

Study - Material

U. G. Sem - 2 (Mathematics)

Paper - CC₃Integral Calculus

Date:

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Q. Integrate: $\int \frac{x^2+1}{x(x^2-1)} dx$

Solⁿ: Here $\frac{x^2+1}{x(x^2-1)} = \frac{x^2+1}{x(x-1)(x+1)}$

Let $\frac{x^2+1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{x+1}$ — ①

$\Rightarrow \frac{x^2+1}{x(x-1)(x+1)} = \frac{A(x-1)(x+1) + Bx(x+1) + Cx(x-1)}{x(x-1)(x+1)}$

$\Rightarrow x^2+1 = A(x^2-1) + B(x^2+x) + C(x^2-x)$

$\Rightarrow x^2+1 = (A+B+C)x^2 + (B-C)x - A$

$\Rightarrow A+B+C = 1$ — ②

$B-C = 0$ — ③

$A = -1$ — ④

② $\Rightarrow B+C = 2$ — ⑤

$$\textcircled{3} + \textcircled{5} \Rightarrow 2B = 2 \Rightarrow \boxed{B=1}$$

$$\textcircled{3} \Rightarrow \boxed{C=1}$$

$$\therefore \textcircled{1} \Rightarrow \frac{x^2+1}{x(x+1)(x-1)} = \frac{-1}{x} + \frac{1}{x+1} + \frac{1}{x-1}$$

$$\begin{aligned} \Rightarrow \int \frac{x^2+1}{x(x^2-1)} dx &= -\int \frac{dx}{x} + \int \frac{dx}{x+1} + \int \frac{dx}{x-1} \\ &= -\log x + \log(x+1) + \log(x-1) \\ &= \log \frac{(x^2-1)}{x} \quad \# \end{aligned}$$

Q Integrate: $\int \frac{x^2+x-1}{x^3+x^2-6x} dx$

Solⁿ: here the given ~~function~~ ^{expression} is

$$\frac{x^2+x-1}{x^3+x^2-6x} = \frac{x^2+x-1}{x(x^2+x-6)} = \frac{x^2+x-1}{x(x-2)(x+3)}$$

$$\text{Let } \frac{x^2+x-1}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3} \quad \text{--- (1)}$$

$$\Rightarrow \frac{x^2+x-1}{x(x-2)(x+3)} = \frac{A(x-2)(x+3) + Bx(x+3) + Cx(x-2)}{x(x-2)(x+3)}$$

$$\Rightarrow x^2+x-1 = A(x^2+x-6) + B(x^2+3x) + C(x^2-2x)$$

$$\Rightarrow x^2 + 2x = (A+B+C)x^2 + (A+3B-2C)x - 6A$$

$$\Rightarrow A+B+C = 1 \quad \text{--- (2)}$$

$$A+3B-2C = 1 \quad \text{--- (3)}$$

$$6A = 1 \Rightarrow \boxed{A = \frac{1}{6}}$$

$$\text{(2)} \Rightarrow B+C = 1 - \frac{1}{6} = \frac{5}{6} \Rightarrow B+C = \frac{5}{6} \quad \text{--- (4)}$$

$$\text{(3)} \Rightarrow 3B-2C = 1 - \frac{1}{6} = \frac{5}{6} \Rightarrow 3B-2C = \frac{5}{6} \quad \text{--- (5)}$$

$$\text{(4)} \Rightarrow 2B+2C = \frac{10}{6} \quad \text{---}$$

$$\text{(5)} \Rightarrow 3B-2C = \frac{5}{6}$$

By adding, we get

$$5B = \frac{15}{6} \Rightarrow \boxed{B = \frac{1}{2}}$$

$$\text{(4)} \Rightarrow C = \frac{5}{6} - B = \frac{5}{6} - \frac{1}{2} = \frac{5-3}{6} = \frac{1}{3}$$

$$\therefore \boxed{C = \frac{1}{3}}$$

$$\therefore \text{(1)} \Rightarrow \frac{x^2+x-1}{x(x+2)(x+3)} = \frac{\frac{1}{6}}{x} + \frac{\frac{1}{2}}{(x-2)} + \frac{\frac{1}{3}}{(x+3)}$$

$$\therefore \int \frac{x^2+x-1}{x^3+x^2-6x} dx = \frac{1}{6} \int \frac{dx}{x} + \frac{1}{2} \int \frac{dx}{(x-2)} + \frac{1}{3} \int \frac{dx}{x+3}$$

$$= \frac{1}{6} \log x + \frac{1}{2} \log(x-2) + \frac{1}{3} \log(x+3) + \#$$

Type-2: Linear and repeated:

Q. Integrate: $\int \frac{x^2}{(x+1)^2(x+2)} dx$

Solⁿ: let $\frac{x^2}{(x+1)^2(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$ (1)

$$\Rightarrow \frac{x^2}{(x+1)^2(x+2)} = \frac{A(x+1)(x+2) + B(x+2) + C(x+1)^2}{(x+1)^2(x+2)}$$

$$\Rightarrow x^2 = A(x^2 + 3x + 2) + B(x+2) + C(x^2 + 2x + 1)$$

$$\Rightarrow x^2 = (A+C)x^2 + (3A+B+2C)x + (2A+2B+C)$$

$$\Rightarrow A+C = 1 \text{ --- (2)}, \quad 3A+B+2C = 0 \text{ --- (3)}, \quad 2A+2B+C = 0 \text{ --- (4)}$$

$$(3) \Rightarrow 3A+B+2C = 0$$

$$(4) \Rightarrow 4A+4B+2C = 0$$

Subtraction, we get

$$A+3B = 0 \Rightarrow A = -3B$$

$$(3) \Rightarrow B = -\frac{1}{3}A$$

$$(2) \Rightarrow C = 1-A$$

$$\therefore (4) \Rightarrow 2A - \frac{2}{3}A + 1 - A = 0$$

$$\Rightarrow \frac{1}{3}A = -1 \Rightarrow A = -\frac{1}{3}$$

$$\therefore B = \frac{1}{9} \quad \& \quad C = \frac{4}{3}$$

$$\therefore (1) \Rightarrow \frac{x^2}{(x+1)^2(x+2)} = \frac{-\frac{1}{3}}{(x+1)} + \frac{\frac{1}{9}}{(x+1)^2} + \frac{\frac{4}{3}}{(x+2)}$$

$$\therefore \int \frac{x^2 dx}{(x+1)^2(x+2)} = -\frac{1}{3} \int \frac{dx}{(x+1)} + \frac{1}{9} \int \frac{dx}{(x+1)^2} + \frac{4}{3} \int \frac{dx}{(x+2)}$$

$$= -\frac{1}{3} \log_e(x+1) + \frac{1}{9} \log_e \frac{1}{(x+1)^2} + \frac{4}{3} \log_e(x+2)$$

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