# Study Material

U.G. Sem - VI
Mathematics
paper - CC13

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## Fourier Transform

Q. Find the Fourier Sine transform of $\frac{e^{-ax}}{x}$.

Solⁿ:- Let $f(x) = \frac{e^{-ax}}{x}$, then its fourier sine transform

i.e. $F_s\{f(x)\} = \int_0^\infty f(x) \sin sx \, dx$

$$\Rightarrow F_s\{f(x)\} = \int_0^\infty \frac{e^{-ax}}{x} \sin sx \, dx$$

differentiate both sides w.r.to $s$, we get

$$\Rightarrow \frac{d}{ds} F_s\{f(x)\} = \int_0^\infty \frac{e^{-ax}}{\cancel{x}} \cos sx \cdot \cancel{x} \, dx$$

$$\Rightarrow \frac{d}{ds} F_s\{f(x)\} = \int_0^\infty e^{-ax} \cos sx \, dx$$

$$\Rightarrow \frac{d}{ds} F_s\{f(x)\} = \frac{a}{s^2+a^2}$$

$\Rightarrow F_s\{f(x)\} = \int \frac{a}{s^2+a^2} ds = a \int \frac{1}{s^2+a^2} ds$

$\Rightarrow F_s\{f(x)\} = \cancel{a} \cdot \frac{1}{\cancel{a}} \tan^{-1}\frac{s}{a} + c$

$\Rightarrow F_s\{f(x)\} = \tan^{-1}\frac{s}{a} + c$ — (1)

But when $s = 0$, $F_s\{f(x)\} = 0$

$\Rightarrow c = 0$

$\Rightarrow F_s\{f(x)\} = \tan^{-1}\frac{s}{a}$ #

Q. Find the Fourier cosine transform of $f(x) = \frac{1}{(1+x^2)}$

Solⁿ.:- ∵ Fourier cosine transform of $f(x)$ is

$$F_c\{f(x)\} = \int_0^\infty f(x) \cos sx \, dx = I_{say}$$

$$\Rightarrow I = \int_0^\infty \frac{1}{1+x^2} \cos sx \, dx = \int_0^\infty \frac{\cos sx}{1+x^2} dx \quad \text{— (1)}$$

$\Rightarrow$ Differentiate w.r.to $s$

$$\Rightarrow \frac{dI}{ds} = \int_0^\infty \frac{-x \sin sx}{1+x^2} dx = \int_0^\infty \frac{-x^2 \sin sx}{x(1+x^2)} dx \quad \text{— (2)}$$

$$\Rightarrow \frac{dI}{ds} = -\int_0^\infty \frac{[(1+x^2)-1]\sin sx}{x(1+x^2)}$$

$$= -\int_0^\infty \frac{\sin sx}{x}dx + \int_0^\infty \frac{\sin sx}{x(1+x^2)}dx$$

$$\Rightarrow \frac{dI}{ds} = -\frac{\pi}{2} + \int_0^\infty \frac{\sin sx}{x(1+x^2)}dx, \text{ as } \int_0^\infty \frac{\sin bx}{x}dx = \frac{\pi}{2} \quad \text{— (3)}$$

again diff. w.r. to S

$$\Rightarrow \frac{d^2I}{ds^2} = 0 + \int_0^\infty \frac{\cancel{x}\cos sx}{\cancel{x}(1+x^2)}dx$$

$$\Rightarrow \frac{d^2I}{ds^2} = \int_0^\infty \frac{\cos sx}{(1+x^2)}dx = I, \text{ by } (1)$$

$$\Rightarrow \frac{d^2I}{ds^2} - I = 0$$

$$\Rightarrow D^2I - I = 0 \quad , \text{ where } D = \frac{dI}{ds}$$

Its Solution is

$$I = C_1e^{s} + C_2e^{-s} \quad \text{— (4)}$$

$$\& \frac{dI}{ds} = C_1e^{s} - C_2e^{-s} \quad \text{— (5)}$$

when $s = 0$, (1) & (4) gives

$$c_1 + c_2 = \int_0^\infty \frac{dx}{1+x^2} = [\tan^{-1} x]_0^\infty$$

$$\Rightarrow c_1 + c_2 = \frac{\pi}{2} \quad \text{—} (6)$$

Also when $s = 0$, (3) & (5) give

$$c_1 - c_2 = -\frac{\pi}{2} \quad \text{—} (7)$$

$$(6) + (7) \Rightarrow c_1 = 0$$

$$\therefore c_2 = -\frac{\pi}{2}$$

$$\therefore F_c\{f(x)\} = I = \frac{\pi}{2} e^{-s} \quad \#$$

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