Study Material

U.G. Sem - VI (Mathematics) Date: 14/5/20

paper - CC 14.

Dr. M. A. Khan
Associate Professor
Dept. of Mathematics
Tata College, Chaibasa

The Inverse Laplace Transform

Def^n:- Let $F(p)$ be the Laplace transformation of a function $f(t)$ i.e. $L\{f(t)\} = F(p)$

Then $f(t)$ is called the inverse Laplace transform of the function $F(p)$ and is written as

$$f(t) = L^{-1}\{F(p)\}$$

L^{-1} is called the inverse Laplace transformation operator.

Table of inverse Laplace transforms

$F(p)$	$L^{-1}\{F(p)\} = f(t)$
$\frac{1}{p}$	1
$\frac{1}{p^{n+1}}$, n = +ve integer	$\frac{t^n}{\lfloor n}$
$\frac{1}{(p-a)}$	e^{at}
$\frac{a}{p^2+a^2}$	$\sin at$
$\frac{p}{p^2+a^2}$	$\cos at$

2020.05.

$\frac{a}{p^2-a^2}$	Sinhat
$\frac{p}{p^2-a^2}$	Coshat

Q. Find (i) $L^{-1}\left\{\frac{1}{p^4}\right\}$ (ii) $L^{-1}\left\{\frac{1}{2p-5}\right\}$

(iii) $L^{-1}\left\{\frac{2p-5}{p^2-9}\right\}$ (iv) $L^{-1}\left\{\frac{3p-8}{4p^2+25}\right\}$

Solⁿ:- (i) $L^{-1}\left\{\frac{1}{p^4}\right\}$

$\because L^{-1}\left\{\frac{1}{p^{n+1}}\right\} = \frac{t^n}{\lfloor n}$

put $n=3$

$\Rightarrow L^{-1}\left\{\frac{1}{p^4}\right\} = \frac{t^3}{\lfloor 3}$

(ii) $L^{-1}\left\{\frac{1}{2p-5}\right\} = L^{-1}\left\{\frac{1}{2(p-5/2)}\right\}$

$= \frac{1}{2} L^{-1}\left\{\frac{1}{p-\frac{5}{2}}\right\} = \frac{1}{2} e^{5/2\, t}$

(iii) We have, $\frac{2p-5}{p^2-9} = \frac{2p}{p^2-9} - \frac{5}{p^2-9}$

$\therefore L^{-1}\left\{\frac{2p-5}{p^2-9}\right\} = 2L^{-1}\left\{\frac{p}{p^2-3^2}\right\} - \frac{5}{3}L^{-1}\left\{\frac{3}{p^2-3^2}\right\}$

$= 2\cosh 3t - \frac{5}{3}\sinh 3t.$

2020.05.14

(iv) We have $\frac{3p-8}{4p^2+25} = \frac{3p}{4p^2+25} - \frac{8}{4p^2+25}$

$$= \frac{3}{4}\frac{p}{p^2+\frac{25}{4}} - 2\frac{1}{p^2+\frac{25}{4}}$$

$$= \frac{3}{4}\frac{p}{p^2+(\frac{5}{2})^2} - 2\frac{1}{p^2+(\frac{5}{2})^2}$$

$$\therefore L^{-1}\left\{\frac{3p-8}{4p^2+25}\right\} = \frac{3}{4}L^{-1}\left\{\frac{p}{p^2+(\frac{5}{2})^2}\right\} - 2L^{-1}\left\{\frac{1}{p^2+(\frac{5}{2})^2}\right\}$$

$$= \frac{3}{4}\cos\frac{5}{2}t - 2\times\frac{2}{5}\sin\frac{5}{2}t$$

$$= \frac{3}{4}\cos\frac{5}{2}t - \frac{4}{5}\sin\frac{5}{2}t. \quad \#$$

(Shifting Theorem):-

Theorem:- If $L^{-1}\{F(p)\} = f(t)$ then

$$L^{-1}\{F(p-a)\} = e^{at}f(t) = e^{at}F^{-1}\{F(p)\}$$

Proof:- We have, $F(p) = \int_0^\infty e^{-pt}f(t)\,dt$

$$\Rightarrow F(p-a) = \int_0^\infty e^{-(p-a)t}f(t)\,dt$$

$$= \int_0^\infty e^{-pt}\{e^{at}f(t)\}\,dt$$

$$\Rightarrow F(p-a) = L\{e^{at}f(t)\}$$

$$\Rightarrow L^{-1}\{F(p-a)\} = e^{at}f(t) = e^{at}L^{-1}\{F(p)\}$$

Q. Find: (i) $L^{-1}\left\{\frac{1}{(p+a)^n}\right\}$ (ii) $L^{-1}\left\{\frac{1}{p^2-6p+10}\right\}$

Solⁿ:- (i) $L^{-1}\left\{\frac{1}{(p+a)^n}\right\} = L^{-1}\left\{\frac{1}{(p-(-a))^n}\right\}$

$= e^{-at} L^{-1}\left\{\frac{1}{p^n}\right\} = e^{-at}\frac{t^{n-1}}{\lfloor n-1}$, by Shifting Theorem. #

(ii) $L^{-1}\left\{\frac{1}{p^2-6p+10}\right\}$

$= L^{-1}\left\{\frac{1}{p^2-2p\cdot 3+3^2+1}\right\} = L^{-1}\left\{\frac{1}{(p-3)^2+1^2}\right\}$

$= e^{3t} L^{-1}\left\{\frac{1}{p^2+1^2}\right\}$, by Shifting Theorem

$= e^{3t}\sin t$

Q Find $L^{-1}\left\{\frac{1}{p^2+8p+16}\right\}$

Solⁿ $L^{-1}\left\{\frac{1}{p^2+8p+16}\right\} = L^{-1}\left\{\frac{1}{p^2+2\cdot p\cdot 4+4^2}\right\}$

$= L^{-1}\left\{\frac{1}{(p+4)^2}\right\} = e^{-4t} L^{-1}\left\{\frac{1}{p^2}\right\}$, by Shifting Theorem

$= e^{-4t}\frac{t}{\lfloor 1} = t e^{-4t}$ #

2020.05.14