Study Material for
Semester - V (Mathematics)
Paper → DSE MATH 502 A
(Theory of Equation)

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Dr. M. A. Khan
Associate Professor
Tata College,
Chaibasa

Topic :- Transformation of Equations

Introduction :- It is often desirable to transform a given equation into another equation when the roots of the transformed equation are connected with those of the given equation by certain assigned relations. ~~that we shall Study in this~~ Some important elementry transformations are as mentioned below.

① Equation whose roots are those of the given equation with signs changed.

Rule :- Change the sign of every alternative term of the given equation, beginning with the second after making the equation complete if it is not so.

Ex :- Change the signs of the roots of the equation

$$x^5 + 4x^3 - 7x^2 + 5 = 0$$

Sol^n :- The given equation may be written as

$$x^5 + 0x^4 + 4x^3 - 7x^2 + 0x + 5 = 0$$

Hence the required equation is

$$x^5 - 0x^4 + 4x^3 + 7x^2 + 0x - 5 = 0$$

i.e. $x^5 + 6x^3 + 7x^2 - 5 = 0$.

2. Multiply the roots of the equation by a given quantity.

Rule:- Multiply the successive coefficients of the given equation beginning with the second by $m, m^2, m^3, \ldots$, after making the equation complete if it is not so.

Ex:- Find the equation whose roots are three times the roots of the equation $3x^3 + 5x^2 + 2x - 10 = 0$

Solution:- The required equation is

$$3x^2 + 3 \cdot 5x^2 + 3^2 \cdot 2x - 3^3 \cdot 10 = 0$$

i.e. $x^3 + 5x^2 + 6x - 90 = 0$.

3. Equation whose roots are the reciprocal of the ~~given~~ roots of the ~~given~~ equation

Rule:- Take the last coefficient of the given equation to be the first coefficient of the transformed equation, last but one to be the second and so on after making the given equation complete if it is not so.

Ex:- If α, β, γ and δ be the roots of the equation

$$x^5 - 10x^3 + 35x^2 - 50x + 24 = 0$$

Then the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ and $\frac{1}{\delta}$ is $24x^4 - 50x^3 + 35x^2 - 10x + 1 = 0$.

4. Increase or diminish the roots of the given equation by a given quantity.

Rule:- Let the given equation be

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$

we want to diminish the roots of the equation by h.

In order to find the successive coefficient of the transformed equation, divide the given equation by $x-h$, divide the quotient thus obtained by $x-h$, and so on. The successive remainders thus obtained are the required coefficients, the first coefficient being the same as that of the given equation.

Ex:- Find the equation whose roots are the roots of the $x^5 + 4x^3 - x^2 + 11 = 0$ each diminish by 2.

Solution:- The given equation may be written as

$$x^5 + 0x^4 + 4x^3 - x^2 + 0x + 11 = 0 \quad \text{—①}$$

In order to diminish the roots of ① by 2 we shall divide the equation by $x-2$ by making use of Synthetic division.

1	0	4	−1	0	11
	2	4	16	30	60
	2	8	15	30	71 = A5

$$\begin{array}{cccc} 2 & 8 & 15 & 20 \\ \underline{2} & \underline{8} & \underline{32} & \underline{94} \\ 4 & 16 & 47 & 124 = A_4 \end{array}$$

$$\begin{array}{ccc} 2 & 12 & 56 \\ \underline{6} & \underline{28} & \underline{103} = A_3 \end{array}$$

$$\begin{array}{cc} 2 & 16 \\ \underline{8} & \underline{44} = A_2 \end{array}$$

$$\begin{array}{c} 2 \\ \underline{10} = A_1 \end{array}$$

Hence the transformed equation is

$$x^5 + 10x^4 + 44x^3 + 103x^2 + 124x + 71 = 0$$

Some questions to solved by the students

Q. ① Change the sign of the roots of the equation $x^5 - 4x^3 + 3x^2 + 1 = 0$

Q. ② Find the equation whose roots are 5 times the roots of the equation

$$5x^3 + 4x^2 + 2x - 20 = 0$$

Q. ③ Find the equation whose roots are the roots of $x^5 + 4x^3 - x^2 + 11 = 0$ each diminished by 3.