

GeoCue Group Support

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Revision 2.0

Introduction

Datums identified only as NAD 83 or WGS 84 are not specific enough to clearly define the reference frame of geodetic data. Additional information is needed that defines the realization or version of a particular datum. In the case of NAD 83, a “datum tag” must be appended to the name, such as NAD 83 (86) or NAD 83 (CORS96); likewise for WGS 84: WGS 84 (G1150), WGS 84(original), etc. NAD 83 (CORS96) and WGS 84 (G1150) are the current versions of these systems. While NAD 83 (86) and WGS 84 (original) were equivalent datums, this is not the case for CORS96 and G1150. A datum transform is required when transforming points between any projected or geographic coordinate systems based on these datums. For these particular datums, the magnitude of the difference is on the order of two meters.

Clearly then, the first step is to know precisely the datum to which your input data is referenced. If your processing will require that this data be transformed to another coordinate system which is not based on the same datum, then you must consider the required datum transform. This note will consider the common case in which input data is referenced to WGS 84 (G1150) and requires being converted to a coordinate system based on NAD 83 (COR96), as these are the current versions of those datums. It is important to note here that for these particular datums, it will also be required to know the date to which the GPS data is processed, also known as the epoch of the data. The will be explained further below.

Several techniques are used for transforming coordinates between datums. Moving between datums such as NAD 83 and NAD 27 can only be accomplished using interpolation from grid-based correction tables. Many other transforms can be accomplished analytically by transforming geographic (spherical) coordinates to geocentric (Cartesian) coordinates and applying a simple shift or possibly a shift, scale, and rotation to change to a new geocentric system (reference frame) and then converting back to geographic coordinates. The latter is the case for WGS 84 (G1150) to NAD 83 (CORS 96) conversions. These type transforms are known by many names: 7-parameter, Helmert, similarity, Bursa-Wolf, Coordinate Frame, etc. The parameters are three shifts, T_x , T_y , and T_z , three rotations, ω_x , ω_y , and ω_z , and a scale, s . Coordinate Frame is the name adopted by ESRI software, which is the engine used in GeoCue for transformations, so we'll use that term from here forward.

The following section provides some background and references as to how to obtain the datum transform for use in GeoCue. You may skip ahead to Equation Set 3 to see immediately the equations required for calculating the parameters or to Table 3 for a set of pre-computed parameter values that may be entered directly into GeoCue's custom datum transform dialogs.

A Seven-Parameter Datum Transform for WGS 84 to NAD 83

Obtaining the required parameters for the Coordinate Frame datum transform is based on several assertions:

- 1) WGS 84 (G1150) is equivalent to ITRF00, the International Terrestrial Reference Frame of 2000, to an accuracy of approximately one centimeter.
- 2) A 14-parameter transform has been defined between ITRF00 and NAD 83 (CORS96)
- 3) For a given instant in time, the 14-parameter transformation may be represented as a 7-parameter Coordinate Frame transform.

While no direct transforms have been defined from WGS 84 (G1150) to NAD 83 (CORS96), as stated in 2) the transform to NAD 83 (CORS96) is defined from ITRF00. Given the assertion in 1), then we can treat WGS 84 (G1150) coordinates as ITRF00 coordinates, thereby creating a path through which the desired transform can be completed. For a justification of assertion 1), see the paper “*A Refinement to the World Geodetic System 1984 Reference Frame*”, by Merrigan, Swift, Wong, and Saffel.

The 14-parameter transform in 2) is specified in “*Transforming Position and Velocities between the International Terrestrial Reference Frame of 2000 and North American Datum of 1983*”, by Tomas Soler and Richard Snay of the National Geodetic Survey (NGS). From that paper:

$$\begin{aligned}
 Tx(t) &= Tx(t_0) + T'x \cdot (t - t_0) \\
 Ty(t) &= Ty(t_0) + T'y \cdot (t - t_0) \\
 Tz(t) &= Tz(t_0) + T'z \cdot (t - t_0) \\
 \omega x(t) &= [\epsilon x(t_0) + \epsilon'x \cdot (t - t_0)] \cdot m_r \\
 \omega y(t) &= [\epsilon y(t_0) + \epsilon'y \cdot (t - t_0)] \cdot m_r \\
 \omega z(t) &= [\epsilon z(t_0) + \epsilon'z \cdot (t - t_0)] \cdot m_r \\
 s(t) &= s(t_0) + s' \cdot (t - t_0)
 \end{aligned}
 \tag{1}$$

The above equations define the seven parameters needed for the Coordinate Frame datum transform. However, the parameters are a function of time, and hence there are an additional seven rate parameters (primed terms). Note that m_r is simply a conversion constant: milliarcs to radians. A consequence of the time dependence is that G1150 coordinates are moving relative to CORS96 coordinates. The motion is primarily due to the movement of the Earth’s crust. This means that a position measured in WGS 84 (G1150) at two different times will have two different coordinates. An interesting note however, is that the CORS96 has been defined in such a way that it is moving at the same rate as the crust of North America relative to G1150 coordinates (everywhere except the West coast). This means that most positions measured in CORS96 remain constant with time. For a complete description of this, refer to “*Horizontal Time-Dependent Positioning*” by Dr. Richard Snay.

The interesting notes above aside, we are still left with fourteen parameters, plus a time variable, for use in constructing a datum transform. However, all the parameters on the right side of each equation above are provided in the Soler/Snay paper, including $t_0=1997$, the defined “epoch time” for ITRF00. Hence, if we hold time constant by selecting a particular value for t , then the right side of each equation

evaluates to a constant and we have seven constant terms which may be used as inputs for a Coordinate Frame transform (assertion 3 above). The table below shows the fourteen parameters from the Soler/Snay paper:

Parameters Adopted for Transformation ITFR00 → NAD 83 (CORS96)			
Parameter	Definition	Units	Value at t_0
$T_x(t_0)$	x-shift	meters	+0.9956
$T_y(t_0)$	y-shift	meters	-1.9013
$T_z(t_0)$	z-shift	meters	-0.5215
$\epsilon_x(t_0)$	x-rotation	mas	+25.915
$\epsilon_y(t_0)$	y-rotation	mas	+9.426
$\epsilon_z(t_0)$	z-rotation	mas	+11.599
$s(t_0)$	scale	ppb	+0.62
T'_x	x-shift rate	meters/year	+0.0007
T'_y	y-shift rate	meters/year	-0.0007
T'_z	z-shift rate	meters/year	+0.0005
ϵ'_x	x-rotation rate	mas/year	+0.067
ϵ'_y	y-rotation rate	mas/year	-0.757
ϵ'_z	z-rotation rate	mas/year	-0.051
s'	scale rate	ppb/year	-0.18
Parameter epoch $t_0 = 1997.0$			
Note: mas \equiv milliarc second. Counterclockwise rotation of axes are assumed positive; 1 ppb = 10^{-3} ppm = 10^{-9}			

Table 1

Time in the above equations is expressed in decimal years. For example, mid-year 2006 would be 2006.5. So, if we have GPS data processed to an epoch of 1 November 2006 (305th day of year), then the epoch time for our data is $t = 2006 + 305/365 = 2006.84$. Using this value, the equations and parameters above, we can compute the following values for a Coordinate Frame datum transform:

$$T_x = 0.9956 + 0.0007 \cdot (2006.84 - 1997) = 1.002 \text{ m}$$

$$T_y = -1.9013 - 0.0007 \cdot (2006.84 - 1997) = -1.908 \text{ m}$$

$$T_z = -0.5215 + 0.0005 \cdot (2006.84 - 1997) = -0.5166 \text{ m}$$

$$\omega_x = [25.915 + 0.067 \cdot (2006.84 - 1997)] \cdot 4.84813681 \cdot 10^{-9} = 1.288 \cdot 10^{-7} \text{ mas} \quad (2)$$

$$\omega_y = [9.426 - 0.757 \cdot (2006.84 - 1997)] \cdot 4.84813681 \cdot 10^{-9} = 9.585 \cdot 10^{-9} \text{ mas}$$

$$\omega_z = [11.599 - 0.051 \cdot (2006.84 - 1997)] \cdot 4.84813681 \cdot 10^{-9} = 5.380 \cdot 10^{-8} \text{ mas}$$

$$s = 0.62 \cdot 10^{-9} - 0.18 \cdot 10^{-9} \cdot (2006.84 - 1997) = -1.151 \cdot 10^{-9} \text{ or } -1.151 \text{ ppb}$$

The values calculated above are the seven parameters for defining a datum transform for converting WGS 84 (G1150) data collected 1 November 2006 to NAD 83 (CORS96) coordinates. In the last section we'll see how these values may be entered as a custom datum transform in GeoCue.

A Coordinate Frame Datum Transform for Use in GeoCue

The parameters above, for the specific time $t = 2006.84$, were calculated using the equations defined in the Soler/Snay paper. While the parameters are correct and equivalent to those needed for the Coordinate Frame transform, some of the units are not the same as required by GeoCue and therefore the parameters must be converted. The table below shows the necessary unit conversions.

Conversion to Parameters with Units Matching Those Required in GeoCue			
Parameter	Units in (1)	Units Required in GeoCue	To Convert to GeoCue
T_x	meters	meters	n/a
T_y	meters	meters	n/a
T_z	meters	meters	n/a
ω_x	radians	arc seconds	divide by $1000 \cdot m_r$
ω_y	radians	arc seconds	divide by $1000 \cdot m_r$
ω_z	radians	arc seconds	divide by $1000 \cdot m_r$
s	parts-per-billion	parts-per-million	divide by 1000
$m_r \equiv \text{milliarcseconds to radians} = 4.84813681 \cdot 10^{-9}$			

Table 2

Using the conversions identified above, the following is a simplified set of equations for calculating the parameters in units as required by GeoCue. Again, t must be specified as a decimal year value.

$$\begin{aligned}
 T_x(t) &= (0.7t - 402.3) / 1000 \\
 T_y(t) &= (-0.7t - 503.4) / 1000 \\
 T_z(t) &= (0.5t - 1520) / 1000 \\
 \omega_x(t) &= (0.067t - 107.884) / 1000 \\
 \omega_y(t) &= (-0.757t + 1521.155) / 1000 \\
 \omega_z(t) &= (-0.051t + 113.446) / 1000 \\
 s(t) &= (-0.18t + 360.08) / 1000
 \end{aligned}
 \tag{3}$$

Using the equations defined in (3) above, the following table shows a collection of parameter sets computed on half-year intervals¹:

GeoCue Transform Parameters WGS84 (G1150) → NAD83 (CORS96) by Year					
	2006.0	2006.5	2007.0	2007.5	2008.0
T_x	1.0019	1.0023	1.0026	1.0030	1.0033
T_y	-1.9076	-1.9080	-1.9083	-1.9087	-1.9090
T_z	-0.5170	-0.5168	-0.5165	-0.5163	-0.5160
ω_x	0.026518	0.026552	0.026585	0.026619	0.026652
ω_y	0.002613	0.002235	0.001856	0.001478	0.001099
ω_z	0.011140	0.011145	0.011089	0.011064	0.011038
s	-0.00100	-0.00109	-0.00118	-0.00127	-0.00136

Table 3

A set of the above parameters may be used in lieu of computing the values each time new data is to be converted. For most of the United States, the accuracy of the transformation will be within one centimeter. Alternatively, an ESRI predefined datum transform is available in GeoCue: NAD_1983_To_WGS84_5. The accuracy of this predefined transform however is limited to about 10-20 centimeters for 2007.0 data (for 1997 data, it is correct to within ≈ 2 cm). The error in this transformation, and any other, including those in Table 3, will increase approximately 1-2 cm/year over most of the U.S. This is of course the reason for having a table of values based on time, in this case, $\frac{1}{2}$ -year increments. When using Table 3, always select the $\frac{1}{2}$ -year increment closest to the date, or epoch, in which the GPS data is processed so as to maintain the highest possible accuracy.

For further discussion of this topic and tools for doing additional analysis, visit the NGS Horizontal Time-Dependent Positioning (HTDP) webpage: <http://www.ngs.noaa.gov/TOOLS/Htdp/Htdp.shtml>. Tools are available at this site for transforming data between the datums described here and several others. Velocities for positions can also be predicted here, as well as transformation of points on different datums to different epochs, a topic not covered in this note.

Entering the Datum Transform into GeoCue

If your GeoCue project does not already have at least one layer based on the WGS 84 datum and one based on the NAD 83 datum, then add the necessary layer(s) now. The Map Coordinate Systems dialog below shows the datum transforms that need to be defined (you may have to click 'Auto Add From Layers' if your dialog does not show the WGS 84 / NAD 83 pairs in the lower left list box).

¹ See the Appendix for parameter data through 2010.5

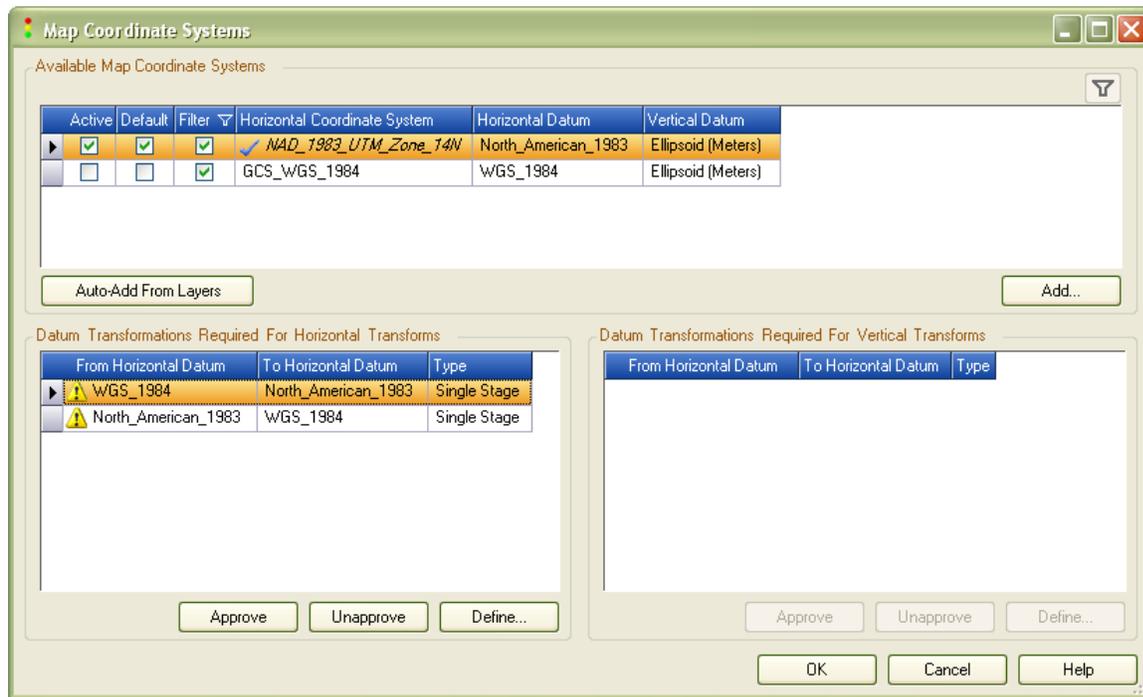


Figure 1

Select the first line (from WGS_1984 to North_American_1983) and click Define. The following dialog will appear:

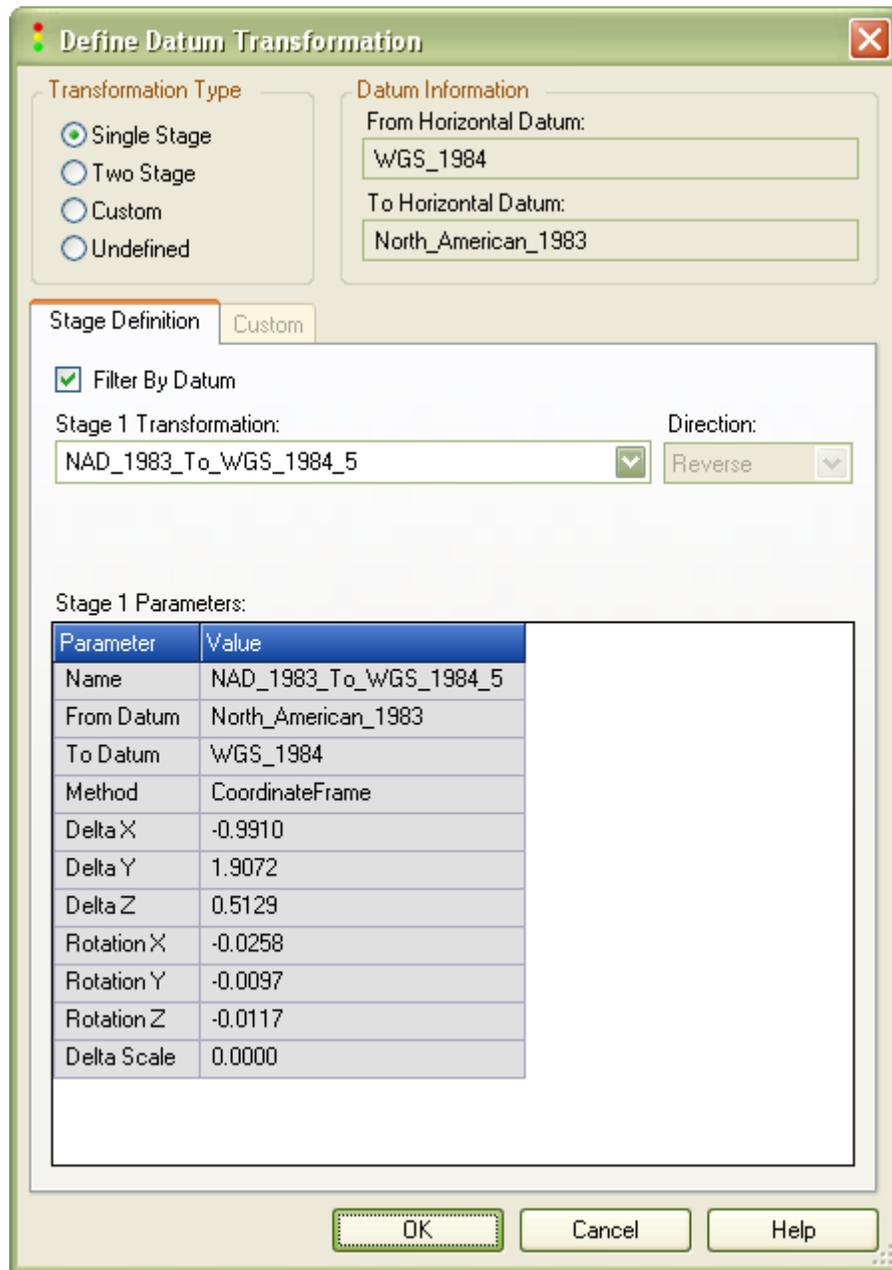


Figure 2

The predefined ESRI datum transform NAD_83_To_WGS_1984_5 (Reverse) is selected by default. As mentioned above, this can be used if your accuracy requirements are within the previously described tolerances. However, to enter an up-to-date transform, such as one from Table 3, you should click the 'Custom' radio button in the upper-left 'Transformation Type' group box. Then select 'Coordinate Frame' as the Transformation Method and enter your parameters. As an example, assume your data has an epoch of November 2006. To make use of Table 3, you should select the parameters for 2007.0,

as this is the closest ½-year increment for your data's epoch. The following dialog shows the result of entering the 2007.0 parameters.

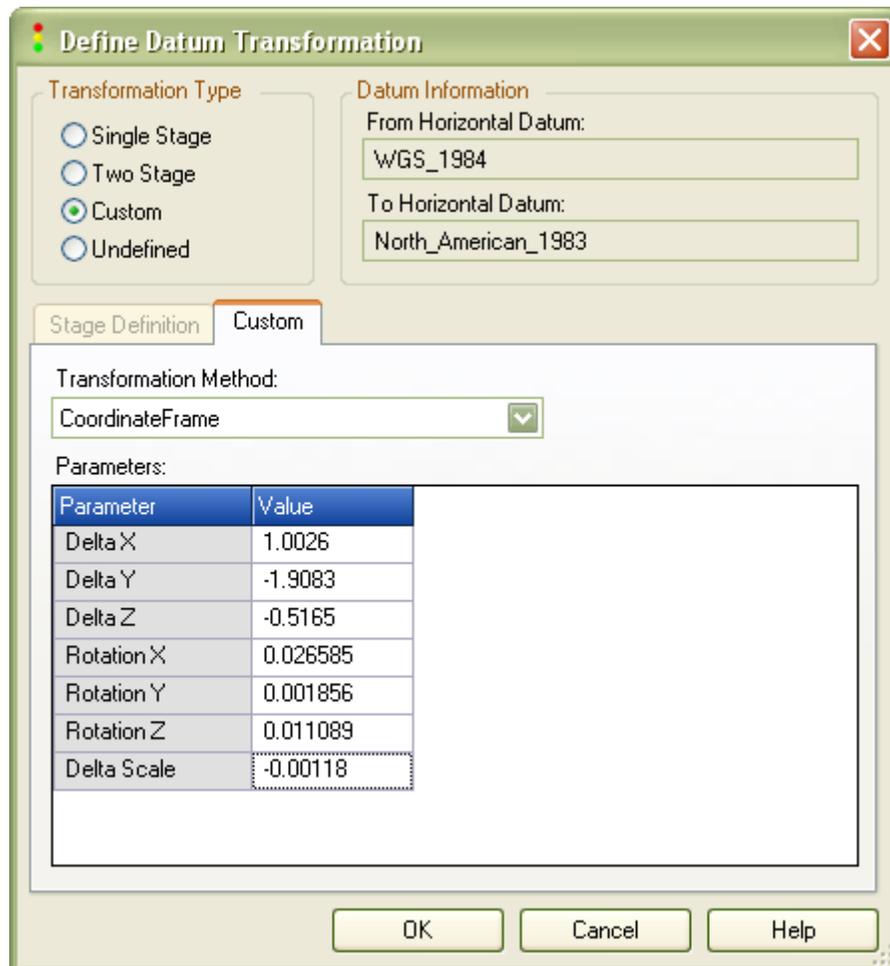


Figure 3

Click 'OK' on the dialog to return to the Map Coordinate Systems dialog. You must then select the remaining line (from North_American_1983 to WGS_1984) and click the 'Define...' button to enter the parameters for the transformation in the opposite direction. As before, select 'Custom' type and Coordinate Frame as the method. This time however, enter the parameters as above, but with all signs **negated**. See the result in the following figure:

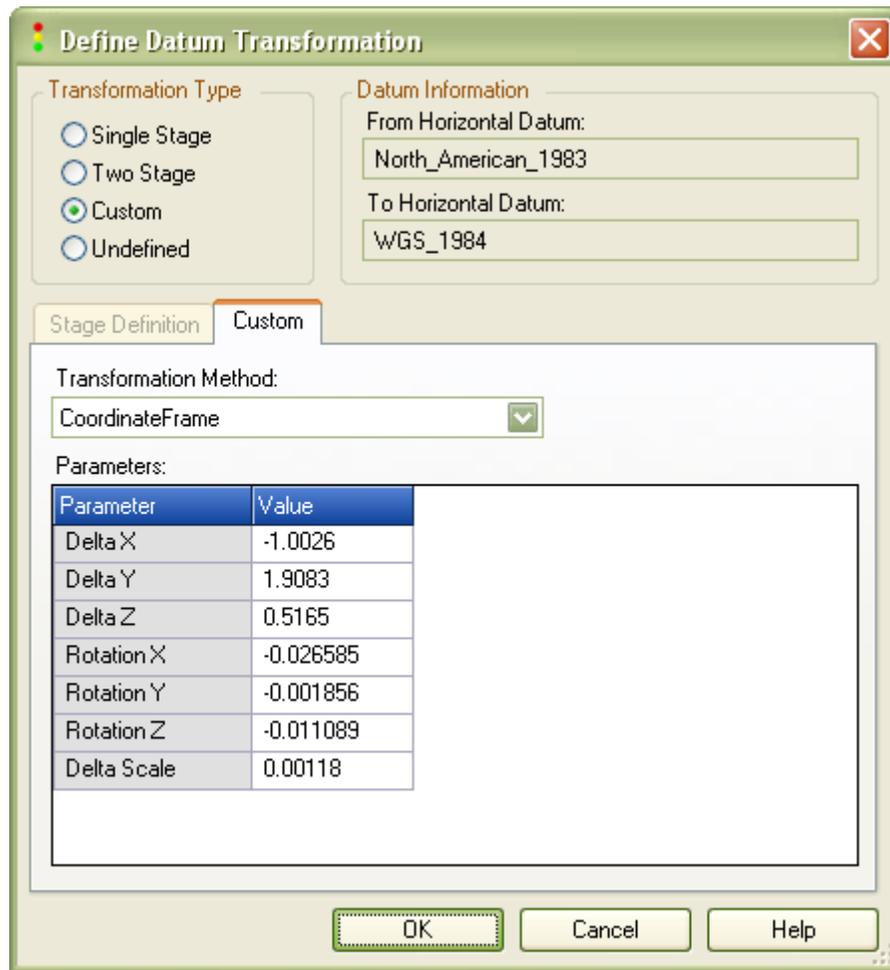


Figure 4

Click 'OK' to complete the GeoCue setup of the necessary datum transforms.

Summary

This note has described how to select parameters for an accurate GeoCue datum transform for transforming data from WGS 84 (G1150), in a given epoch, to NAD 83 (CORS) in the same epoch. The epoch of your input data is the date for which the positional coordinates are valid. This is determined by your GPS processing workflow. You must know the epoch of the data resulting from that processing to properly select the required datum transform parameters.

Once the epoch is determined and WGS 84 and NAD 83 layers are created in GeoCue, the Map Coordinate Systems dialog is used to either select an existing transform, or to enter parameters defining custom datum transforms. For the case of custom datum transforms as defined by the parameters from either the equations in (3), or from Table 3, two datum transforms must be defined: one from WGS 84

to NAD 83 and a second one from NAD 83 to WGS 84. The parameters for the second one are just the negatives of the corresponding parameters for the first.

Note that there is a single transform pair (WGS 84 to NAD 83, and vice-versa) for all layers in a GeoCue project. It is important to be aware of this if you have input data or control in multiple WGS 84 epochs. Discussions and design are underway to determine how to more easily support multiple epochs, as well as how to avoid the current situation in which custom datum transform parameters must be entered for each GeoCue project.

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APPENDIX**Additional Transform Parameter Sets through Mid-2010**

GeoCue Transform Parameters WGS84 (G1150) → NAD83 (CORS96) by Year					
	2008.5	2009	2009.5	2010	2010.5
T_x	1.0036	1.0040	1.0043	1.0047	1.0050
T_y	-1.9093	-1.9097	-1.9100	-1.9104	-1.9107
T_z	-0.5158	-0.5155	-0.5152	-0.5150	-0.5148
ω_x	0.026686	0.026719	0.026753	0.026786	0.026820
ω_y	0.000720	0.000342	-0.000036	-0.000415	-0.000793
ω_z	0.011013	0.010987	0.010961	0.010936	0.010910
s	-0.00145	-0.00154	-0.00163	-0.00172	-0.00181

GeoCue Transform Parameters WGS84 (G1150) → NAD83 (CORS96) by Year					
	2011	2011.5	2012	2012.5	2013
T_x	1.0054	1.0057	1.0061	1.0065	1.0068
T_y	-1.9111	-1.9114	-1.9118	-1.9122	-1.9125
T_z	-0.5145	-0.5143	-0.5140	-0.5138	-0.5135
ω_x	0.026853	0.026886	0.026920	0.026954	0.026987
ω_y	-0.001172	-0.001551	-0.001929	-0.002308	-0.002686
ω_z	0.010885	0.010860	0.010834	0.010809	0.010783
s	-0.00190	-0.00199	-0.00208	-0.00217	-0.00226