

Sem - IV : Economics - Honours.
 Compulsory: Mathematical Economics
 Application of Mathematics in Economics:

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Elasticity of Demand:

By definition, elasticity is the quantitative relationship between price and Demand. It measures the degree of responsiveness of Demand due to some change in its price.

Symbolically:

$$e_d = \text{Elasticity of Demand} = \frac{\text{Relative change in Demand}}{\text{Relative change in Price}} \left(\frac{\Delta Q}{Q} \div \frac{\Delta P}{P} \right)$$

$$\Rightarrow e_d = (-) \frac{\frac{\text{Change in Demand}}{\text{Original Demand}}}{\frac{\text{Change in Price}}{\text{Original Price}}}$$

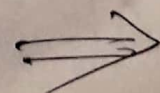
$$= (-) \frac{\frac{\text{Final Demand} - \text{Initial Demand}}{\text{Initial Demand}}}{\frac{\text{Final Price} - \text{Initial Price}}{\text{Initial Price}}}$$

$$= (-) \frac{\frac{Q_1 - Q}{Q}}{\frac{P_1 - P}{P}} = (-) \frac{\frac{40}{Q}}{\frac{4P}{P}}$$

$$= - \frac{40}{Q} \times \frac{P}{4P} = (-) \frac{40}{4P} \cdot \frac{P}{Q}$$

$$e_d \text{ at } \bar{P} = \lim_{\Delta P \rightarrow 0} \left\{ (-) \frac{40}{4P} \cdot \frac{P}{Q} \right\}$$

$$= \lim_{\Delta P \rightarrow 0} \frac{40}{4P} \left(- \frac{P}{Q} \right)$$



$$\Rightarrow ed = (-) \frac{p}{e} \cdot \frac{dq}{dp},$$

Alternatively, Marginal Function

$$|ed| = \frac{\text{Marginal Function}}{\text{Average Function}},$$

~~Q~~ Illustration :

$$x = 25 - 4p + p^2$$

$$\therefore \frac{dx}{dp} = -4 + 2p$$

$$\therefore ed = (-) \frac{p}{x} \cdot \frac{dx}{dp}$$

$$= (-) \frac{p}{25 - 4p + p^2} \cdot (-4 + 2p)$$

$$= \frac{4p + 2p^2}{25 - 4p + p^2}$$

The numerical estimate of ed depends upon the given value of p .

Alternatively,

$$|ed| = \frac{\text{Marginal Function}}{\text{Average Function}}$$

$$M.F = \frac{dx}{dp} = -4 + 2p$$

$$A.F = \frac{x}{p} = \frac{25 - 4p + p^2}{p}$$



$$|e_d| = \frac{25 - 4P + P^2}{P} \times (-4 + 2P)$$

$$|e_d| = \frac{-4 + 2P}{25 - 4P + P^2} \times P$$

$$= \frac{(-4 + 2P)P}{25 - 4P + P^2}$$

$$\Rightarrow e_d = \frac{(-) (-4 + 2P)P}{25 - 4P + P^2}$$

$$= \frac{4P - 2P^2}{25 - 4P + P^2} \quad \text{Ans}$$

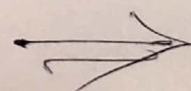
Relationship among Average Revenue, Marginal Revenue and the Elasticity of Demand:

$$AR = \frac{R}{Q} = \frac{P \cdot Q}{Q} = P.$$

$$MR = \frac{dR}{dQ}.$$

$$\therefore R = P \cdot Q.$$

$$\therefore MR = \frac{d}{dQ} (P \cdot Q)$$



Section IV

28

$$\begin{aligned} \Rightarrow MR &= \frac{d}{dQ} (P \cdot Q) \\ &= Q \cdot \frac{dP}{dQ} + P \cdot \frac{dQ}{dQ} \\ &= P + Q \cdot \frac{dP}{dQ} \\ &= P \left(1 + \frac{Q}{P} \cdot \frac{dP}{dQ} \right) \\ &= P \left[1 - \frac{-1}{\frac{P}{Q} \cdot \frac{dQ}{dP}} \right] \quad \left\{ + = - \times - \right. \\ &= P \left[1 - \frac{1}{ed} \right] \quad \left\{ ed = - \frac{P}{Q} \cdot \frac{dQ}{dP} \right. \\ \frac{MR}{P} &= \frac{AR \left(\frac{ed - 1}{ed} \right)}{ed} = \frac{AR \cdot ed - AR}{ed} \end{aligned}$$

$$\begin{aligned} \Rightarrow AR \cdot ed - AR &= MR \cdot ed \\ \Rightarrow AR \cdot ed - MR \cdot ed &= AR \\ \Rightarrow ed (AR - MR) &= AR \\ \Rightarrow ed &= \frac{AR}{AR - MR} \quad \underline{\underline{\text{Established}}} \end{aligned}$$