Study Material

Dr. M. A. Khan
Associate Prof,
Dept. of Mathematics
Tata College,
Chaibasa

U.G. Sem - 2 (Mathematics)
paper - CC_3
Integral Calculus

Type - 3 : Quadratic and non-repeated

Q. Integrate $\int \frac{x\,dx}{(1+x)(1+x^2)}$

Solⁿ:- Let $\frac{x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$ — (1)

$$\Rightarrow \frac{x}{(1+x)(1+x^2)} = \frac{A(1+x^2) + (Bx+C)(1+x)}{(1+x)(1+x^2)}$$

$$\Rightarrow x = A(1+x^2) + Bx^2 + Cx + Bx + C$$

$$\Rightarrow x = (A+B)x^2 + (B+C)x + (A+C)$$

$\Rightarrow$ $A+B = 0 \Rightarrow B = -A$ — (2)

$B+C = 1$ — (3)

$A+C = 0 \Rightarrow C = -A$ — (4)

(3) $\Rightarrow -2A = 1 \Rightarrow \boxed{A = -\frac{1}{2}}$

$\boxed{B = \frac{1}{2}}$ & $\boxed{C = \frac{1}{2}}$

$\therefore (1) \Rightarrow \frac{x}{(1+x)(1+x^2)} = \frac{-\frac{1}{2}}{1+x} + \frac{\frac{1}{2}x+\frac{1}{2}}{1+x^2}$

$\therefore \int \frac{x\,dx}{(1+x)(1+x^2)} = -\frac{1}{2}\int \frac{dx}{1+x} + \frac{1}{2}\int \frac{(x+1)\,dx}{1+x^2}$

$= -\frac{1}{2}\log(1+x) + \frac{1}{2}\left[\frac{1}{2}\int \frac{2x\,dx}{1+x^2} + \int \frac{dx}{1+x^2}\right]$

$= -\frac{1}{2}\log(1+x) + \frac{1}{4}\int \frac{d(1+x^2)}{1+x^2} + \frac{1}{2}\int \frac{dx}{1+x^2}$

$= -\frac{1}{2}\log(1+x) + \frac{1}{4}\log(1+x^2 + \frac{1}{2}\tan^{-1} x.$

Q <u>Integrate:-</u> $\int \frac{dx}{1+x^3}$

<u>Solⁿ:-</u> Let $\frac{1}{1+x^3} = \frac{1}{(1+x)(x^2-x+1)} = \frac{A}{1+x} + \frac{Bx+C}{x^2-x+1}$ — (1)

$\Rightarrow \frac{1}{1+x^3} = \frac{A(x^2-x+1)+(Bx+C)(x+1)}{(x+1)(x^2-x+1)}$

$\Rightarrow 1 = A(x^2-x+1) + Bx^2 + Cx + Bx + C$

$\Rightarrow 1 = (A+B)x^2 + (-A+B+C)x + (A+C)$

$\Rightarrow A+B = 0 \Rightarrow B = -A$ — (2)

$-A+B+C = 0 \Rightarrow -2A+C = 0$ — (3)

$A+C = 1$ — (4)

$④ - ⑤ \Rightarrow 3A = 1 \Rightarrow \boxed{A = \frac{1}{3}}$

$B = -\frac{1}{3} \quad C = \frac{2}{3}$

$$\therefore ① \Rightarrow \frac{1}{1+x^3} = \frac{\frac{1}{3}}{1+x} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2 - x + 1}$$

$$\Rightarrow \int \frac{dx}{1+x^3} = \frac{1}{3}\int \frac{dx}{1+x} - \frac{1}{3}\int \frac{x-2}{x^2-x+1}\,dx$$

$$= \frac{1}{3}\log(1+x) - \frac{1}{3}\left[\frac{1}{2}\int \frac{2x-4}{x^2-x+1}\,dx\right]$$

$$= \frac{1}{3}\log(1+x) - \frac{1}{3}\left[\frac{1}{2}\int \frac{2x-1-3}{x^2-x+1}\,dx\right]$$

$$= \frac{1}{3}\log(1+x) - \frac{1}{6}\int \frac{2x-1}{x^2-x+1}\,dx + \frac{1}{2}\int \frac{dx}{x^2 - 2\cdot x\frac{1}{2} + \frac{1}{4} + \frac{3}{4}}$$

$$= \frac{1}{3}\log(1+x) - \frac{1}{6}\int \frac{d(x^2-x+1)}{x^2-x+1} + \frac{1}{2}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{3}\log(1+x) - \frac{1}{6}\log(x^2-x+1) + \frac{1}{\cancel{2}}\,\frac{1}{\frac{\sqrt{3}}{\cancel{2}}}\tan^{-1}\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{3}\log(1+x) - \frac{1}{6}\log(x^2-x+1) + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}}$$

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Type-4: Quadratic and repeated

Q. Integrate $\int \frac{x\,dx}{(x^2+a^2)(x^2+b^2)}$

Solⁿ:- Let $\frac{x}{(x^2+a^2)(x^2+b^2)} = \frac{Ax+B}{x^2+a^2} + \frac{Cx+D}{x^2+b^2}$ — ①

$$\Rightarrow \frac{x}{(x^2+a^2)(x^2+b^2)} = \frac{(Ax+B)(x^2+b^2)+(Cx+D)(x^2+a^2)}{(x^2+a^2)(x^2+b^2)}$$

$$\Rightarrow x = Ax^3 + Bx^2 + Ab^2x + Bb^2 + Cx^3 + Dx^2 + Ca^2x + Da^2$$

$$\Rightarrow x = (A+C)x^3 + (B+D)x^2 + (Ab^2+Ca^2)x + (Bb^2+Da^2)$$

$\Rightarrow$ $A+C=0 \Rightarrow C=-A$

$B+D=0 \Rightarrow D=-B$

$Ab^2+Ca^2=1 \Rightarrow A(b^2-a^2)=1 \Rightarrow A=\frac{1}{b^2-a^2}$

$Bb^2+Da^2=0 \Rightarrow B(b^2-a^2)=0 \Rightarrow B=0$

$\Rightarrow D=0$ & $C=\frac{1}{a^2-b^2}$

$$\therefore ① \Rightarrow \frac{x}{(x^2+a^2)(x^2+b^2)} = \frac{\frac{1}{b^2-a^2}x+0}{x^2+a^2} + \frac{\frac{1}{a^2-b^2}x+0}{x^2+b^2}$$

$$\Rightarrow \int \frac{x\,dx}{(x^2+a^2)(x^2+b^2)} = \frac{1}{b^2-a^2}\frac{1}{2}\int\frac{2x}{x^2+a^2} + \frac{1}{a^2-b^2}\frac{1}{2}\int\frac{2x}{x^2+b^2}$$

$$= -\frac{1}{2(a^2-b^2)}\log(x^2+a^2) + \frac{1}{2(a^2-b^2)}\log(x^2+b^2)$$

$$= \frac{1}{2(a^2-b^2)}\log\frac{x^2+b^2}{x^2+a^2}$$ #

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