Sem-IV : Economics - Honours. 9/7/2020 (1)

$\Rightarrow$ Mathematical-Economics. Dr. P.C. Bhakat

5 $y = -x^3 + \frac{9}{2}x^2 - 6x + 6$

$\frac{dy}{dx} = -3x^2 + 9x - 6 = f'(x)$ —— (i)

$\frac{d^2y}{dx^2} = -6x + 9 = f''(x)$ —— (ii)

(i) F.O.C : NC: $\frac{dy}{dx} = 0$

$\Rightarrow -3x^2 + 9x - 6 = 0 \Rightarrow -3(x^2 - 3x + 2) = 0$

$\Rightarrow \cancel{-3(x^2 - 3x + 6) = 0} \Rightarrow x^2 - 3x + 2 = \frac{0}{-3} = 0$

$\Rightarrow \cancel{x^2 - 3x + 6 = 0}$ $\quad x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x-2) - 1(x-2) = 0$

$\Rightarrow (x-2)(x-1) = 0$

$\Rightarrow x - 2 = 0$ or $x = 2$ $\quad \left\{ \begin{array}{l} x - 1 = 0 \\ x = 1 \end{array} \right.$

$\Rightarrow x = 1, 2,$

Verification and conclusion:

$\because \frac{d^2y}{dx^2} = -6x + 9$

$= -6 \times 1 + 9 \quad \{$ at $x = 1$

$= -6 + 9$

$= 3$

$\Rightarrow d^2y/dx^2 = 3 > 0$ —— (ii) $\Rightarrow$

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$\because \dfrac{dy}{dx} = 0$ — (i)

and $\dfrac{d^2y}{dx^2} > 0$ — (ii)

$\therefore$ The given function has a Minima at $x = 1$

~~Further at $x = 2$~~

$\cancel{\because \dfrac{d^2y}{dx^2} = f''(x) = -6x + 9 = 0}$

$\cancel{= -6\times 2 + 9 = 0}$

$\cancel{\Rightarrow -6x = -9}$ $\cancel{= -12 + 9}$

$\cancel{\therefore x = \dfrac{9}{6} = \dfrac{3}{2}}$

Further at $x = 2$

$$\frac{d^2y}{dx^2} = f''(x) = -6x + 9$$

$= -6\times 2 + 9$ $\{x = 2$

$= -12 + 9$

$= -3$

$\Rightarrow \dfrac{d^2y}{dx^2} < 0$ It satisfies the S.O.C = S.C of a Maxima at $x = 2$

$\therefore \dfrac{dy}{dx} = 0$ and $\dfrac{d^2y}{dx^2} < 0$

$\therefore$ The given function has a Maxima at $x = 2$

$\Rightarrow$ Maxima at $x = 2$, Minima at $x = 1$ } Ans

(3) Show that $y = x\sqrt{1+x}$ has a minimum value and $y = -x\cdot\sqrt{1+x}$ has maximum value. Why is it so?

Solution:

Given

$$y = x\sqrt{1+x}$$

$$\Rightarrow y = x(1+x)^{1/2} = u\cdot v \quad \left\{ \begin{array}{l} u = x \\ v = (1+x)^{1/2} \end{array} \right.$$

$$\therefore \frac{dy}{dx} = (1+x)^{1/2}\cdot\frac{d}{dx}(x) + x\cdot\left[\frac{d}{dx}(1+x)^{1/2}\right]$$

$$= (1+x)^{1/2}\times 1 + x\cdot\left[\frac{1}{2}(1+x)^{\frac{1}{2}-1}\cdot\frac{d}{dx}(1+x)\right]$$

$$= (1+x)^{1/2} + \frac{x}{2}(1+x)^{-1/2}\times 1$$

$$= \frac{1+x + x(1+x)^{-1/2}(1+x)^{1/2}}{2(1+x)^{1/2}}$$

$$= \frac{1+x + x\,\cancel{(1+x)}^{0}}{2(1+x)^{1/2}} = \frac{1+2x}{2(1+x)^{1/2}}$$

$\therefore$ ~~$\frac{dy}{dx} = 0$ and~~

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left\{\frac{1+2x}{2(1+x)^{1/2}}\right\} = \frac{d}{dx}\left(\frac{u}{v}\right)$$

$\Rightarrow$

$$\frac{d^2y}{dx^2} = \frac{2(1+x)^{1/2}\frac{d}{dx}(1+2x) - (1+2x)\frac{d}{dx}\left\{2(1+x)^{1/2}\right\}}{\left\{2(1+x)^{1/2}\right\}^2}$$ (4)

$$= \frac{2(1+x)^{1/2}\times 2 - (1+2x)\left\{\frac{1}{2}\times 2\,(1+x)^{-1/2}\cdot\frac{d}{dx}(1+x)\right\}}{4\,(1+x)}$$

$$= \frac{4(1+x)^{1/2} - (1+2x)(1+x)^{-1/2}}{4\,(1+x)}$$

It appears to be positive.

$\because$ $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

This Function has a Minima.

$\therefore$

Again for $y = -x\sqrt{1+x}$

$$\frac{d^2y}{dx^2} = (-)\,\frac{4(1+x)^{1/2} - (1+2x)(1+x)^{-1/2}}{4\,(1+x)}$$

is obviously negative and hence bears a Maxima because it satisfies both the conditions, Necessary (First Order) condition and the Sufficient (Second Order) condition simultaneously

i.e., (I) $\frac{dy}{dx} = 0$

and (II) $\frac{d^2y}{dx^2} < 0$

Ans