ES120 Spring 2018 – Section 4 Notes

Matheus Fernandes

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Problem 1:

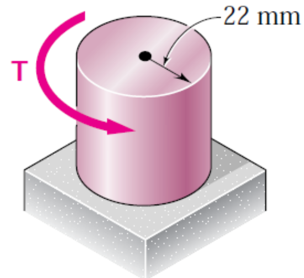


Figure 1

For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude T=1.5 kN·m.

Solution 1

In order to determine the maximum shear stress caused by a torque onto the shaft we must convert the physical torque (which is equivalent to a force) to a stress. The relationship between torque and maximum shear stress is given by:

$$\tau_{max} = \frac{Tr}{J}, \quad (1)$$

where J is the polar moment of inertia for a cylinder as described per the document at the end of the notes. We can then obtain the equation for J as:

$$J = \frac{\pi}{2}c^4 \quad (2)$$

such that we can solve for τ_{max} (where r and c for this problem are the same) as

$$\boxed{\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(1500)}{\pi(0.022)^3} = 89.7\text{MPa}} \quad (3)$$

Problem 2:

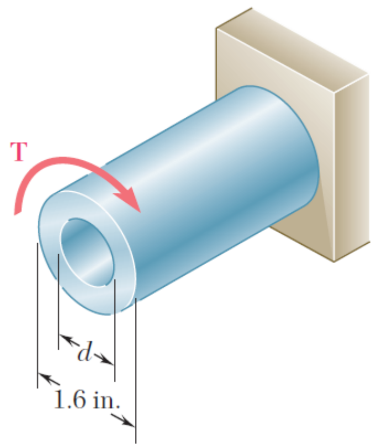


Figure 2

Knowing that the internal diameter of the hollow shaft shown is d=0.9 in., determine the maximum shearing stress caused by a torque of magnitude T=9 kip·in.

Solution 2

Similar to the previous problem we know that the maximum shear stress is dictated by the equation

$$\tau_{max} = \frac{Tr}{J}, \quad (4)$$

where now our radii differ in that for an annulus. So our c's become

$$c_1 = 0.5 * 0.9 = 0.45 \text{ in} \quad (5)$$

$$c_2 = 0.5 * 1.6 = 0.8 \text{ in} \quad (6)$$

Our second moment of an annulus can be calculated by subtracting the moment of the inner from the outer such that:

$$J = \frac{\pi}{2}(c_2^4 - c_2^4) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4 \quad (7)$$

Applying that in to the equation for τ_{max} (remembering that r is the outer radius) we get

$$\boxed{\tau_{max} = \frac{(9)(0.8)}{0.5790} = 12.44 \text{ ksi}} \quad (8)$$

Problem 3:

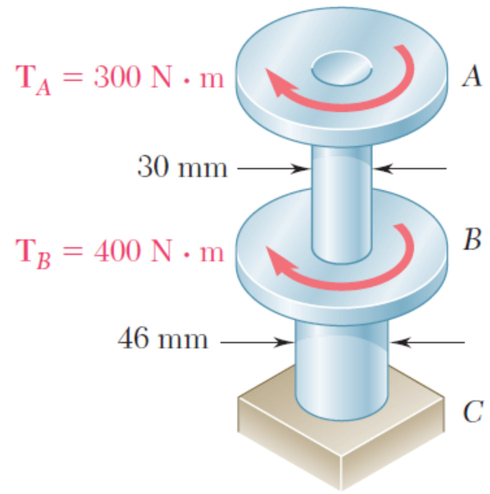


Figure 3

The torques shown are exerted on pulleys A and B. Knowing that each shaft is solid, determine the maximum shearing stress (a) in shaft AB, (b) in shaft BC.

Solution 3

This problem is very similar to the ones we have previously looked at however, in this case we need to make sure we keep in mind how the torques change over the shaft.

Part A

For the AB shaft it is quite easy to figure out the torque. We can once again pull out our handy equations

$$\tau_{max} = \frac{Tr}{J} \tag{9}$$

$$J = \frac{\pi}{2}c^4 \text{ (from second moment of inertia table)} \tag{10}$$

where again $r = c = 0.015$ m for this problem are the same. We can see from the schematic that the only torque acting on this section of the shaft is T_A so that we can obtain the following:

$$\tau_{max} = \frac{Tr}{J} = \frac{(2)(300)}{\pi(0.015)^3} \tag{11}$$

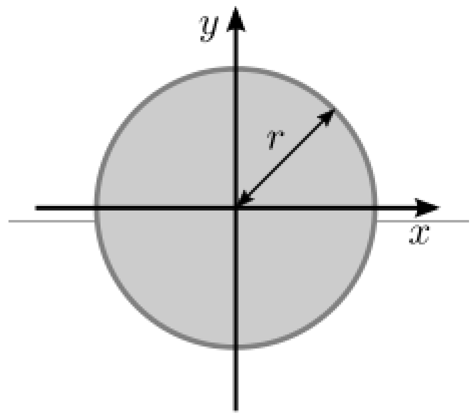
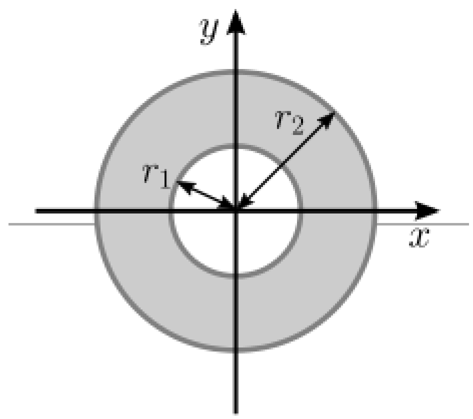
$$\boxed{\tau_{max} = 56.6 \text{ MPa}} \tag{12}$$

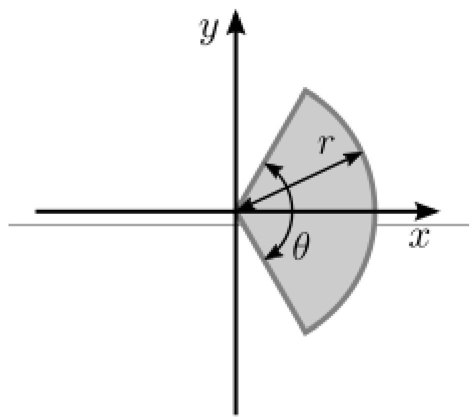
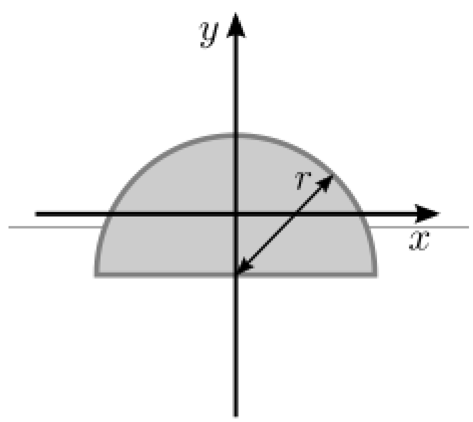
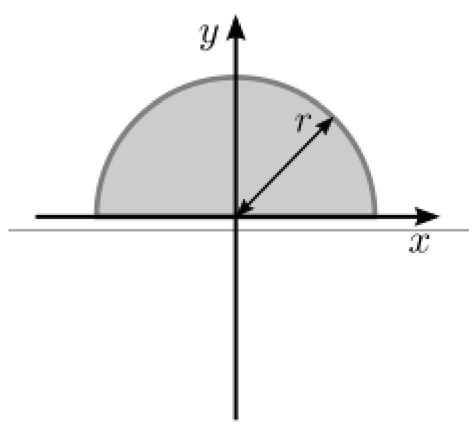
Part B

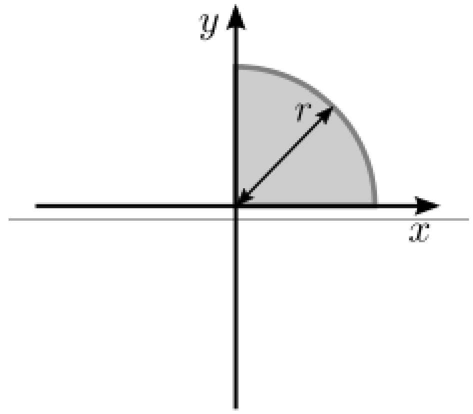
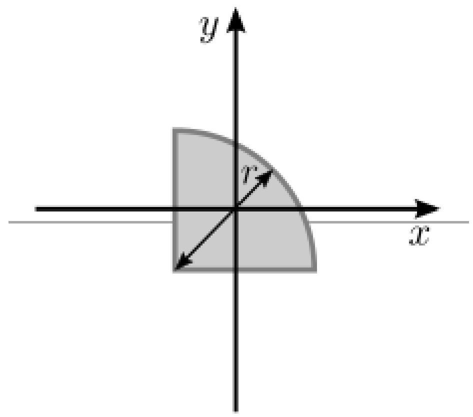
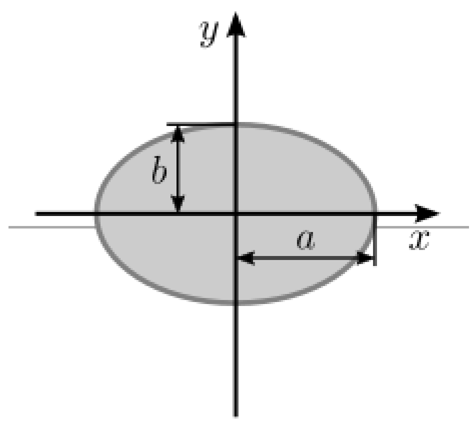
For this section of the shaft, we know that $r = c = 0.023$ m have to be the same again due to the uniform cross-section, however, we know that torque now has to change to $T_{total} = T_A + T_B$. So that the equation above becomes:

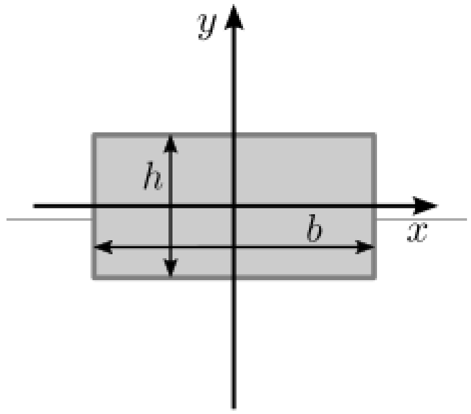
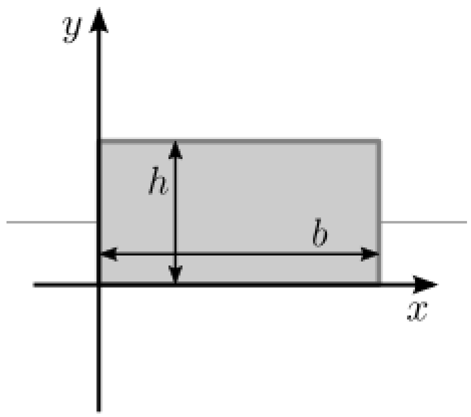
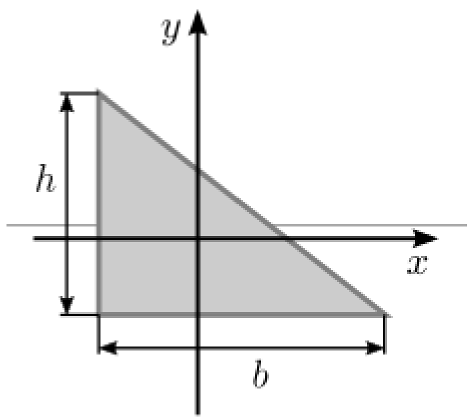
$$\tau_{max} = \frac{Tr}{J} = \frac{(2)(700)}{\pi(0.023)^3} \tag{13}$$

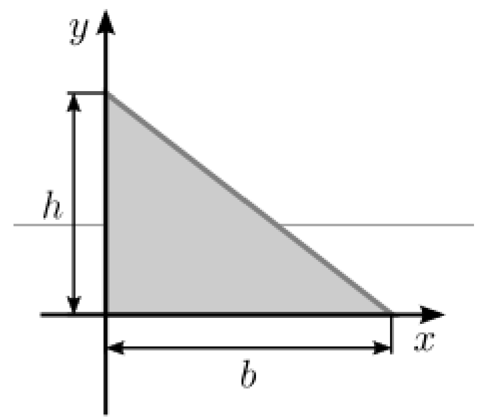
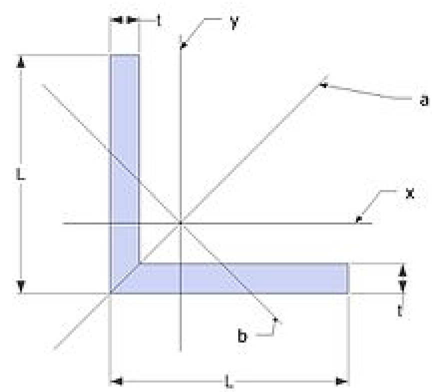
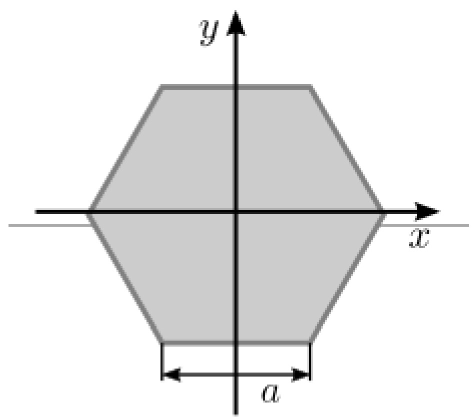
$$\boxed{\tau_{max} = 36.6 \text{ MPa}} \tag{14}$$

Description	Figure	Area moment of inertia	Comment
A filled circular area of radius *r*		$I_x = \frac{\pi}{4} r^4$ $I_y = \frac{\pi}{4} r^4$ $I_z = \frac{\pi}{2} r^4$ [1]	I_z is the Polar moment of inertia.
An annulus of inner radius r_1 and outer radius r_2		$I_x = \frac{\pi}{4} \left({r_2}^4 - {r_1}^4 \right)$ $I_y = \frac{\pi}{4} \left({r_2}^4 - {r_1}^4 \right)$ $I_z = \frac{\pi}{2} \left({r_2}^4 - {r_1}^4 \right)$	For thin tubes, $r \equiv r_1 \approx r_2$ and $r_2 \equiv r_1 + t$. So, for a thin tube, $I_x = I_y \approx \pi r^3 t$. I_z is the Polar moment of inertia.
A filled circular sector of angle θ in radians and radius *r* with respect to an axis through the centroid of the sector and the center of the circle		$I_x = (\theta - \sin\theta) \frac{r^4}{8}$	This formula is valid only for 0 ≤ θ ≤ π

			
A filled semicircle with radius *r* with respect to a horizontal line passing through the centroid of the area		$I_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 \approx 0.1098 r^4$ $I_y = \frac{\pi r^4}{8}$ [2]	
A filled semicircle as above but with respect to an axis collinear with the base		$I_x = \frac{\pi r^4}{8}$ $I_y = \frac{\pi r^4}{8}$ [2]	I_x: This is a consequence of the parallel axis theorem and the fact that the distance between the x axes of the previous one and this one is $\frac{4r}{3\pi}$
A filled quarter circle with radius *r* with the			

axes passing through the bases		$I_x = \frac{\pi r^4}{16}$ $I_y = \frac{\pi r^4}{16}$ [3]	
A filled quarter circle with radius *r* with the axes passing through the centroid		$I_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4 \approx 0.0549 r^4$ $I_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4 \approx 0.0549 r^4$ [3]	This is a consequence of the parallel axis theorem and the fact that the distance between these two axes is $\frac{4r}{3\pi}$
A filled ellipse whose radius along the *x*-axis is *a* and whose radius along the *y*-axis is *b*		$I_x = \frac{\pi}{4} a b^3$ $I_y = \frac{\pi}{4} a^3 b$	
A filled rectangular area with a base width of			

b and height *h*		$I_x = \frac{bh^3}{12}$ $I_y = \frac{b^3h}{12}$ [4]	
A filled rectangular area as above but with respect to an axis collinear with the base		$I_x = \frac{bh^3}{3}$ $I_y = \frac{b^3h}{3}$ [4]	This is a result from the parallel axis theorem
A filled triangular area with a base width of *b* and height *h* with respect to an axis through the centroid		$I_x = \frac{bh^3}{36}$ $I_y = \frac{b^3h}{36}$ [5]	
A filled triangular area as above but with			This is a

respect to an axis collinear with the base		$I_x = \frac{bh^3}{12}$ $I_y = \frac{b^3h}{12}$ [5]	consequence of the parallel axis theorem
An equal legged angle, commonly found in engineering applications		$I_x = I_y = \frac{t(5L^2 - 5Lt + t^2)(L^2 - Lt + t^2)}{12(2L - t)}$ $I_{(xy)} = \frac{L^2t(L-t)^2}{4(t-2L)}$ $I_a = \frac{t(2L-t)(2L^2 - 2Lt + t^2)}{12}$ $I_b = \frac{t(2L^4 - 4L^3t + 8L^2t^2 - 6Lt^3 + t^4)}{12(2L-t)}$	$I_{(xy)}$ is the often unused product of inertia, used to define inertia with a rotated axis
A filled regular hexagon with a side length of *a*		$I_x = \frac{5\sqrt{3}}{16}a^4$ $I_y = \frac{5\sqrt{3}}{16}a^4$	The result is valid for both a horizontal and a vertical axis through the centroid, and therefore is also valid for an axis with arbitrary direction that passes through the origin.