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Study Material

U.G. Sem - VI (Mathematics)

paper - CC14

The inverse Laplace Transform :-

Q. Evaluate: $L^{-1}\left(\dfrac{p+1}{p^2+6p+25}\right)$

Soln:- We have $L^{-1}\left\{\dfrac{p+1}{p^2+6p+25}\right\}$

$$= L^{-1}\left\{\frac{(p+3)-2}{(p+3)^2+16}\right\}$$

$$= L^{-1}\{F(p+3)\}$$

$$= e^{-3t} F(p), \text{ using shifting theorem.}$$

$$= e^{-3t} L^{-1}\left\{\frac{p-2}{p^2+16}\right\}$$

$$= e^{-3t} L^{-1}\left[\frac{p}{p^2+4^2} - 2\,\frac{1}{p^2+4^2}\right]$$

$$= e^{-3t}\left[L^{-1}\left\{\frac{p}{p^2+4^2}\right\} - 2\,L^{-1}\left\{\frac{1}{p^2+4^2}\right\}\right]$$

$$= e^{-3t}\left[\cos 4t - \cancel{2}\,\frac{1}{\cancel{4}\,2}\sin 4t\right]$$

$$= e^{-3t}\left[\cos 4t - \frac{1}{2}\sin 4t\right]$$

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Q. Evaluate (i) $L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\}$ (ii) $L^{-1}\left\{\frac{3p+7}{p^2-2p-3}\right\}$

Solⁿ:- (i) $L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\} = L^{-1}\left\{\frac{3p-2}{(p-2)^2+16}\right\}$

$= L^{-1}\left\{\frac{3(p-2)+4}{(p-2)^2+4^2}\right\}$

$= L^{-1}\left\{\frac{3(p-2)}{(p-2)^2+4^2}\right\} + L^{-1}\left\{\frac{4}{(p-2)^2+4^2}\right\}$

$= e^{2t} L^{-1}\left\{\frac{3p}{p^2+4^2}\right\} + e^{2t} L^{-1}\left\{\frac{4}{p^2+4^2}\right\}$

by Shifting theorem

$= e^{2t}\, 3\cos 4t + e^{2t}\sin 4t.$

(ii) Evaluate $L^{-1}\left\{\frac{3p+7}{p^2-2p-3}\right\}$

$= L^{-1}\left\{\frac{3p+7}{(p-1)^2-4}\right\} = L^{-1}\left\{\frac{3(p-1)+10}{(p-1)^2-4}\right\}$

$= L^{-1}\left\{\frac{3(p-1)}{(p-1)^2-4}\right\} + 10\, L^{-1}\left\{\frac{1}{(p-1)^2-4}\right\}$

$= e^{t} L^{-1}\left\{\frac{3p}{p^2-4}\right\} + 10\, e^{t} L^{-1}\left\{\frac{1}{p^2-4}\right\}$

$= e^{t}\, 3\cosh 2t + 10\, e^{t}\, \frac{1}{2}\sinh 2t.$

$= e^{t}\left\{3\cosh 2t + 5\sinh 2t\right\}$

Q. Prove that $L^{-1}\left\{\dfrac{4p+5}{(p-1)^2(p+2)}\right\} = 3te^t + \frac{1}{3}e^t - \frac{1}{3}e^{-2t}$

Solⁿ:- We break $\dfrac{4p+5}{(p-1)^2(p+2)}$ into partial fractions.

Let $\dfrac{4p+5}{(p-1)^2(p+2)} = \dfrac{A}{(p-1)} + \dfrac{B}{(p-1)^2} + \dfrac{C}{p+2}$ — (1)

$$= \frac{A(p-1)(p+2) + B(p+2) + C(p-1)^2}{(p-1)^2(p+2)}$$

$\Rightarrow 4p+5 = A(p-1)(p+2) + B(p+2) + C(p-1)^2$ — (2)

$\Rightarrow 4p+5 = A(p^2+p-2) + B(p+2) + C(p^2-2p+1)$

$\Rightarrow 4p+5 = (A+C)p^2 + (A+B-2C)p + (-2A+2B+C)$

$\Rightarrow A+C = 0, \Rightarrow A = -C$ — (3)

$A+B-2C = 4 \Rightarrow B-3C = 4$ — (4)

$-2A+2B+C = 5 \Rightarrow 2B+3C = 5$ — (5)

(4)+(5) $\Rightarrow 3B = 9 \Rightarrow B = 3$

(4) $\Rightarrow 3C = B-4 = -1 \Rightarrow C = -\frac{1}{3}$

$\therefore A = \frac{1}{3}$

$\therefore$ (1) $\Rightarrow \dfrac{4p+5}{(p-1)^2(p+2)} = \dfrac{\frac{1}{3}}{(p-1)} + \dfrac{3}{(p-1)^2} + \dfrac{-\frac{1}{3}}{p+2}$

$\Rightarrow L^{-1}\left\{\dfrac{4p+5}{(p-1)^2(p+2)}\right\} = \frac{1}{3}L^{-1}\left\{\dfrac{1}{(p-1)}\right\} + 3L^{-1}\left\{\dfrac{1}{(p-1)^2}\right\} - \frac{1}{3}L^{-1}\left\{\dfrac{1}{p+2}\right\}$

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$$= \frac{1}{3} e^t L^{-1}\left\{\frac{1}{p}\right\} + 3 e^t \left\{\frac{1}{p^2}\right\} - \frac{1}{3} e^{-2t} L^{-1}\left\{\frac{1}{p}\right\}$$

$$= \frac{1}{3} e^t (1) + 3 e^t (t) - \frac{1}{3} e^{-2t} (1)$$

$$= \frac{1}{3} e^t + 3te^t - \frac{1}{3} e^{-2t}.$$

Q. Find : $L^{-1}\left\{\frac{p}{(p^2+a^2)^2}\right\}$

Solⁿ:- We have, $L^{-1}\left\{\frac{1}{p^2+a^2}\right\} = \frac{1}{a}\sin at$

$$\therefore L^{-1}\left\{\frac{d}{dp}\left(\frac{1}{p^2+a^2}\right)\right\} = \frac{1}{a}(-1)\, t \sin at$$

as $L^{-1}\left\{\frac{d}{dp}F(p)\right\} = (-1)t f(t) = (-1)t\, L^{-1}\{F(p)\}$.

$$\Rightarrow L^{-1}\left\{\frac{-2p}{(p^2+a^2)^2}\right\} = -\frac{1}{a} t \sin at$$

$$\Rightarrow L^{-1}\left\{\frac{p}{(p^2+a^2)^2}\right\} = \frac{1}{2a} t \sin at.$$

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