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REAL NUMBERS
CLASS 10
ASSIGNMENT 1 SOLUTIONS

1. The maximum number of boxes is the HCF of 1134 and 1215.
   $1134 = 2 \times 3^4 \times 7$
   $1215 = 3^5 \times 5$
   HCF = $3^4 = 81$
   Hence, the maximum number of boxes = 81

2. The time when the two bells will ring together again is the LCM of 40 and 60.
   $40 = 2^3 \times 5$
   $60 = 2^2 \times 3 \times 5$
   LCM = $2^3 \times 3 \times 5 = 120$
   Required time = 120 minutes = 2 hours
   Hence, the two bells will ring together again at 9 + 2 = 11 a.m.

3. Let a be any positive integer and $b = 6$.
   By Euclid's Division Lemma
   $a = 6q + r, 0 \leq r < 6$
   Possible values of $r = 0, 1, 2, 3, 4, 5$
   When $r = 0, a = 6q + 0 = 2(3q) = 2m$, where $m = 3q$; which is even.
   When $r = 1, a = 6q + 1 = 2(3q) + 1 = 2m + 1$, where $m = 3q$; which is odd.
   When $r = 2, a = 6q + 2 = 2(3q + 1) = 2m$, where $m = 3q + 1$; which is even.
   When $r = 3, a = 6q + 3 = 6q + 2 + 1 = 2(3q + 1) + 1 = 2m + 1$, where $m = 3q + 1$; which is odd.
   When $r = 4, a = 6q + 4 = 2(3q + 2) = 2m$, where $m = 3q + 2$; which is even.
   When $r = 5, a = 6q + 5 = 6q + 4 + 1 = 2(3q + 2) + 1 = 2m + 1$, where $m = 3q + 2$; which is odd.
   Thus, when $a = 6q + 1$ or $6q + 3$ or $6q + 5$, then a is odd.
   Hence, every odd integer is of the form $6q + 1$ or $6q + 3$ or $6q + 5$.

4. Let n be any positive integer and $b = 3$
By Euclid's Division Lemma
$n = 3q + r, 0 \leq r < 3$
Possible values of $r = 0, 1, 2$
Case 1: When $r = 0$
$n = 3q + 0 = 3q$, which is divisible by 3.
$n + 2 = 3q + 2$, which is not divisible by 3.
$n + 4 = 3q + 4 = 3q + 3 + 1 = 3(q + 1) + 1 = 3m + 1$, where
$m = q + 1$; which is not divisible by 3.
Case 2: When $r = 1$
$n = 3q + 1$, which is not divisible by 3.
$n + 2 = 3q + 1 + 2 = 3q + 3 = 3(q + 1) = 3m$, where $m = q + 1$;
which is divisible by 3.
$n + 4 = 3q + 1 + 4 = 3q + 5 = 3q + 3 + 2 = 3(q + 1) + 2 = 3m + 2$,
where $m = q + 1$; which is not divisible by 3.
Case 3: When $r = 2$
$n = 3q + 2$, which is not divisible by 3.
$n + 2 = 3q + 2 + 2 = 3q + 4 = 3q + 3 + 1 = 3(q + 1) + 1 = 3m + 1$,
where $m = q + 1$; which is not divisible by 3.
$n + 4 = 3q + 2 + 4 = 3q + 6 = 3(q + 2) = 3m$, where $m = q + 2$;
which is divisible by 3.
Hence, one and only one out of $n, n + 2, n + 4$ is divisible by 3, where n is any positive integer.

5. Let a number a be of the form $4q + 2, q \in N$.
$a = 4q + 2 = 2(2q + 1)$
Thus, a is the product of 2 and some odd integer. For a to be a perfect square, the number $2q + 1$ must have 2 as one of its prime factors, which is not possible as $2q + 1$ is odd.
Hence, a number of the form $4q + 2, q \in N$ cannot be a perfect square.

6. Any odd positive integer is of the form $4q + 1$ or $4q + 3$
$(4q + 1)^2 = 16q^2 + 8q + 1 = 8(2q^2 + q) + 1 = 8m + 1$, where
$m = 2q^2 + q$
$(4q + 3)^2 = 16q^2 + 24q + 9 = 16q^2 + 24q + 8 + 1$
$= 8(2q^2 + 3q + 1) + 1 = 8m + 1$, where $m = 2q^2 + 3q + 1$

Hence, the square of any odd positive integer is of the form $8m + 1$, where $m \epsilon N$.

7. **Case 1**: When n is an even number.
Let $n = 2q$
$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2(2q^2 - q) = 2m$, where $m = 2q^2 - q$; which is even.
Case 2: When n is an odd number.
Let $n = 2q + 1$
$n^2 - n = (2q + 1)^2 - (2q + 1) = 4q^2 + 4q + 1 - 2q - 1$
$= 4q^2 + 2q = 2(2q^2 + q) = 2m$, where $m = 2q^2 + q$; which is even.
Since, any positive integer is either even or odd, hence in each case $n^2 - n$ is always even.

8. Let three consecutive positive integers be $a, a + 1, a + 2$.
Let a be any positive integer and $b = 3$
By Euclid's Division Lemma
$a = 3q + r, 0 \leq r < 3$
Possible values of $r = 0, 1, 2$
Case 1: When $r = 0$
$a = 3q + 0 = 3q$, which is divisible by 3.
$a + 1 = 3q + 1$, which is not divisible by 3.
$a + 2 = 3q + 2$, which is not divisible by 3.
Case 2: When $r = 1$
$a = 3q + 1$, which is not divisible by 3.
$a + 1 = 3q + 1 + 1 = 3q + 2$, which is not divisible by 3.
$a + 2 = 3q + 1 + 2 = 3q + 3 = 3(q + 1) = 3m$, where $m = q + 1$; which is divisible by 3.
Case 3: When $r = 2$
$a = 3q + 2$, which is not divisible by 3.
$a + 1 = 3q + 2 + 1 = 3q + 3 = 3(q + 1) = 3m$, where $m = q + 1$; which is divisible by 3.
$a + 2 = 3q + 2 + 2 = 3q + 4 = 3q + 3 + 1 = 3(q + 1) + 1 = 3m + 1$, where $m = q + 1$; which is not divisible by 3.
Hence, only one out of every three consecutive positive integers is divisible by 3.

9. Let a be any positive integer and $b = 3$
By Euclid's Division Lemma
$a = 3q + r, 0 \le r < 3$
Possible values of $r = 0, 1, 2$
Case 1: When $r = 0, a = 3q + 0 = 3q$
Squaring both sides
$a^2 = 9q^2 = 3(3q^2) = 3k$, where $k = 3q^2$
Case 2: When $r = 1, a = 3q + 1$
Squaring both sides
$a^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3k + 1$, where $k = 3q^2 + 2q$
Case 3: When $r = 2, a = 3q + 2$
Squaring both sides
$a^2 = (3q + 2)^2 = 9q^2 + 12q + 4 = 9q^2 + 12q + 3 + 1 = 3(3q^2 + 4q + 1) + 1 = 3k + 1$, where $k = 3q^2 + 4q + 1$
Hence, the square of any positive integer is of the form $3k$ or $3k + 1$, where $k \epsilon N$.

10. Using Euclid's Division Lemma
$2346 = 1794 \times 1 + 552, r \neq 0$
$1794 = 552 \times 3 + 138, r \neq 0$
$552 = 138 \times 4 + 0$
HCF of 1794 and 2346 is 138.
Using Euclid's Division Lemma
$4761 = 138 \times 34 + 69, r \neq 0$
$138 = 69 \times 2 + 0$
HCF of 138 and 4761 is 69.
Hence, HCF of $1794, 2346$ and 4761 is 69.