

ES120 Spring 2018 – Section 2 Notes

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Problem 1:

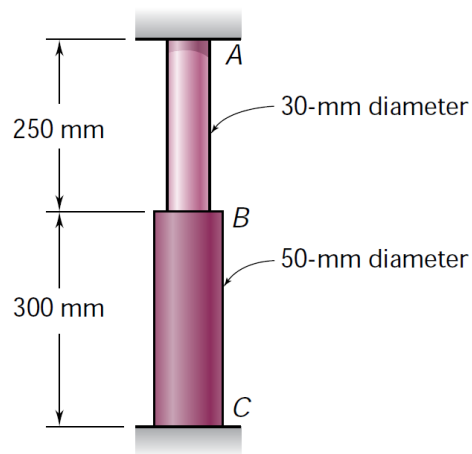


Figure 1

A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 200 \text{ GPa}, \alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$) and portion BC is made of brass ($E_b = 105 \text{ GPa}, \alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$). Knowing that the rod is initially unstressed, determine the compressive force induced in ABC when there is a temperature rise of 50°C .

Solution 1

Let's first find the cross-sectional area of AB and BC:

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (30[\text{mm}])^2 \Rightarrow 706.86 \times 10^{-6} \text{m}^2 \quad (1)$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (50[\text{mm}])^2 \Rightarrow 1.9635 \times 10^{-3} \text{m}^2 \quad (2)$$

Now, let's assume the device is not constrained and that it is free to move. Let's calculate the free thermal expansion of the rods as if there was nothing acting on them.

$$\delta_t = L_{AB} \alpha_s (\Delta T) + L_{BC} \alpha_b (\Delta T) \quad (3)$$

$$\delta_t = (0.250)(11.7 \times 10^{-6})(50) + (0.300)(20.9 \times 10^{-6})(50) \quad (4)$$

$$\delta_t = 459.75 \times 10^{-6} \text{m} \quad (5)$$

Now let's ignore the fact that there is any thermal expansion at all, and just consider an arbitrary shortening due to some load P , namely,

$$\begin{aligned}\delta_P &= \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}} \\ &= \frac{0.250P}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{0.300P}{(105 \times 10^9)(1.9635 \times 10^{-3})} \\ &= 3.2235 \times 10^{-9} P\end{aligned}\quad (6)$$

Now we know that the net deflection of both the thermal expansion and this arbitrary force must be zero. Thus the deflection due to the thermal expansion must be transferred as elastic potential energy into the constrained device. In other words...

$$\delta_T = \delta_P \quad (7)$$

Thus equating both of the things we found above we get a closed form solution for P, namely

$$3.2235 \times 10^{-9} P = 459.75 \times 10^{-6} \quad (8)$$

$$P = 142.62 \times 10^{-3} N = 142.62 kN \quad (9)$$

Problem 2:

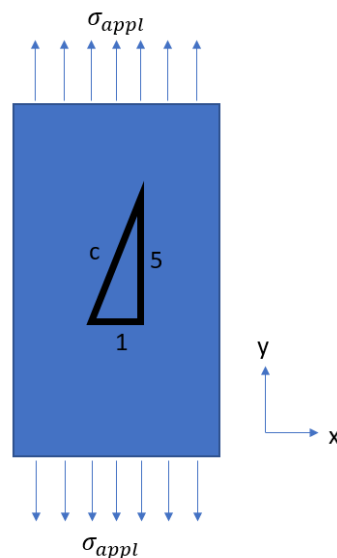


Figure 2

Assuming a triangle is drawn in a block of material which material properties are given by $E = 100$ GPa and $\nu = 100 MPa$, determine the length of the hypotenuse of the triangle (c) in the block's deformed state.

Solution 2

For this, we analyze what is known as Hooke's law in the field of linear elasticity. The Hooke's law equations provide you with an insightful relationship between stress and strain and how the couple in different directions. The Hooke's law equations are given as

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) \quad (10)$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x - \nu\sigma_z) \quad (11)$$

$$\epsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_y - \nu\sigma_x) \quad (12)$$

Here based on the applied stress and current state of the geometry, we can obtain the value for stresses as:

$$\sigma_x = \sigma_z = 0 \quad (13)$$

$$\sigma_y = \sigma_{app} \quad (14)$$

We can now plug in what we know above to eq. (10) and eq. (11) to obtain,

$$\epsilon_x = \frac{-\nu\sigma_{app}}{E} \text{ and} \quad (15)$$

$$\epsilon_y = \frac{\sigma_{app}}{E}, \quad (16)$$

respectively. Note, we have ignored eq. (12), simply because the triangle lays on the XY plane, so we do not care about the deformation in the z direction.

We now know that the total length of each triangle in the deformed state will become as illustrated

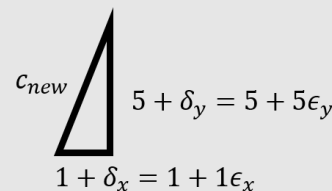


Figure 3: Illustration of deformed triangle. Not to scale!

Therefore, we can solve for each side of the triangle using what we found in eqs. (15) and (16) to be:

$$L_y = 5 + 5\epsilon_y = 5 + 5\frac{\sigma_{app}}{E} = 5.005 \quad (17)$$

$$L_x = 1 + \delta_x = 1 - \frac{\nu\sigma_{app}}{E} = 0.9995 \quad (18)$$

So to obtain the new length on the triangle we use these values in conjunction with Pythagorean theorem, such that:

$$C_{new} = \sqrt{5.005^2 + 0.9995^2} \approx 5.104 \quad (19)$$