Study Materials for
Sem - V (Mathematics)
Paper → DSEMATH 502A.
(Theory of equation)

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Topic :- Removal of terms

One of the most important applications of the transformation is to remove the certain specific term from an equation.

Let us consider the equation

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$ — ①

In order to remove the second or third term the roots of of the given equation diminished by ① by h where h ~~can~~ is obtained by

i) for removal of second term use the equation

$na_0h + a_1 = 0$ i.e. $h = -\frac{a_1}{na_0}$

ii) For removal of third term use the equation

$\frac{n(n-1)}{1\cdot 2}a_0h^2 + (n-1)a_1h + a_2 = 0$

Ex:- Transform the equation $x^4 + 8x^3 + x - 5 = 0$ into one in which the second term is missing

Sol^n. :- $\because$ $x^4 + 8x^3 + x - 5 = 0$ may be written as

$$x^4 + 8x^3 + 0x^2 + x - 5 = 0 \longrightarrow (2)$$

In order to ~~d~~ remove the second term let the roots of (2) is diminished ~~the~~ by h where h may be obtained by

$$na_0 h + a_1 = 0 \Rightarrow 4 \cdot 1 \cdot h + 8 = 0 \Rightarrow h = -2.$$

$\therefore$ ~~using~~ making use of synthetic division

1	8	0	1	−5
	−2	−12	24	−50
	6	−12	25	−55
	−2	−8	40	0
	4	−20	65	
	−2	−4		
	2	−24		
	−2			
	0			

Therefore the transformed equation is

$$\cancel{y^4 - 24y^2} \quad y^4 + 0y^3 - 24y^2 + 65y - 55 = 0$$

$$\Rightarrow y^4 - 24y^2 + 65y - 55 = 0$$

Solve the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing the second term.

Solⁿ:- Here Given equation is

$$x^4 + 20x^3 + 143x^2 + 430x + 462 = 0 \quad \text{--- (1)}$$

here $a_0 = 1$, $a_1 = 20$, $a_2 = 143$, $a_3 = 430$, $a_4 = 462$

$\therefore$ ~~Here~~ $n a_0 h + a_1 = 0$ gives $4 \cdot 1 \cdot h + 20 = 0$

$\Rightarrow h = -5$

Here we diminish the roots by -5

$\therefore$ making use of Synthetic division

$$\begin{array}{ccccc}
1 & 20 & 143 & 430 & 462 \\
 & -5 & -75 & -340 & -450 \\ \hline
 & 15 & 68 & 90 & 12 \\
 & -5 & -50 & -90 & \\ \hline
 & 10 & 18 & 0 & \\
 & -5 & -25 & & \\ \hline
 & 5 & -7 & & \\
 & -5 & & & \\ \hline
 & 0 & & &
\end{array}$$

Hence the transformed equation is

$y^4 + 0y^3 - 7y^2 + 0y + 12 = 0 \Rightarrow y^4 - 7y^2 + 12 =$

$\Rightarrow y^4 - 3y^2 - 4y^2 + 12 = 0 \Rightarrow y^2(y^2-3) - 4(y^2-3)$

$\Rightarrow (y^2-4)(y^2-3) = 0 \Rightarrow y^2 = 4$ or $y^2 = 3$

$\Rightarrow y = \pm 2$ or $\pm\sqrt{3}$

Hence $x = y + h = 2-5, -2-5, \sqrt{3}-5, -\sqrt{3}-5$

$\therefore x = -7, -3, -5+\sqrt{3}, -5-\sqrt{3}$ #

Equation with binomial coefficients

It is often found convenient to write the equation in the form

$$a_0 x^n + n a_1 x^{n-1} + \frac{n(n-1)}{\lfloor 2} a_2 x^{n-2} + \cdots + \frac{n(n-1)}{\lfloor 2} a_{n-2} x^2$$
$$+ n a_{n-1} x + a_n = 0 \quad \text{--- (1)}$$

Here the successive coefficients are obtained by multiplying $a_0, a_1, a_2, \ldots a_{n-1}, a_n$ respectively by $1, n, \frac{n(n-1)}{\lfloor 2} \ldots \frac{n(n-1)}{\lfloor 2}, n, 1$ which are the successive coefficients in the expansion of $(1+x)^n$.

Let us denote the L.H.S of (1) by $f_n(x)$ so that

$$f_n(x) = a_0 x^n + n a_1 x^{n-1} + \frac{n(n-1)}{\lfloor 2} a_2 x^{n-2} + \cdots + \frac{n(n-1)}{\lfloor 2} a_{n-2} x^2$$
$$+ n a_{n-1} x + a_n \quad \text{--- (2)}$$

replacing n by $n-1, n-2, \ldots 4, 3, 2, 1, 0$, we get

$$f_{n-1}(x) = a_0 x^{n-1} + (n-1) a_1 x^{n-2} + \cdots + (n-1) a_{n-2} x + a_{n-1}$$

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$$f_4(x) = a_0 x^4 + 4 a_1 x^3 + 6 a_2 x^2 + 4 a_3 x + a_4$$
$$f_3(x) = a_0 x^3 + 3 a_1 x^2 + 3 a_2 x + a_3$$
$$f_2(x) = a_0 x^2 + 2 a_1 x + a_2$$
$$f_1(x) = a_0 x + a_1$$
$$f_0(x) = a_0.$$

Q. Reduce the cubic $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ in the form $x^3 + 3Hx + G = 0$.

Solⁿ:- Let $f_3(x) = a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ — ①

We diminish the roots of ① by h, so that the transformed equation will be in the form

$$a_0y^3 + 3f_1(h)y^2 + 3f_2(h)y + f_3(h) = 0$$

i.e. $A_0y^3 + 3A_1y^2 + 3A_2y + A_4 = 0$ —— ②

where $y = x-h$

If the second term is absent in transformed equation, we must have $f_1(h) = 0$

i.e. $f_1(h) = a_0h + a_1 = 0 \Rightarrow h = -\frac{a_1}{a_0}$

$$\therefore A_2 = f_2(h) = a_0h^2 + 2a_1h + a_2$$

$$= a_0\left(-\frac{a_1}{a_0}\right)^2 + 2a_1\left(-\frac{a_1}{a_0}\right) + a_2$$

$$= \frac{a_1^2}{a_0} - \frac{2a_1^2}{a_0} + a_2 = -\frac{a_1^2}{a_0} + a_2$$

$$\Rightarrow A_2 = \frac{a_0a_2 - a_1^2}{a_0}$$

and $A_3 = f_3(h) = a_0h^3 + 3a_1h^2 + 3a_2h + a_3$

$$= a_0\left(-\frac{a_1}{a_0}\right)^3 + 3a_1\left(-\frac{a_1}{a_0}\right)^2 + 3a_2\left(-\frac{a_1}{a_0}\right) + a_3$$

$$= -\frac{a_1^3}{a_0^2} + \frac{3a_1^3}{a_0^2} - \frac{3a_1a_2}{a_0} + a_3$$

$$\Rightarrow A_3 = \frac{-a_1^3 + 3a_1^3 - 3a_0a_1a_2 + a_0^2a_3}{a_0^2}$$

$$\Rightarrow A_3 = \frac{2a_1^3 - 3a_0a_1a_2 + a_0^2a_3}{a_0^2}$$

$\therefore$ (2) can be written as

$$a_0y^3 + 0y^2 + 3\frac{(a_0a_2 - a_1^2)}{a_0}y + \left(\frac{2a_1^3 - 3a_0a_1a_2 + a_0^2a_3}{a_0^2}\right) = 0$$

$$\Rightarrow a_0y^3 + 3\frac{H}{a_0}y + \frac{G}{a_0^2} = 0 \quad \text{— (3)}$$

where $H = a_0a_2 - a_1^2$ & $G = 2a_1^3 - 3a_0a_1a_2 + a_0^2a_3$

$$(3) \Rightarrow y^3 + \frac{3H}{a_0^2}y + \frac{G}{a_0^3} = 0$$

multiplying the roots of (3) by a_0

$$\Rightarrow \boxed{z^3 + 3Hz + G = 0}$$

where $z = a_0y$

Relation between x, y, z

$\because z = a_0y = a_0(x-h)$, where $y = x-h$

$$\Rightarrow z = a_0(x-h) = a_0\left(x + \frac{a_1}{a_0}\right)$$

$$\Rightarrow \boxed{z = a_0x + a_1}$$