Study Materials for
U.G. Semester - V (Mathematics)
paper → DSE MATH 502A
(Theory of Equation)

Date -
Dr. M. A.
Associate
Deptt. of Ma
Tata Colle
Chaibas

Topic :- Cardon's Method

Q. Discuss the Cardon's method of solving the cubic equation.

Solⁿ:- Let the cubic equation be

$$a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0 \quad \text{--- (1)}$$

Let the given equation be transformed into the form

$$z^3 + 3Hz + G = 0 \quad \text{--- (2)}$$

where $z = a_0x + a_1$ and H and G have usual meanings.

Let $z = p^{1/3} + q^{1/3}$ be the solution of (2 --- (3)

$$\Rightarrow z^3 = p + q + 3p^{1/3}q^{1/3}(p^{1/3} + q^{1/3})$$

$$\Rightarrow z^3 = (p+q) + 3p^{1/3}q^{1/3}z.$$

$$\Rightarrow z^3 - 3p^{1/3}q^{1/3}z - (p+q) = 0 \quad \text{--- (4)}$$

Since (2) & (4) are identical, therefo

Comparing their coefficients

$\Rightarrow -3p^{1/3}q^{1/3} = 3H \Rightarrow pq = -H^3$

and $-(p+q) = G \Rightarrow p+q = -G$

Therefore, p, q are the roots of the quadratic equation

$$t^2 + Gt - H^3 = 0 \quad - (5)$$

$$\Rightarrow t = \frac{-G \pm \sqrt{G^2+4H^3}}{2}$$

Therefore we can take

$$\left.\begin{aligned} p &= \frac{-G + \sqrt{G^2+4H^3}}{2} \\ \& \; q &= \frac{-G - \sqrt{G^2+4H^3}}{2} \end{aligned}\right\} - (6)$$

Now cube roots of $p = p^{1/3}, p^{1/3}\omega, p^{1/3}\omega^2$

& cube roots of $q = q^{1/3}, q^{1/3}\omega, q^{1/3}\omega^2$

Three values of z will be

$z = p^{1/3}+q^{1/3}$, $p^{1/3}\omega + q^{1/3}\omega^2$ & $p^{1/3}\omega^2+q^{1/3}\omega$

From these values of z, the value of x can be obtained by the relation $z = a_0x+a_1$.

Q. Solve $x^3 - 9x + 28 = 0$ by Cardan's method

Soln:- $\because x^3 - 9x + 28 = 0$ — (1)

The given equation is in the ~~form~~ standard form $x^3 + 3Hx + G$ — (2)

$\Rightarrow 3H = -9$ & $G = 28$

$\Rightarrow H = -3$ & $G = 28$

Let $x = p^{1/3} + q^{1/3}$ — (3)

$\Rightarrow x^3 = p + q + 3p^{1/3}q^{1/3}(p^{1/3} + q^{1/3})$

$\Rightarrow x^3 = (p+q) + 3p^{1/3}q^{1/3}x$

$\Rightarrow x^3 - 3p^{1/3}q^{1/3}x - (p+q) = 0$ — (4)

but (2) & (4) are identical

$\Rightarrow -3p^{1/3}q^{1/3} = -9 \Rightarrow p^{1/3}q^{1/3} = 3$

$\Rightarrow pq = 27$

& $p + q = -28$

Hence p, q are the roots of the equa

$t^2 - (p+q)t + pq = 0$

$\Rightarrow t^2 + 28t + 27 = 0$ — (5)

$\Rightarrow t^2+27t+t+27=0$

$\Rightarrow t(t+27)+(t+27)=0$

$\Rightarrow (t+1)(t+27)=0$

$\Rightarrow t=-1, -27.$

take $p=-27$ & $q=-1$

Hence roots of ① are

$p^{1/3}+q^{1/3} = (-27)^{1/3}+(-1)^{1/3} = -3-1=-4$

$p^{1/3}\omega + q^{1/3}\omega^2 = \omega(-3)+\omega^2(-1)$

$= -3\omega - \omega^2 = \cancel{2\omega}$

$= -2\omega - (\omega+\omega^2) = -2\omega+1$ as $\omega^2+\omega+1$

$= -2\left(\frac{-1+\sqrt{3}i}{2}\right)+1$

$= 1-\sqrt{3}i+1 = 2-\sqrt{3}i$

also $p^{1/3}\omega^2 + q^{1/3}\omega = -3\omega^2-\omega$

$= -2\omega^2-(\omega^2+1)$

$= -2\omega^2+1$

$= -2\left(\frac{-1-\sqrt{3}i}{2}\right)+1$

$= 1+\sqrt{3}i+1 = 2+\sqrt{3}i$

$\therefore$ The roots of the given equation is

$-4,\ 2+\sqrt{3}i,\ 2-\sqrt{3}i$

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