alpha_2)) \\
(12(\alpha_1 + \alpha_3) - 63)(\alpha_3 - \alpha_2) \\
+ (24\alpha_3 + 15\alpha_1 + 9\alpha_2)(\alpha_2 - \alpha_1) \\
+ 7(\alpha_3^2 - \alpha_2^2) + 9(\alpha_1\alpha_3^2 - \alpha_2^2) \end{array} \right\}
\]

\[
E \left[ \tilde{p}_2 \mid \alpha_2 \leq \frac{\alpha_1 + \alpha_3}{2} \right] = E \left[ \tilde{p}_2 \mid \alpha_2 > \frac{\alpha_1 + \alpha_3}{2} \right] + \frac{\left(\frac{\alpha_1 + \alpha_3}{2} - \alpha_2\right)}{48|\alpha_2 - M_2|} \left\{ \begin{array}{l}
72\alpha_2 - 19(\alpha_1 + \alpha_3)^2 - 28\alpha_2^2 \\
+ (60 - 44\alpha_2)(\alpha_1 + \alpha_3) \end{array} \right\}
\]

\[
E[\tilde{p}_3] = \frac{(\alpha_2 - \alpha_1)(5\alpha_1 + 11\alpha_2 - 3(\alpha_2^2 - \alpha_1^2))}{16(\alpha_2 - M_3)}
\]

44. It is easy to verify that $E[\tilde{p}_3] = E[\tilde{p}_2 \mid \alpha_2 > (\alpha_1 + \alpha_3)/2]$ when $\alpha_3 = \alpha_2$. 
Then, it follows that

\[ \frac{\partial E[\tilde{p}_3]}{\partial \alpha_1} = \frac{(-9\alpha_1^2 + 6\alpha_1\alpha_2 - 10\alpha_1 + 3\alpha_2^3 - 6\alpha_2)}{16(\alpha_2 - M_3)} < 0 \]

In addition, we need to prove that \( E[\tilde{p}_2 | \alpha_2 \leq (\alpha_1 + \alpha_3)/2]_{\alpha_1 = \alpha_2 + \Delta, \alpha_3 = \alpha_2 - \Delta} \) decreases with \( \Delta \) and \( E[\tilde{p}_2 | \alpha_2 > (\alpha_1 + \alpha_3)/2]_{\alpha_1 = \alpha_2 + \Delta, \alpha_3 = \alpha_2 - \Delta} \) increases with \( \Delta \). As the steps are analogous we just show it for the first. We have that

\[
\frac{\partial E \left[ \tilde{p}_2 \mid \alpha_2 > \frac{\alpha_1 + \alpha_3}{2} \right]_{\alpha_1 = \alpha_2 + \Delta, \alpha_3 = \alpha_2 - \Delta}}{\partial \Delta} \]

\[
= - \frac{\left( 71\Delta^2 + 84\Delta\alpha_1 - 32\Delta\alpha_3 + 48\Delta_3^2\alpha_3^2 \right)}{32(\alpha_2 - M)}
\]

which is negative because

\[
\frac{\partial E \left[ \tilde{p}_2 \mid \alpha_2 > \frac{\alpha_1 + \alpha_3}{2} \right]_{\alpha_1 = \alpha_2 + \Delta, \alpha_3 = \alpha_2 - \Delta}}{\partial \Delta} \bigg|_{\Delta=0} < 0
\]

and

\[
\frac{\partial^2 E \left[ \tilde{p}_2 \mid \alpha_2 > \frac{\alpha_1 + \alpha_3}{2} \right]_{\alpha_1 = \alpha_2 + \Delta, \alpha_3 = \alpha_2 - \Delta}}{\partial \Delta^2} < 0
\]

b. For (i) we need to show that there exists \( \alpha_2^* \) such that for all \( \alpha_2 < \alpha_2^* \) then \( E[\tilde{p}_3] < E[\tilde{p}_2 | \alpha_2 < (\alpha_1 + \alpha_3)/2] \) but that follows after we note that

\[
E[\tilde{p}_3] - E \left[ \tilde{p}_2 \mid \alpha_2 \frac{\alpha_1 + \alpha_3}{2} \right] < 0
\]

\[
\Leftrightarrow \alpha_2^* + \frac{(21\alpha_1 + 19\alpha_3 - 39)}{7} \alpha_2
\]

\[
+ \frac{(9\alpha_1^2 + 21\alpha_1\alpha_3 - 42\alpha_1 + 19\alpha_3^2 - 63\alpha_3 + 48)}{7} > 0
\]

\[
\Leftrightarrow (\alpha_2 - \alpha_20)(\alpha_2 - \alpha_21) > 0.
\]
Hence if \( \alpha_2 < \alpha_{20} = \alpha_z^* \), it follows that \( E[\tilde{p}_3] < E[\tilde{p}_2 | \alpha_2 < (\alpha_1 + \alpha_3)/2] \).

For (ii) If \( \alpha_1 \) is close enough to \( \alpha_2 \), then we know that \( 2\alpha_2 - \alpha_1 \in [\alpha_2, \alpha_3] \). Hence because it is true that (1) when \( \alpha_3 = 1 \) then \( E[\tilde{p}_3] < E[\tilde{p}_2 | \alpha_2 < (\alpha_1 + \alpha_3)/2] \); (2) when \( \alpha_3 > \alpha_2 \) but \( \alpha_3 < \alpha_z^* \) then \( E[\tilde{p}_3] > E[\tilde{p}_2 | \alpha_2 < (\alpha_1 + \alpha_3)/2] \) when \( \alpha_3 < \alpha_z^* \) as we wanted to prove. \( \square \)

References


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45. It is true after we notice that \( E[\tilde{p}_3] - E[\tilde{p}_2 | \alpha_2 < (\alpha_1 + \alpha_3)/2] |_{\alpha_1 = \alpha_2 = \alpha, \alpha_3 = 1} = (-15\alpha_2^2 - 17\alpha_3^3 + 81\alpha - 49)/96(\alpha_2 - M_2) < 0 \) for all values of \( \alpha \in [0, 1] \). Then because of continuity when \( \alpha_1 = \alpha_2 - \varepsilon \) still holds \( E[\tilde{p}_3] < E[\tilde{p}_2 | \alpha_2 < (\alpha_1 + \alpha_3)/2] \).

46. From numerical simulations in which we cover all values of \( \{\alpha_1, \alpha_2, \alpha_3\} \) we find that \( E[\tilde{p}_3] - E[\tilde{p}_2 | \alpha_2 > 0 \) when \( \alpha_3 = 2\alpha_2 - \alpha_1 \).

47. We know that \( \frac{\partial E[\tilde{p}_3 | \alpha_2 < (\alpha_1 + \alpha_3)/2]}{\partial \alpha_3 |_{\alpha_3 = 1} > 0 \) because \( \frac{\partial^2 E[\tilde{p}_3 | \alpha_2 < (\alpha_1 + \alpha_3)/2]}{\partial \alpha_3^2} < 0 \) and \( \frac{\partial E[\tilde{p}_3 | \alpha_2 < (\alpha_1 + \alpha_3)/2]}{\partial \alpha_3 |_{\alpha_3 = 1} > 0 \).