YOU MUST MEMORIZE THESE!

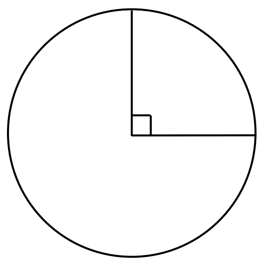
Problems relevant to the following formulas appear on PSAT/SAT and PLAN/ACT Mathematics Tests. Memorizing these formulas and knowing how to apply them will ensure success on the tests.

I. CIRCLES

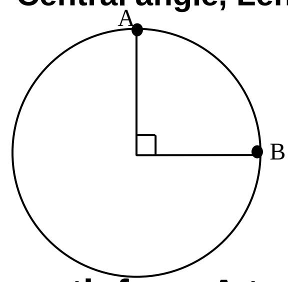
A. CIRCUMFERENCE: $C = 2\pi r$ or $C = \pi d$ (when $\pi = 3.14$ or $\frac{22}{7}$)

B. AREA: $A=\pi r^2$

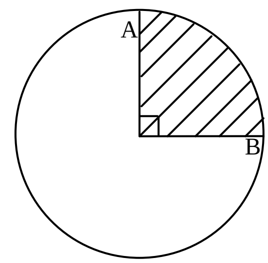
C. EQUIVALENT RATIOS: **Central angle, Length of arc, Area of sector**



$90° = \frac{1}{4}$ of $360°$



Length from A to B $= \frac{1}{4}$ of circle's circumference

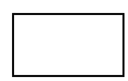


Area of Sector $= \frac{1}{4}$ of circle's area

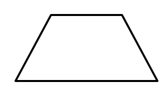
D. General Equation is $x^2 + y^2 = r^2$ or $(x- h)^2 + (y – k)^2 = r^2$ where r is the radius and (h, k) is the center

II. TWO-DIMENSIONAL FIGURES

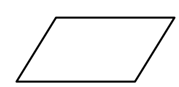
A. RECTANGLE: Area = length x width
$A = lw$



B. TRAPEZOID: Area = $\frac{1}{2}$ height x sum of the bases
$A = \frac{1}{2} h (b_1+b_2)$



C. PARALLELOGRAM: Area = length of base x height
$A = bh$



D. SUM OF ANGLES IN FOUR-SIDED FIGURE = 360° or (n-2)180

III. RECTANGULAR SOLIDS

A. VOLUME: Volume = length x width x height
$V = lwh$

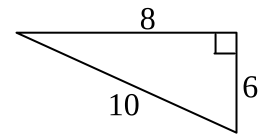
B. SURFACE AREA = AREA OF THE SURFACES
SA = area of the front + area of back + area of top + area of bottom + area of left side + area of right side

IV. TRIANGLES

A. AREA OF "GARDEN VARIETY" TRIANGLE

$A = \frac{1}{2}bh$

B. AREA OF RIGHT TRIANGLE



Quite Often Area = $\frac{1}{2}$(LEG)(LEG)

$A = \frac{1}{2}(6)(8) = 24$

C. AREA OF EQUILATERAL TRIANGLE

$$A = \frac{s^2\sqrt{3}}{4}$$

V. TRIANGLES – THOSE THAT ARE RIGHT

A. $a^2 + b^2 = c^2$ Pythagorean Theorem

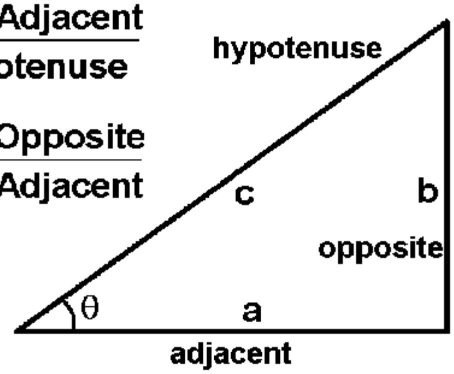
1. Pythagorean Triplets
   - 3-4-5
   - 6-8-10
   - 5-12-13
   - 7-24-25
   - 8-15-17

B. SIN, COS, TAN or "CHIEF SOHCAHTOA"

$$\text{SIN}\,\theta = \frac{\text{Side Opposite}}{\text{Hypotenuse}}$$

$$\text{COS}\,\theta = \frac{\text{Side Adjacent}}{\text{Hypotenuse}}$$

$$\text{TAN}\,\theta = \frac{\text{Side Opposite}}{\text{Side Adjacent}}$$



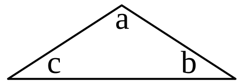
$\text{SIN}\angle A = \text{COS}\angle B$

$\text{COS}\angle A = \text{SIN}\angle B$

$$\text{COT}\angle A = \frac{1}{\text{TAN}\angle A}$$

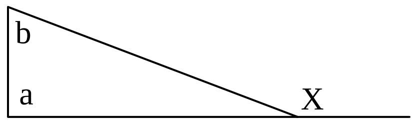
VI. TRIANGLES – ADDITIONAL "GOTTA KNOWS"

A.



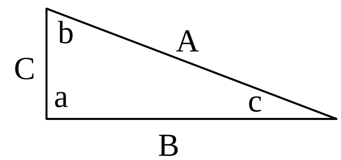
Sum of angles = a + b + c = 180°

B.



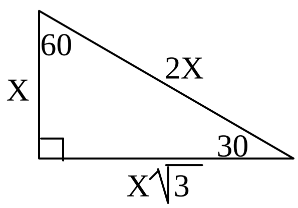
Measure of exterior angle = sum of measures of 2 remote interior angles
X = a + b

C. Longest side is opposite largest angle
If c < b < a, then C < B < A

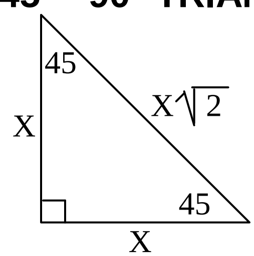


VII. TRIANGLES THAT ARE SO SPECIAL

30° - 60° - 90° TRIANGLE



45° - 45° - 90° TRIANGLE



***IMPORTANT COROLLARY**
Length of side opposite the 30° is $\frac{1}{2}$ the length of the hypotenuse

VIII. SLOPE UPSIDE DOWN, BACKWARD AND FORWARD

A. STANDARD FORM EQUATION

$$Ax+By+C = 0$$

slope = $-\frac{A}{B}$

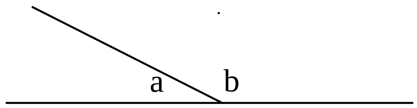
B. SLOPE INTERCEPT FORM EQUATION

$$y = mx+b$$

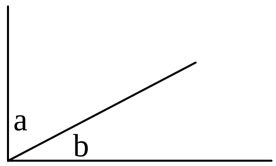
m = slope of the line
b = y-intercept (where x = 0)

C. SLOPE = $\frac{y_2 - y_1}{X_2 - x_1}$ **(rise) / (run)**

IX. ANGLE SUMS



SUM OF SUPPLEMENTARY ∠'S = 180°
$a + b = 180°$



SUM OF COMPLIMENTARY ∠'S = 90°
$a + b = 90°$

X. AVERAGES

IF AN AVERAGE IS GIVEN IN A PROBLEM:
THE SUM OF THE ITEMS = THE NUMBER OF ITEMS TIMES THE AVERAGE

$$S = NA$$

XI. OTHER MUSTS

A. DISTANCE FORMULA $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

1. MIDPOINT FORMULA $\left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$

B. OTHER DISTANCE
DISTANCE TRAVELED = RATE TIMES TRAVEL TIME

$$D = RT$$

C. QUADRATIC FORMULA $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

D. RULES FOR EXPONENTS

1. **When the bases are the same, to multiply - add the exponents**
   $(x^2)(x^6) = x^8$
2. **When the bases are the same, to divide - subtract the exponents** $\frac{x^8}{x^6} = x^2$
3. **To raise to a power - multiply exponents**
   $(x^2)^6 = x^{12}$
4. **Negative exponents: perform the normal operation (raise the quantity to the power) and then INVERT**
   $2^3 = 8$, **THEREFORE** $2^{-3} = \frac{1}{8}$, $(x+2)^{-2} = \frac{1}{(x+2)^2}$

E. FOIL AND FACTOR

Multiply: $(x+a)(x+b) = x^2+bx+ax+ab$

(first terms; outside terms; inside terms; last terms)

Important quadratics to know how to factor:

$(x + y)^2 = x^2 + 2xy + y^2$; $(x – y)^2 = x^2 – 2xy + y^2$;

$x^2 – y^2 = (x + y)(x – y)$

F. SYSTEMS OF EQUATIONS

Can be solved by either substitution or linear combination

SOLVE:

$$2x + y = 15$$
$$4x + 3y = 37$$

By Substitution:

Step 1: $2x + y = 15 \longrightarrow y = 15 – 2x$
Step 2: (substitution) $4x + 3(15 – 2x) = 37$
Step 3: (distribute) $4x – 6x + 45 = 37$
Step 4: (subtract 45 from both sides) $-2x = -8$
Step 5: (multiply both sides by –1) $2x = 8$
Step 6: (divide both sides by 2) $x = 4$
Step 7: substitute back into the original equation
$2(4) + y = 15$
Step 8: $y = 7$

By Linear Combination:

Step 1: multiply $2x + y = 15$ by –2 $\longrightarrow$ $-4x – 2y = -30$
Step 2: add

$$\begin{array}{r} -4x - 2y = -30 \\ \underline{4x + 3y = 37} \\ y = 7 \end{array}$$

Step 3: substitute $y = 7$ back into the original equation
Step 4: using properties of equality, solve for x:
$2x + 7 = 15 \longrightarrow 2x = 8 \longrightarrow x = 4$

G. SERIES PROBLEMS

Arithmetic Sequence – the difference of each term is a number. Use the formula $t_n = t_1 + d(n - 1)$ where t_1 is the first term and d is the common difference.

For example, find the 100th term in the sequence 3,7,11,15 …

$t_{100} = 3 + 4(99)$
$t_{100} = 399$

Geometric Sequence – where the terms are multiplied. Use the formula $t_n = (t_1)(r^{n-1})$ where t_1 is the first term and r is the common ratio.

For example, find the 10th term in the sequence 5, 10, 20, 40 …

$t_{10} = (5)(2^{10-1})$
$t_{10} = (5)(512)$
$t_{10} = 2560$

Summation (Arithmetic) – S_n of the first n terms

$S_n = \frac{n}{2}(a_1 + a_n)$ **where n is a positive integer**

Summation (Geometric)

$S_n = \frac{a_1(1 - r^n)}{1 - r}$ **where r is not equal to 1**