Introduction to Algebraic Fractions
Addition and Subtraction

Big Idea

- The Lowest Common Multiple is used to find a common denominator.
- Common denominators must be found in order to add or subtract fractions because of the order of operations.

Key Knowledge

Students need to know how to find the LCM of algebraic expressions and must be able to do simple multiplication with algebraic expressions.

Pro-Tip

Writing with additional space between characters will lend itself to additional neatness on the page. This can really help a student use their work as a record of thinking. This can keep them on track if distract and allow them to examine their process to find errors, instead of starting over.

In the last section we saw both how to reduce Algebraic Fractions, which if you recall, are also called Rational Expressions, but also how the math works. Because of the relationship between division and multiplication, and multiplication's commutative property, we can reduce Algebraic Fractions like the one below:

$$\frac{3a+6}{12a^2-9}$$

When dealing with Algebraic Fractions, your work is not done until you've reduced completely. The example above is the unfinished answer to the first problem we will do to introduce addition and subtraction of Algebraic Fractions.

$$\frac{2a-5}{12a^2-9}+\frac{a+11}{12a^2-9}$$

Since these have a common denominator, we just add the like terms in the numerator. Note, in math when we say add, it means combine with addition and subtraction.

$$\frac{2a-5+a+11}{12a^2-9}$$

Combining like terms, we end up with:

$$\frac{3a+6}{12a^2-9}$$

But since each term has a factor of 3, we can reduce each term by 3:

$$\frac{3a+6}{12a^2-9} \rightarrow \frac{\cancel{3}a+\cancel{3}\cdot 2}{\cancel{3}\cdot 4\cdot a^2-3\cdot\cancel{3}} = \frac{a+2}{4a^2-3}.$$

Our answer has four total terms. While some share a common factor, not all four terms share a common factor, so we are finished.

If the denominators are the same, you just combine the numerators.

With subtraction it is slightly trickier. Let's simplify a problem and see how this works.

$$5 - 4 + 3$$

$$5 - (4 + 3)$$

The two expressions above are not the same. The first equals four, while the second is -2. The parenthesis make a group of the four and three, which is being subtracted from the five. The four and the three are both being subtracted from the five. But, great care is in order here. It is easy to mess up these signs.

$$5 - (4 - 3) = 4$$

Because 5 – 4 - -3 is 5 – 4 + 3.

Remember that fraction bars also create groups in Algebra. So, instead of parenthesis, you will see:

$$\frac{5}{x} - \frac{4+3}{x}$$

This is the same as:

$$\frac{5-(4+3)}{x} \text{ but not } \frac{5-4+3}{x}.$$

An example with variables would be:

$$\frac{x^3-5x^4}{9} - \frac{5x^4-x^3}{9}$$

Note: Exponents are repeated multiplication, they do not change from addition. Also, in order to be like terms, the variables and exponents must be the same. x^3 and x^5 are not like terms.

$$\frac{x^3-5x^4-\left(5x^4-x^3\right)}{9}$$

Caution and care are in order when dealing with subtraction and Algebraic Fractions!

$$\frac{x^3-5x^4-5x^4+x^3}{9}$$

Combining like terms we get

$$\frac{2x^3-10x^4}{9}.$$

Since there is not a common factor between all terms, we are done.

Adding and subtracting Algebraic Fractions with unlike denominators involves finding the LCM (lowest common multiple) of each denominator. We will restrict our denominators to monomials for now, as to keep this appropriate for beginning Algebra students.

Let's begin with the denominators *a* and *b*. All we know is that *a* and *b* are numbers that cannot be zero, but we don't know their exact value. So, we assume they are relatively prime, making their LCM their product, $a\times b$.

$$\frac{3}{a}+\frac{2}{b}$$

We arrive at common denominators through multiplication. Don't get confused here, we are multiplying by a number that equals one if it were reduced, but we don't want to reduce until we are finished. Also, note, that since e are multiplying by one, the expression will look different but will have the same value. It is not unlike the difference between a twenty dollar bill versus a ten and two five dollar bills.

Our denominator will be *ab*. To change *a* into *ab*, we multiply by *b*. We multiply *b* by *a*, writing the variables in alphabetical order will help to recognize that they are the same. (Sometimes students write *ab* and then *ba*. While they're the same, the order in which they're written can confuse you.)

$$\frac{b}{b}\cdot\frac{3}{a}+\frac{2}{b}\cdot\frac{a}{a}$$

$$\frac{3b+2a}{ab}, \text{or } \frac{2a+3b}{ab}$$

In many respects, adding or subtracting Algebraic Fractions is easier because there is less calculation taking place.

Let's walk through an uglier problem involving subtraction and negative signs.

$$\frac{7x^2-2y}{5x^2y}-\frac{4x-2}{5x^2}$$

We need to find the LCM of $5x^2y$ and $5x^2$. The LCM will be $5x^2y$. So we need to multiply the second fraction by y over y.

$$\frac{7x^2-2y}{5x^2y}-\frac{4x-2}{5x^2}\cdot\frac{y}{y}$$

Now we are multiplying the entire group of $4x-2$ by y, not just whatever term is written next to the y.

$$\frac{7x^2-2y}{5x^2y}-\frac{4xy-2y}{5x^2y}$$

Normally I would not write the step above, but did so to help make sure you understand why the fraction on the right is $4xy$, not just $4x$. We have to distribute the y to the entire group.

Now with care for that negative sign, let's put it all together.

$$\frac{7x^2-2y-(4xy-2y)}{5x^2y}$$

which will become:

$$\frac{7x^2-2y-4xy+2y}{5x^2y}$$

Combining like terms, we get:

$$\frac{7x^2-4xy}{5x^2y}.$$

Before we can say we're finished we need to check for a common factor (GCF) that could be divided out. These don't always exist but if one does, and you had the answer correct up to that point, it would be a shame to mess up the last little step, so check.

$$\frac{7x^{\cancel{2}} - 4\cancel{x}\cancel{y}}{5x^{\cancel{2}}y} = \frac{7x - 4y}{5xy}.$$

In review, you need a common denominator which will be the LCM of the denominators. You must take care to both distribute property in the numerator, and watch for sign errors, especially with subtraction, when combining like terms. The last thing is to check for a common factor between all terms when you've finished combining like terms.

Practice Problems
Instructions: Add or Subtract as indicated

1. $\frac{3x}{5}+\frac{2}{a}$

2. $\frac{4}{9a}+7$

3. $\frac{4}{9a}+\frac{7}{9}$

4. $\frac{4}{9a}+\frac{7}{a}$

5. $\frac{2x-1}{3x^2}-\frac{2x-1}{x^2}$