Compulsory : Mathematical Economics 3.7.2020 (1)

Sem – IV

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A little bit Revision before starting the application of it in Economics:

(a) $P = Q^{2.6} \Rightarrow \frac{dp}{dQ} = 2.6Q^{2.6-1} = 2.6\,Q^{1.6}$

(b) $C = Q^{-1} \Rightarrow \frac{dc}{dQ} = -1\,Q^{-1-1} = -\frac{1}{Q^2}$

(c) $C = \frac{Q^6}{Q^{-4.5}} = Q^6 \cdot Q^{4.5} = Q^{6+4.5} = Q^{10.5}$

$\Rightarrow \frac{dc}{dQ} = (1 \times 10.5)\,Q^{10.5-1} = 10.5\,Q^{9.5}$

(d) $C = 0.5Q^3 - 12Q^2 + 300$

$\frac{dc}{dQ} = 1.5Q^2 - 24Q.$

(e) $Q = \frac{12}{L^{2.5}} = 12L^{-2.5}$

$\frac{dQ}{dL} = (12 \times -2.5)\,L^{-2.5-1}$

$= -30\,L^{-3.5}$

(f) $p = \frac{1}{Q^5} = Q^{-5}$

$\frac{dp}{dQ} = (1 \times -5)\,Q^{-5-1} = -5Q^{-6}$

$= -\frac{5}{Q^6}$

(g) $C = 0.2Q^3 - 15Q^2 + 175Q + 1000$

$\frac{dc}{dQ} = 0.6Q^2 - 30Q + 175$

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Higher Derivatives:

(1) $p = 25Q^4 - 10Q^2 + 200$

$$\frac{dp}{dQ} = 100Q^3 - 20Q$$

$$\frac{d^2p}{dQ^2} = \frac{d}{dQ}\left(\frac{dp}{dQ}\right) = \frac{d}{dQ}\left(100Q^3 - 20Q\right)$$
$$= 300Q^2 - 20$$

$$\frac{d^3p}{dQ^3} = \frac{d}{dQ^2}\left(\frac{dp}{dQ}\right) = \frac{d}{dQ^2}\left(300Q^2 - 20\right)$$
$$= 600Q.$$

Application:

Let us now start to study how derivatives are used in Economics.

At the beginning, let us refer the well known Market Equilibrium approach in the context of profit maximisation.

By definition, we know that profit is the difference between ~~Total Cost~~ Total Revenue and Total cost.

$$\text{Profit} = \text{Total Revenue} - \text{Total Cost}$$

$$\Rightarrow \quad \pi = R - C \qquad \left\{ \begin{array}{l} R = P \cdot Q = f(Q) \\ C = f(Q) \end{array} \right.$$

$$\therefore \pi = f(Q)$$

According to the necessary condition of profit maximisation:

$$\frac{d\pi}{dQ} = 0 \quad \left\{ \text{First Order Condition.} \right.$$

$$\Rightarrow \frac{d}{dQ}(R - C) = 0$$

$$\Rightarrow \frac{dR}{dQ} - \frac{dC}{dQ} = 0 \qquad \left\{ \begin{array}{l} MR = \frac{dR}{dQ} \\ MC = \frac{dC}{dQ} \end{array} \right.$$

$\Rightarrow$

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$MR - MC = 0$

$\Rightarrow MR = 0 + MC = MC$

$\Rightarrow$ Marginal Revenue = Marginal cost

The F.O.C.: N.C. needs that there must be equality between Marginal Revenue and Marginal cost for maximisation of profit.

But mere satisfaction of the F.O.C.: N.C can not ensure the ultimate realization of the maximum profit; it requires the satisfaction of ~~the~~ its sufficient condition simultaneously.

(S.C) : Second Order Condition (S.O.C)

$$\Rightarrow \frac{d^2\pi}{dQ^2} < 0$$

$$\Rightarrow \frac{d}{dQ}\left(\frac{d\pi}{dQ}\right) < 0 \qquad \left\{ \because \frac{d\pi}{dQ} = MR - MC \text{ as obtained above} \right.$$

$$\Rightarrow \frac{d}{dQ}(MR - MC) < 0$$

$$\Rightarrow \frac{d}{dQ}(MR) - \frac{d}{dQ}(MC) < 0$$

$$\Rightarrow \frac{d}{dQ}(MR) < 0 + \frac{d}{dQ}(MC)$$

$$\Rightarrow \frac{d}{dQ}(MR) < \frac{d}{dQ}(MC)$$

$=$ Slope of the MR $<$ Slope of the MC

$\Rightarrow$ The MC-curve must cut the MR-curve from below.

We now need to study this concept of profit maximization in the language of graph. Graphic presentation of the concept of profit-maximization will illustrate the initial requirement of Market Equilibrium in detail.

$\Rightarrow$