THE SUCCESS STORY CONTINUES.

300
300
JEE Main

DUGGINENI VENKATA PANEEESH
KARANAM Lokesh

295
300
JEE Main

VUTUKURI VINAY MOHAN
KUCHIPUDI RAHUL DEEPAK
KHUSHANG SINGLA

Congratulating Our Students For Their Exceptional Performance In The JEE Main 2021 (July) as per the NTA Preliminary Key.
PHYSICS  

Max Marks: 100

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

1. In the given potentiometer circuit arrangement, the balancing length AC is measured to be 250 cm. When the galvanometer connection is shifted from point (1) to point (2) in the given diagram, the balancing length becomes 400 cm. The ratio of the emf of two cells, \( \frac{\varepsilon_1}{\varepsilon_2} \) is:

\[
\begin{align*}
(1) & \quad \frac{4}{3} \\
(2) & \quad \frac{8}{5} \\
(3) & \quad \frac{5}{3} \\
(4) & \quad \frac{3}{2}
\end{align*}
\]

KEY: 3

SOL:– \( \varepsilon_1 = 250 \times \text{Potential Gradient} \)

\[
\varepsilon_1 + \varepsilon_2 = 400 \times \text{Potential Gradient}
\]

\[
\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2} = \frac{250}{400} \Rightarrow \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2} = \frac{5}{8}
\]

\[
\Rightarrow 8 \varepsilon_1 = 5 \varepsilon_1 + 5 \varepsilon_2 \Rightarrow 3 \varepsilon_1 = 5 \varepsilon_2
\]

\[
\frac{\varepsilon_1}{\varepsilon_2} = \frac{5}{3}
\]
2. A force \( \vec{F} = (40\hat{i} + 10\hat{j}) \) N acts on a body of mass 5 kg. If the body starts from rest, its position of vector \( \vec{r} \) at time \( t = 10 \) s, will be:

(1) \( (400\hat{i} + 100\hat{j})m \)  
(2) \( (100\hat{i} + 400\hat{j})m \)  
(3) \( (100\hat{i} + 100\hat{j})m \)  
(4) \( (400\hat{i} + 400\hat{j})m \).

**KEY:** 1  
**SOL:**  
\[
\ddot{a} = \frac{\vec{F}}{m} = 8\hat{i} + 2\hat{j} \\
\vec{r} = ut + \frac{1}{2}at^2 \\
\vec{r} = 0 + \frac{1}{2}(8\hat{i} + 2\hat{j})(10)^2 \\
\vec{r} = 400\hat{i} + 100\hat{j}
\]

3. The instantaneous velocity of a particle moving in a straight line is given as \( \vec{v} = \alpha t + \beta t^2 \), where \( \alpha \) and \( \beta \) are constants. The distance travelled by the particle between 1s and 2s is:

(1) \( \frac{\alpha}{2} + \frac{\beta}{3} \)  
(2) \( \frac{3}{2} \alpha + \frac{7}{2} \beta \)  
(3) \( 3\alpha + 7\beta \)  
(4) \( \frac{3}{2} \alpha + \frac{7}{3} \beta \)

**KEY:** 4  
**SOL:**  
\[
\vec{v} = \alpha t + \beta t^2 
\Rightarrow \frac{d\vec{r}}{dt} = \alpha t + \beta t^2
\]

\[
\int_0^2 \frac{d\vec{r}}{dt} \, dt = \int_0^2 (\alpha t + \beta t^2) \, dt
\]

\[
r = \left[ \frac{\alpha t^2}{2} + \frac{\beta t^3}{3} \right]^2
\]

\[
= \left[ 2\alpha + \frac{8\beta}{3} \right] - \left[ \frac{\alpha}{2} + \frac{\beta}{3} \right]
\]

\[
= \frac{3\alpha}{2} + \frac{7\beta}{3}
\]

4. The relation between time \( t \) and distance \( x \) for moving body is given as \( t = mx^2 + nx \), where \( m \) and \( n \) are constants. The retardation of the motion is: (where \( \vec{v} \) stands for velocity)

(1) \( 2\vec{v}^3 \)  
(2) \( 2\, m\vec{v}^3 \)  
(3) \( 2\, m\vec{v}^3 \)  
(4) \( 2\, n^2\vec{v}^3 \)

**KEY:** 3
3

| PAGE |

SOL:

\[ t = mx^2 + nx \]
\[ \frac{dt}{dx} = 2mx + n \]
\[ g = \frac{1}{2mx + n} \]
\[ a = g \frac{dg}{dx} = v \frac{-2m}{(2mx + n)^2} = -2mg^3 \]

Hence Retardation = \(2mg^3\)

5. A heat engine has an efficiency of \(\frac{1}{6}\). When the temperature of sink is reduced by 62° C, its efficiency get doubled. The temperature of the source is:

(1) 124° C  
(2) 62° C  
(3) 99° C  
(4) 37° C

KEY: 3

SOL:

\[ n = 1 - \frac{T_2}{T_1} = \frac{1}{6} \quad (I) \]
\[ 1 - \frac{T_2 - 62}{T_1} = \frac{1}{3} \quad (II) \]

Eqn (II) – (I)

\[ \frac{62}{T_1} = 1 - \frac{1}{3} = \frac{1}{6} \]

\[ T_1 = 62 \times 6 = 372K \text{ or } 99° C \]

6. If \(q_f\) is the free charge on the capacitor plates and \(q_b\) is the bound charge on the dielectric slab of dielectric constant \(k\) placed between the capacitor plates, then bound charge \(q_b\) can be expressed as:

(1) \(q_b = q_f \left(1 + \frac{1}{\sqrt{k}}\right)\)
(2) \(q_b = q_f \left(1 - \frac{1}{k}\right)\)
(3) \(q_b = q_f \left(1 + \frac{1}{k}\right)\)
(4) \(q_b = q_f \left(1 - \frac{1}{\sqrt{k}}\right)\)

KEY: 2

SOL:

\[ E_{net} = \frac{E_0}{K} \]
\[ \frac{q_f}{E_0} \cdot \frac{q_b}{E_0} = \frac{q_f}{KE_0} \]
\[ \Rightarrow q_b = q_f \left(1 - \frac{1}{K}\right). \]
7. An electron moving with speed $\nu$ and a photon moving with speed $c$, have same $D$-Broglie wave length. The ratio of kinetic energy of electron to that of photon is:

\[(1) \frac{\nu}{3c} \quad (2) \frac{2c}{\nu} \quad (3) \frac{\nu}{2c} \quad (4) \frac{3c}{\nu}\]

KEY: 3

SOL:

$$\lambda_e = \frac{h}{p} \Rightarrow KE_e = \frac{1}{2} m \theta^2 = \frac{p \theta}{2}$$

$$KE_{\text{photon}} = \frac{hc}{\lambda} = pc$$

$$\frac{KE_e}{KE_{\text{photon}}} = \frac{p \theta / 2}{2c} = \frac{\theta}{2c}$$

8. In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position.

\[(1) \frac{3}{4} \quad (2) \frac{1}{3} \quad (3) \frac{1}{4} \quad (4) \frac{1}{2}\]

KEY: 1

SOL:

For $x = \pm A/2, \theta = \omega \sqrt{A^2 - x^2}$

$$\Rightarrow KE = \frac{1}{2} m \omega^2 (A^2 - x^2) = \frac{1}{2} m \omega^2 A^2 \left( \frac{3}{4} \right)$$

$$\frac{KE}{\text{Total energy}} = \frac{3}{4}$$

9. Two ions having same mass have charges in the ratio 1:2. They are projected normally in a uniform magnetic field with their speeds in the ratio 2:3. The ratio of the radii of their circular trajectories is:

\[(1) \ 4:3 \quad (2) \ 1:4 \quad (3) \ 2:3 \quad (4) \ 3:1\]

KEY: 1

SOL: $2q_1 = q_2$ and $q_1 = \frac{2}{3} q_2, m_1 = m_2$

$$r = \frac{m \theta}{qB}, \text{ Hence } \frac{r_1}{r_2} = \left( \frac{m_1}{m_2} \right) \left( \frac{q_2}{q_1} \right) \left( \frac{q_1}{q_2} \right)$$

$$\Rightarrow \frac{r_1}{r_2} = (2) \left( \frac{2}{3} \right) = \frac{4}{3}$$
10. The force is given in terms of time $t$ and displacement $x$ by the equation

$$F = A \cos Bx + C \sin Dt$$

The dimensional formula of $\frac{AD}{B}$ is:

(1) $[M^1 L T^{-2}]$    (2) $[M^0 L T^{-1}]$    (3) $[M^2 L^2 T^{-3}]$    (4) $[M L^2 T^{-3}]$.

**KEY:** 4

**SOL:**

$$F = A \cos Bx + C \sin Dt$$

Dimension of $A = [ML^{-2}]$

Dimension of $D = [T^{-1}]$

Dimension of $B = [L^{-1}]$

Dimension of $\frac{AD}{B} = [ML^{-2} T^{-1}] [T^{-1}]$

$$= [ML^2 T^{-3}]$$

11. A balloon was moving upwards with a uniform velocity of $10m/s$. An object of finite mass is dropped from the balloon when it was at a height of $75m$ from the ground level. The height of the balloon from the ground when object strikes the ground was around:

(takes the value of $g$ as $10m/s^2$)

(1) 200m    (2) 300m    (3) 125m    (4) 250m.

**KEY:** 3

**SOL:**

Concept: - Body thrown vertically upwards from top of a tower

$$h = -ut + \frac{1}{2} gt^2$$

$u = 10m/sec$

$h = 75m$

$g = 10 m/sec^2$

$75 = -10t + St^2$

$t^2 - 2t - 15 = 0$

Solve $t = 5 sec$

In 5 sec balloon will go to further height $h^I = ut$

$$= 10 \times 5 = 50m$$

Hence distance between balloon and object after 5 sec is

$75 + 50 = 125m$
12. Two spherical soap bubbles of radii \( r_1 \) and \( r_2 \) in vacuum combine under isothermal conditions. The resulting bubble has a radius equal to:

\[
\frac{r_1 + r_2}{2}, \quad \sqrt{r_1^2 + r_2^2}, \quad \sqrt{r_1 r_2}, \quad \frac{r_1 r_2}{r_1 + r_2}
\]

Key: 2

SOL:

Isothermal condition Boyles law holds good vacuum pressure outside is 0

\[
Here \ p_m = \frac{4T}{r}
\]

Only apply \( PV_1 + P_2V_2 = PV \)

\[
\frac{4T}{r_1} \times \frac{4}{3}\pi r_1^3 + \frac{4T}{r_2} \times \frac{4}{3}\pi r_2^3 = \frac{4T}{r} \times \frac{4}{3}\pi r^3
\]

\[
r_1^2 + r_2^2 = r^2
\]

Radius of single bubble \( r_1^2 + r_2^2 = r^2 \)

\[
r = \sqrt{r_1^2 + r_2^2}
\]

13. When radiation of wavelength \( \lambda \) is incident on a metallic surface, the stopping potential of ejected photoelectrons is 4.8\( V \). If the same surface is illuminated by radiation of double the previous wavelength, then the stopping potential becomes 1.6\( V \). Then the threshold wavelength of the metal is:

\[
(1) \ 8\lambda \quad (2) \ 4\lambda \quad (3) \ 2\lambda \quad (4) \ 6\lambda
\]

Key: 2

SOL:

Einstein's photoelectric equation

\[
\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV
\]

Using this

\[
\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + e \times 4.8 \quad \ldots \ldots (1)
\]

\[
\frac{hc}{2\lambda} = \frac{hc}{\lambda_0} + e \times 1.6 \quad \ldots \ldots (2)
\]

\[
\frac{2(\lambda_0 - \lambda)}{\lambda_0 - 2\lambda} = 3
\]

\[
2\lambda_0 - 2\lambda = 3\lambda_0 - 6\lambda
\]

Threshold wave length \( \lambda_0 = 4\lambda \)
14. A 10 Ω resistance is connected across 220 V – 50 HZ AC supply. The time taken by the current to change from its maximum value to the rms value is:

\[ \text{(1) } 3.0 \text{ms} \quad \text{(2) } 4.5 \text{ms} \quad \text{(3) } 1.5 \text{ms} \quad \text{(4) } 2.5 \text{ms}. \]

Key: 4

SOL: Concept – AC source across resistor time taken from maximum current\( \left( i_0 \right) \) of RMS current\( \left( \frac{i_0}{\sqrt{2}} \right) \)

Here equation is

\[ i = i_0 \cos \omega t \]

\[ \frac{i_0}{\sqrt{2}} = i_0 \cos \omega t \]

\[ \cos \omega t = \frac{1}{\sqrt{2}} \]

\[ 2\pi \times 50t = \frac{\pi}{4} \]

\[ t = 2.5 \text{ ms} \]

15. A prism of refractive index \( \mu \) and angle of prism \( A \) is placed in the position of minimum angle of deviation. If minimum angle of deviation is also \( A \), then in terms of refractive index value of \( A \) is:

\[ \text{(1) } \sin^{-1} \left( \sqrt{\frac{\mu - 1}{2}} \right) \quad \text{(2) } \cos^{-1} \left( \frac{\mu}{2} \right) \]

\[ \text{(3) } \sin^{-1} \left( \frac{\mu}{2} \right) \quad \text{(4) } 2 \cos^{-1} \left( \frac{\mu}{2} \right) \]

Key: 4

SOL: Prism in minimum deviation position

\[ \mu = \frac{\sin \left( \frac{A + dm}{2} \right)}{\sin \left( \frac{A}{2} \right)} \]

Given \( dm = A \)

\[ \mu = \frac{\sin A}{\sin \left( \frac{A}{2} \right)} \]

\[ = \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{\sin \frac{A}{2}} \]

\[ = 2 \cos \frac{A}{2} \]

\[ \mu = 2 \cos \frac{A}{2} \]

\[ A = 2 \cos^{-1} \left( \frac{\mu}{2} \right) \]
16. A ray of light entering from air into a denser medium of refractive index \( \frac{4}{3} \), as shown in figure. The light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle \( \theta \) should be equal to:

\[
\begin{align*}
(1) \sin^{-1} \frac{\sqrt{7}}{3} & \quad (2) \sin^{-1} \frac{\sqrt{7}}{4} \\
(3) \sin^{-1} \frac{\sqrt{5}}{3} & \quad (4) \sin^{-1} \frac{\sqrt{5}}{4}.
\end{align*}
\]

Key: 1

**SOL:** Total internal reflection as per the figure \( \theta^{11} + \theta^1 = 90^0 \)

Apply Snell’s law

\[1 \times \sin \theta = \frac{4}{3} \times \sin \theta^1\]

If light undergoes TIR at surface (2)

\[\theta^1 > C\]

\[\sin \theta^1 > \sin C\]

\[\sin(90 - \theta^1) > \sin C\]

\[\cos \theta^1 > \sin C\]

\[1 - \left(\frac{3}{4} \sin \theta\right)^2 > \sin^2 C\]

\[1 - \frac{9}{16} \sin^2 \theta > \sin^2 C\]

\[\frac{9\sin^2 \theta}{16} < \frac{7}{16}\]

\[\sin \theta < \frac{\sqrt{7}}{3}\]

17. The given potentiometer has its wire of resistance 10Ω. When the sliding contact is in the middle of the potentiometer wire, the potential drop across 2Ω resistor is:

\[
\begin{align*}
(1) \frac{40}{9} V & \quad (2) 5V \\
(3) \frac{40}{11} V & \quad (4) 10V.
\end{align*}
\]
Key: 1
SOL: Potentiometer Redraw circuit

\[ \begin{align*}
V_1 &= \frac{10}{7} V \\
V_2 &= \frac{2}{9} \times 20 = \frac{40}{9} V
\end{align*} \]

18. Two vectors \( \vec{X} \) and \( \vec{Y} \) have equal magnitude. The magnitude of \( (\vec{X} - \vec{Y}) \) is \( n \) times the magnitude of \( (\vec{X} + \vec{Y}) \). The angle between \( \vec{X} \) and \( \vec{Y} \) is:

\[
(1) \cos^{-1} \left( \frac{-n^2 - 1}{n^2 - 1} \right) \\
(2) \cos^{-1} \left( \frac{n^2 - 1}{-n^2 - 1} \right) \\
(3) \cos^{-1} \left( \frac{n^2 + 1}{-n^2 - 1} \right) \\
(4) \cos^{-1} \left( \frac{n^2 + 1}{n^2 - 1} \right)
\]

Key: 2
SOL:

Given \( |\vec{X}| = |\vec{Y}| \Rightarrow X^2 = Y^2 \)

\[
|\vec{X} - \vec{Y}| = n|\vec{X} + \vec{Y}|
\]

\[
X^2 + Y^2 - 2XY = n^2 (X^2 + Y^2 + 2X \cdot Y)
\]

\[
(1 - n^2)(X^2 + Y^2) = (1 + n^2)2XY \cos \theta
\]

\[
\cos \theta = \frac{1-n^2}{1+n^2}
\]

19. Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is \( W \), the weight of the same object on that planet will be:

\[
(1) 2W \quad (2) \sqrt{2}W \quad (3) \frac{1}{2}W \quad (4) W
\]

Key: 3
SOL: Acceleration due to gravity

\[
g = \frac{GM}{R^2} = \frac{G \times \frac{4}{3} R^3 P}{R^2}
\]

\[
g = \frac{4}{3} \pi GRP
\]
Two ideal electric dipoles A and B, having their dipole moment $p_1$ and $p_2$ respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of $37^0$ with the axis.

The ratio of the dipole moment of A and B, $\frac{p_1}{p_2}$ is: (take $\sin 37^0 = \frac{3}{5}$)

Key: 3

SOL:

$$\tan 37^0 = \frac{E_2}{E_1}$$

where $E_1 = K \frac{2p_1}{r^3}$ axial line

$E_2 = K \frac{p_2}{r_3}$ equitorial line

$$\tan 37^0 = \frac{p_2}{2p_1}$$

$$\frac{3}{4} = \frac{p_2}{2p_1}$$

$$\frac{p_1}{p_2} = \frac{2}{3}$$
This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A 16Ω wire is bend to form a square loop. A 9V supply having internal resistance of 1Ω is connected across one of its sides. The potential drop across the diagonals of the square loop is _______×10⁻¹ V.

**KEY:** 45

**SOL:**

\[ \Rightarrow \text{Reff}=4 \, \Omega \]
\[ i = \frac{9}{4} A, \text{also } i = \frac{4}{16} = \frac{9}{16} A \]
\[ \text{P.D across diagonal } = i (4+4) \]
\[ = \frac{9}{16} (8) = 4.5 \, V \]
\[ \therefore V_{AC} = 45 \times 10^{-1} \, V \]

22. A force of \( F = (5y + 20) \hat{j} \) N acts on a particle. The work done by this force when the particle is moved from \( y = 0 \) m to \( y = 10 \) m is _______J.

**KEY:** 450

\[ W = \int_{0}^{10} F_y \, dy \]

**SOL:**

\[ W = \int_{0}^{10} (5y + 20) \, dy \]
\[ W = \left[ \frac{5y^2}{2} + 20y \right]_{0}^{10} \]
\[ W = 450J. \]

23. Two circuits are shown in the figure (a)& (b). At a frequency of _______rad/s the average power dissipated in one cycle will be same in both the circuits.

**KEY:** 500
SOL:

\[ P_1 = P_2 \]
\[ \frac{220 \times 220}{5} = 220 \times \frac{220}{Z} \left( \frac{5}{Z} \right) \Rightarrow Z^2 = 25 \]
\[ Z = 5 = R \]

Circuit is in resonance

\[ \omega = \frac{1}{\sqrt{LC}} \]
\[ \omega = \frac{1}{\sqrt{0.1 \times 40 \times 10^{-6}}} = \frac{1}{2 \times 10^{-3}} = \frac{1000}{2} = 500 \text{ rad/s} . \]

24. A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is __________ \( \pi \times 10^{-1} \) Nm.

KEY: 4

SOL:

\[ \tau = I \alpha \]
\[ \tau = I \left[ \frac{\omega_2 - \omega_1}{\Delta t} \right] \]
\[ \tau = \frac{MR^2}{2} \left| \frac{0 - 600 \times \frac{2\pi}{60}}{10} \right| \]
\[ = \frac{10 \times 400 \times 10^{-4}}{2} (2\pi) \]
\[ = 4\pi \times 10^{-1} \text{ Nm} \]

25. From the given data, the amount of energy required to break the nucleus of aluminum \( ^{27}_{13} Al \) is __________ \( x \times 10^{-3} \) J.

Mass of neutron=1.00866 u.

Mass of proton= 1.00726 u

Mass of Aluminum nucleus= 27.18846 u

(Assume 1u corresponding to \( x \) J of energy)

(Round off to the nearest integer)
26. A system consists of two types of gas molecules A and B having same number density $2 \times 10^{25} / \text{m}^3$. The diameter of A and B are $10 \text{A}^0$ and $5 \text{A}^0$ respectively. They suffer collision at room temperature. The ratio of average distance covered by the molecule A to that of B between two successive collision is ______ $\times 10^{-2}$.

KEY: 25

SOL: 

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{D_2}{D_1}\right)^2$$

Mean free path $\alpha = \frac{1}{(\text{Diameter})^2}$

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{D_2}{D_1}\right)^2 \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{25}{100} = \frac{1}{4} = 25 \times 10^{-2}.$$ 

27. The nuclear activity of a radioactive element becomes $\left(\frac{1}{8}\right)^{th}$ of its initial value in 30 years. The half-life of radioactive element is ______ years.

KEY: 10

SOL: 

$$A = \frac{A_0}{2^n}$$

Given $A = \frac{A_0}{8} \Rightarrow n = 3$

$\therefore t = nT_{1/2}$

$30 = 3T_{1/2} \Rightarrow T_{1/2} = 10 \text{ Years}$

28. A light beam of wavelength $500 \text{ nm}$ is incident on a metal having work function of $1.25 \text{ eV}$, placed in a magnetic field of intensity B. The electrons emitted perpendicular to the magnetic field B, with maximum kinetic energy are bent into circular arc of radius $30 \text{ cm}$. The value of B is ______ $\times 10^{-7}$ T. Given $hc = 20 \times 10^{-26} \text{ J} - \text{m}$, mass of electron= $9 \times 10^{-31} \text{ kg}$.

KEY: 125
SOL:

\[ R = \frac{\sqrt{2mK}}{Be} \Rightarrow B = \frac{\sqrt{2mK}}{Re} \quad (1) \]

as \( K = \frac{hc}{\lambda} - W_0 \)

\[ K = \frac{20 \times 10^{-26}}{500 \times 10^{-9}} - 1.25 \times 1.6 \times 10^{-19} \]

\[ K = 2 \times 10^{-19} J \quad (2) \]

From (1) & (2)

\[ B = \frac{\sqrt{2 \times 9 \times 10^{-31} \times 2 \times 10^{-19}}}{30 \times 10^{-2} \times 1.6 \times 10^{-19}} \]

\[ B = 125 \times 10^{-7} T \]

29. A message signal of frequency 20 kHz and peak voltage of 20 volt is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 volt. The modulation index will be _________.

KEY: 1

SOL: Modulation index = \( \frac{A_m}{A_c} \)

\[ \mu = \frac{20}{20} \]

30. In a semiconductor, the number density of intrinsic charge carriers at 27°C is \( 1.5 \times 10^{16} / m^3 \). If the semiconductor is doped with impurity atom, the hole density increases to \( 4.5 \times 10^{22} / m^3 \). The electron density in the semiconductor is \( _____ \times 10^9 / m^3 \).

KEY: 5

SOL: \( N_i^2 = N_e N_h \)

\[ \left(1.5 \times 10^{16}\right)^2 = N_e \left(4.5 \times 10^{22}\right) \]

\[ N_e = \frac{10^{10}}{2} = 5 \times 10^9 \]
31. In the following the correct bond order sequence is:

1) $O_2 > O_2^+ > O_2^{2-} > O_2^-$  
2) $O_2^{2-} > O_2^- > O_2 > O_2^+$  
3) $O_2^+ > O_2 > O_2^{2-} > O_2$  
4) $O_2^+ > O_2 > O_2 > O_2^{2-}$

Key: 4

SOL: No of electrons

<table>
<thead>
<tr>
<th>Bond Order</th>
<th>$O_2^+$</th>
<th>$O_2$</th>
<th>$O_2^-$</th>
<th>$O_2^{2-}$</th>
<th>$O_2$</th>
<th>$O_2^-$</th>
<th>$O_2$</th>
<th>$O_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5</td>
<td>2</td>
<td>1.5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ C_6H_5NO_2 \xrightarrow{Sn+HCl} A \xrightarrow{C_6H_5N_2Cl} P \] (Yellow coloured compound)

32. Consider the above reaction, the Product “P” is:

1) \[ \text{N=N--N} \quad \text{NH}_2 \]
2) \[ \text{N=N} \quad \text{H} \quad \text{N} \quad \text{H} \]
3) \[ \text{N=N} \quad \text{NH}_2 \quad \text{N} \]
4) \[ \text{N=N} \quad \text{H} \quad \text{N} \quad \text{H} \]

Key: 1
Given below are two statements:

Statement I: Chlorofluoro carbons breakdown by radiation in the visible energy region and release chlorine gas in the atmosphere which then reacts with stratospheric ozone.

Statement II: Atmospheric ozone reacts with nitric oxide to give nitrogen and oxygen gases, which add to the atmosphere.

For the above statements choose the correct answer from the options given below:

1) Both statement I and II are false
2) Both statement I and II are correct
3) Statement I is correct but statement II is false
4) Statement I is incorrect but statement II is true

Key: 1

SOL: Statement I – $\text{CF}_2 + \text{UV} \rightarrow \text{Cl}$

Statement II - $\text{O}_3 + \text{NO} \rightarrow \text{NO}_2 + \text{O}_2$

Both statement I and II are false

34. The spin only magnetic moments (in BM) for free $\text{Ti}^{3+}$, $\text{V}^{2+}$ and $\text{Sc}^{3+}$ ions respectively are (At. No. Sc: 21 ; Ti : 22 ; V : 23)

1) 0, 3.87, 1.73  
2) 3.87, 1.73, 0  
3) 1.73, 3.87, 0  
4) 1.73, 0, 3.87

Key: 3

SOL: $\text{Ti}^{3+}(Z=22)(\text{Ar})3d^1$

No’of unpaired electron (n)=1

Magnetic moment $\mu = \sqrt{n(n+2)BM}$

$\mu = \sqrt{1(1+2)} = 1.73$

$\text{V}^{2+}(Z=23)(\text{Ar})3d^3$  

$n = 3$

$\mu = \sqrt{3(3+2)BM} = 3.87BM$

$\text{Sc}^{3+}(Z=21) \ (\text{Ar})3d^1$  

$n = 0$

$\mu = 0$
35. Identify the species having one $\pi$-bond and maximum number of canonical forms from the following:

1) $CO_3^{2-}$  
2) $O_2$  
3) $SO_2$  
4) $SO_3$

Key: 1

SOL:

36. The correct decreasing order of densities of the following compounds is:

(A)  
(B)  
(C)  
(D)

1) $(C) > (D) > (A) > (B)$  
2) $(A) > (B) > (C) > (D)$  
3) $(C) > (B) > (A) > (D)$  
4) $(D) > (C) > (B) > (A)$

Key: 4

SOL: Density $\propto$ M.wt

37. What is the major product “P” of the following reaction?

1)  
2)  
3)  
4)  

Key: 4
SOL:

\[
\text{NaNO}_2 + \text{HCl} \quad 278K \quad \text{NH}_2 \quad \text{CH}_3 \quad \text{N}_2^0 \quad \text{OH}
\]

38. Identify the process in which change in the oxidation state is five:
   1) \( \text{CrO}_4^{2-} \rightarrow \text{Cr}^{3+} \)
   2) \( \text{MnO}_4^{-} \rightarrow \text{Mn}^{2+} \)
   3) \( \text{C}_2\text{O}_4^{2-} \rightarrow 2\text{CO}_2 \)
   4) \( \text{Cr}_2\text{O}_7^{2-} \rightarrow 2\text{Cr}^{3+} \)

Key: 2
SOL: \( \text{MnO}_4^{-} + 8\text{H}^+ + 5\text{e}^- \rightarrow \text{Mn}^{2+} + 4\text{H}_2\text{O} \)

39. A biodegradable polyamide can be made from:
   1) Hexamethylene diamine and adipic acid
   2) Glycine and isoprene
   3) Glycine and amino caproic acid
   4) Styrene and caproic acid

Key: 3
SOL: \( \text{Nylon} \; -2- \text{nylon} \; 6- \)  
\( \text{glycine}[\text{H}_2\text{N}-(\text{CH}_2)_5\text{COOH}] \) and Amino caproic acid 
\( [\text{H}_2\text{N}(\text{CH}_2)_5\text{COOH}] \)

40. A reaction of benzonitrile with one equivalent \( \text{CH}_3\text{MgBr} \) followed by hydrolysis produces a yellow liquid “P”. the compound “P” will give positive ________

   1) Iodoform test
   2) Schiff’s test
   3) Ninhydrin’s test
   4) Tollen’s test

Key: 1
SOL:
41. Which one of the following is correct structure for cytosine?

1) 

\[
\begin{array}{c}
\text{H}_2\text{C} \\
\text{N} \\
\text{O} \\
\text{H} \\
\end{array}
\]

2) 

\[
\begin{array}{c}
\text{H}_2\text{N} \\
\text{N} \\
\text{O} \\
\text{H} \\
\end{array}
\]

3) 

\[
\begin{array}{c}
\text{NH}_2 \\
\text{N} \\
\text{O} \\
\text{H} \\
\end{array}
\]

4) 

\[
\begin{array}{c}
\text{H}_2\text{C} \\
\text{N} \\
\text{O} \\
\text{H} \\
\end{array}
\]

Key: 3

SOL:

\[
\begin{array}{c}
\text{NH}_2 \\
\text{N} \\
\text{O} \\
\text{H} \\
\end{array}
\]

Cytosine

42. Maleic anhydride

Maleic anhydride can be prepared by:

1) Treating trans-but-2-enedioic acid with alcohol and acid
2) Treating cis-but-2-enedioic acid with alcohol and acid
3) Heating cis-but-2-enedioic acid
4) Heating trans-but-2-enedioic acid

Key: 3
SOL:

43. Match List I with List II

<table>
<thead>
<tr>
<th>List –I</th>
<th>List – II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example of Colloids</td>
<td>Classification</td>
</tr>
<tr>
<td>(a) Cheese</td>
<td>(i) dispersion of liquid in liquid</td>
</tr>
<tr>
<td>(b) Pumice stone</td>
<td>(ii) dispersion of liquid in gas</td>
</tr>
<tr>
<td>(c) Hair Cream</td>
<td>(iii) dispersion of gas in Solid</td>
</tr>
<tr>
<td>(d) Cloud</td>
<td>(iv) dispersion of liquid in Solid</td>
</tr>
</tbody>
</table>

Choose the most appropriate answer from the options given below:

1) \( a-iv, b-iii, c-ii, d-i \)
2) \( a-iv, b-iii, c-i, d-ii \)
3) \( a-iii, b-iv, c-i, d-ii \)
4) \( a-iv, b-i, c-iii, d-ii \)

Key: 2

SOL: a) Cheese is a colloid of liquid in solid also called as gel  
    b) Pumice stone is a colloid of gas (air) in solid  
    c) Hair cream is a colloid of liquid in liquid also called as Emulsions  
    d) Cloud is a colloid of liquid(water drops) in gas(Air)

44. Match List I with List II

<table>
<thead>
<tr>
<th>List –I</th>
<th>List – II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements</td>
<td>Properties</td>
</tr>
<tr>
<td>(a) Li</td>
<td>(i) Poor water solubility of ( Li^- ) salt</td>
</tr>
<tr>
<td>(b) Na</td>
<td>(ii) Most abundant element in cell fluid</td>
</tr>
<tr>
<td>(c) K</td>
<td>(iii) Bicarbonate salt used in fire extinguisher</td>
</tr>
<tr>
<td>(d) Cs</td>
<td>(iv) Carbonate salt decomposes easily on heating</td>
</tr>
</tbody>
</table>

Choose the correct answer from the options given below:

1) \( a-iv, b-iii, c-ii, d-i \)
2) \( a-iv, b-ii, c-iii, d-i \)
3) \( a-i, b-ii, c-iii, d-iv \)
4) \( a-i, b-iii, c-ii, d-iv \)

Key: 1
SOL:

a) \( \text{Li}_2\text{CO}_3 \) is less stable due to high polarising power of \( \text{Li}^+ \) ion and weakens the C–O bond and forms \( \text{Li}_2\text{O} \) & \( \text{CO}_2 \)

b) \( \text{NaHCO}_3 \) gives \( \text{CO}_2 \) in the presence of heat (reduce the availability of \( \text{O}_2 \)) hence it stops fire

\[
2\text{NaHCO}_3 \rightarrow \text{Na}_2\text{CO}_3 + \text{CO}_2 + \text{H}_2\text{O}
\]

c) Most abundant metal ion in cell fluid is \( \text{K}^+ \)

d) CsI has very less heat of hydration due to large size of cation and anion

45. Which among the following is the strongest acid?

1) \( \text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_3 \)  
2)  
3)  
4)  

Key: 4

SOL:

Acidic strength \( \alpha \) Stability of conjugate base

1) Conjugate base is less stable

2) unstable

3) is an anti aromatic

4) most stable among these four C Bases due to Aromatic nature

46.

\[
\text{Br} \quad \text{CHO} \quad \text{EtOHexcess} \quad \text{dry HCl gas} \quad \text{(()major product)} \quad \text{(()major product)}
\]

[Where \( \text{Et} \Rightarrow -\text{C}_2\text{H}_5 \) \( \text{Bu} \Rightarrow (\text{CH}_3)_3\text{C}^- \)] Consider the reaction sequence, product “A” and product “B” formed respectively are:
1) \[
\text{Br} \quad \text{Et} \quad \text{O} \quad \text{Et} \quad \text{BuO} \quad \text{Et} \quad \text{O} \quad \text{Et}
\]

2) \[
\text{EtO} \quad \text{Et} \quad \text{O} \quad \text{Et} \quad \text{H}_2\text{C} \quad \text{EtO} \quad \text{Et} \quad \text{Et}
\]

3) \[
\text{Br} \quad \text{Et} \quad \text{Et} \quad \text{H}_2\text{C} \quad \text{EtO} \quad \text{Et} \quad \text{Et}
\]

4) \[
\text{EtO} \quad \text{Et} \quad \text{EtO} \quad \text{O} \quad \text{Bu}
\]

Key: 3 or 1

(Note: key given by NTA (3))

Reason: - reaction conditions are not properly mentioned

SOL:

47. Which one of the following metals forms interstitial hydride easily?

1) Cr \hspace{1cm} 2) Mn \hspace{1cm} 3) Fe \hspace{1cm} 4) Co

Key: 1

SOL: In periodic table, 7, 8 and 9 groups i.e, Mn, Fe and Co groups does not forms interstitial hydrides Therefore Cr can form easily
48. Match List I with List II (Both having metallurgical terms)

**List – I**
- (a) Concentration of Ag ore
- (b) Blast furnace
- (c) Blister copper
- (d) Froth floatation method

**List – II**
- (i) Reverberatory furnace
- (ii) Pig iron
- (iii) Leaching with dilute NaCN solution
- (iv) Sulfide ores

Choose the correct answer from the options given below:
1) $a - iv, b - iii, c - ii, d - i$
2) $a - iv, b - i, c - iii, d - ii$
3) $a - iii, b - iv, c - i, d - ii$
4) $a - iii, b - ii, c - i, d - iv$

Key: 4

**SOL:**
- a) Ag ore is concentrated by leaching and extracted by Mac Arthur Forest Cyanide process
- b) In blast furnace Pig iron formed
- c) In the extraction of Cu, blisters of SO2 are released in reverberatory furnace
- d) Froth floatation technique is used to concentrate Sulphide ores

49. The ionic radii of $F^-$ and $O^{2-}$ respectively are 1.33 Å and 1.4 Å, while the covalent radius of N is 0.74 Å. The correct statement for the ionic radius of $N^{3-}$ from the following is:

1) It is smaller than $O^{2-}$ and $F^-$, but bigger than of N
2) It is smaller than $F^-$ and N
3) It is bigger than $O^{2-}$ and $F^-$
4) It is bigger than $F^-$ and N, but smaller than of $O^{2-}$

Key: 3

**SOL:** $F^-, O^{2-}$ and $N^{3-}$ are isoelectronic species therefore radius decreases as Z(atomic number) Increases

50. Which one of the following metal complexes is most stable?

1) $[Co(en)_2(NH_3)_2]Cl_2$
2) $[Co(en)(NH_3)_4]Cl_2$
3) $[Co(NH_3)_6]Cl_2$
4) $[Co(en)_3]Cl_2$

Key: 4

**SOL:** stability of complex is α no of chelating ligands
i.e, poly dentate ligands i.e, three en groups in $[Co(en)_3]Cl_2$
51. A system does 200 J of work and at the same time absorbs 150 J of heat. The magnitude of the change in internal energy is ___________J. (Nearest integer)

Key: 50

SOL: \( q = 150 \text{J} \) \( \omega = -200 \text{J} \) \( \Delta E = q + \omega = 150 + (-200) = -50 \text{J} \)

52. Consider the above chemical reaction. The total number of stereo Isomers possible for product ‘P’ is __________

Key: 2

SOL: cis alkene on anti addition gives racemic mixture

53. For a chemical reaction \( A \rightarrow B \), it was found that concentration of B is increased by \( 0.2 \text{molL}^{-1} \) in 30 min. The average rate of the reaction is __________\( \times 10^{-1} \text{mol L}^{-1} \text{h}^{-1} \). (in nearest integer)

Key: 4

SOL: average rate = \( + \frac{d[B]}{dt} \) \( dt = 30 \text{ mts} = 0.5 \text{ hrs} \)

\( = \frac{0.2}{0.5} = 0.4 = 4 \times 10^{-1} \)

54. Assuming that \( \text{Ba(OH)}_2 \) is completely ionized in aqueous solution under the given conditions the concentration of \( H_2O^+ \) ions in 0.005M aqueous solution of \( \text{Ba(OH)}_2 \) at \( 298 K \) is __________\( \times 10^{-12} \text{mol L}^{-1} \). (Nearest integer)

Key: 1

SOL:

\( \text{Ba(OH)}_2 \rightleftharpoons \text{Ba}^{2+} + 2\text{OH}^- \)

\( 5 \times 10^{-3} \)

\( 5 \times 10^{-3} \) \( 1 \times 10^{-3} \)

\( : [\text{OH}^-] = 10^{-2} \text{M} : [\text{H}_2\text{O}^+] = \frac{k_w}{[\text{OH}^-]} = \frac{10^{-14}}{10^{-2}} = 10^{-12} \)
55. Number of electrons present in 4f orbital of Ho$^{3+}$ ion is __________ (given atomic No. of Ho=67)

Key: 10

SOL: Electronic configuration of Ho → [Xe]$_{64}$ 4f$^{11}$5d$^0$6s$^2$

∴ Ho$^{3+}$ → [Xe]$_{64}$ 4f$^{10}$

56. An LPG cylinder gas at a pressure of 300kPa at 27°C. The cylinder can withstand the pressure of 1.2×10$^6$ Pa. The room in which the cylinder is kept catches fire. The minimum temperature at which the bursting of cylinder will take place is __________ °C. (Nearest integer)

Key: 927

SOL: \[
\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \text{[Gaylussac's law]} \\
\frac{300 \times 10^3 \text{ Pa}}{300} = \frac{1.2 \times 10^6 \text{ Pa}}{T_2} \quad \Rightarrow T_2 = 1200 \text{K} = 927°C
\]

57. 0.8 g of an organic compound was analysed by Kjeldahl’s method for the estimation of nitrogen. If the percentage of nitrogen in the compound was found to be 42%, then _______ mL of 1M H$_2$SO$_4$ would have been neutralized by the ammonia evolved during the analysis.

Key: 12

SOL: \[
\%N = \frac{1.4 \times V \times N}{w} \quad 42 = \frac{1.4 \times V \times 2}{0.8} \quad \therefore V = \frac{2 \times 0.8}{1.4 \times 2} = 12 \text{ml}
\]

58. When 300 g of a substance ‘X’ is dissolved in 100 g of CCl$_4$, it raises the boiling point by 0.60 K. The molar mass of the substance ‘X’ is __________ g mol$^{-1}$. (Nearest integer)

Key: 250

SOL: \[
\Delta T_b = K_b \times m = 5 \times \left( \frac{w}{M \cdot \text{wt}} \right)_{\text{solvent}} \times 1000 \quad 0.6 = 5 \times \frac{3}{M \cdot \text{wt}_{\text{solvent}}} \times 1000 \\
M \cdot \text{wt}_{\text{solvent}} = \frac{3000}{60} \times 5 \quad = 250 \text{g/mol}
\]

59. An accelerated electron has a speed of 5×10$^6$ m/s$^{-1}$ with an uncertainty of 0.02%. The uncertainty in finding its location while in motion is $x$×10$^{-9}$. The value of $x$ is __________ (Nearest integer)

Key: 58

SOL: \[
\Delta v = 5 \times 10^6 \times 0.02 \frac{100}{100} = 1000 \text{ m/sec} \\
\left[ \frac{\Delta x \cdot m \Delta v}{h} \right] = \frac{h}{4\pi} \\
\therefore \Delta x = \frac{h}{4\pi \cdot m \cdot \Delta v} = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 1000} = 58 \times 10^{-9}
\]

60. The number of significant figures in 0.00340 is ________

Key: 3

SOL: Zero’s preceding to first non-zero digit
61. Let \( X \) be a random variable such that the probability function of a distribution is given by \( \Pr(X = 0) = \frac{1}{2}, \Pr(X = j) = \frac{1}{3^j} \) \((j = 1, 2, 3, ..., \infty)\) Then the mean of the distribution and \( \Pr(X \text{is positive and even}) \) respectively are:

1) \( \frac{3}{4} \) and \( \frac{1}{8} \)  
2) \( \frac{3}{4} \) and \( \frac{1}{9} \)  
3) \( \frac{3}{8} \) and \( \frac{1}{8} \)  
4) \( \frac{3}{4} \) and \( \frac{1}{16} \)

Key: 1

**SOL:**

\[
P(x) = \frac{1}{2}, \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^2}, \frac{1}{3^3}, \ldots.
\]

Mean = \( \sum_{i=0}^{\infty} X_i P_i = 0 + \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{4}{3^4} + \ldots \)

\[
= \frac{1}{3} \left( 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots \right) = \frac{1}{3} \left( 1 - \frac{1}{3} \right)^{-2}
\]

\[
= \frac{1}{3} \cdot \frac{9}{4} = \frac{3}{4}
\]

\[
\Pr(X \text{is positive and even}) = \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \ldots
\]

\[
= \frac{1}{3^2} \left( 1 + \frac{1}{9} + \frac{1}{9^2} + \ldots \right) = \frac{1}{9} \left( 1 - \frac{1}{9} \right)^{-1} = \frac{9}{8} = \frac{1}{8}
\]

62. Let \( y = y(x) \) be the solution of the differential equation \( xdy = (y + x^3 \cos x) \, dx \) with \( y(\pi) = 0 \) then \( y(\frac{\pi}{2}) \) is equal to:

1) \( \frac{\pi^2}{4} - \frac{\pi}{2} \)  
2) \( \frac{\pi^2}{2} + \frac{\pi}{4} \)  
3) \( \frac{\pi^2}{4} + \frac{\pi}{2} \)  
4) \( \frac{\pi^2}{2} - \frac{\pi}{4} \)

Key: 3

**SOL:**

\[
x \, dy - x \, dx = x^3 \cos x \, dx
\]

\[
\frac{x \, dy - x \, dx}{x^2} = (x \cos x) \, dx
\]

\[
\int x \, \sin x \, dx = \int x \cos x \, dx
\]

\[
y = x \sin x - (-\cos x) + c
\]
\[ y = x^2 \sin x + x \cos x + x \]

\[ y(\frac{\pi}{2}) = \frac{\pi^2}{4} + \frac{\pi}{2} \]

63. The value of \( \cot \frac{\pi}{24} \) is:

1) \( \sqrt{2} - \sqrt{3} - 2 + \sqrt{6} \)
2) \( 3\sqrt{2} - \sqrt{3} - \sqrt{6} \)
3) \( \sqrt{2} + \sqrt{3} + 2 + \sqrt{6} \)
4) \( \sqrt{2} + \sqrt{3} + 2 - \sqrt{6} \)

Key: 3

SOL:
\[ \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{2\cos^2 \theta}{2\sin \theta \cos \theta} \]

\[ \therefore \cot \frac{\pi}{24} = \frac{1 + \cos \frac{\pi}{12}}{\sin \frac{\pi}{12}} \left( \sin \frac{\pi}{12} = \frac{\sqrt{3} - 1}{2\sqrt{2}} \right) \& \left( \cos \frac{\pi}{12} = \frac{\sqrt{3} + 1}{2\sqrt{2}} \right) \]

\[ = \frac{1 + \cos 2\theta}{\sin 2\theta} \]

\[ \therefore \cot \frac{\pi}{24} = \frac{1 + \cos \frac{\pi}{12}}{\sin \frac{\pi}{12}} \left( \sin \frac{\pi}{12} = \frac{\sqrt{3} - 1}{2\sqrt{2}} \right) \& \left( \cos \frac{\pi}{12} = \frac{\sqrt{3} + 1}{2\sqrt{2}} \right) \]

\[ = \frac{1 + \frac{\sqrt{3} + 1}{2\sqrt{2}}}{\frac{\sqrt{3} - 1}{2\sqrt{2}}} = \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{3} - 1} = \frac{(2\sqrt{2} + \sqrt{3} + 1)(\sqrt{3} + 1)}{2} \]

\[ = \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2} = \sqrt{6} + \sqrt{2} + \sqrt{3} + 2 \]

64. Let \( a, b \) and \( c \) be distinct positive numbers. If the Vectors

\( \hat{a}i + \hat{a}j + \hat{c}k, \hat{i} + \hat{k} \) and \( \hat{c}i + \hat{c}j + \hat{b}k \) are co-planar then \( c \) is equal to:

1) \( \frac{2}{\frac{1}{a} + \frac{1}{b}} \)
2) \( \frac{1 + \frac{1}{b}}{a} \)
3) \( \sqrt{ab} \)
4) \( \frac{a + b}{2} \)

Key: 3

SOL:

\[
\begin{vmatrix}
  a & a & c \\
  1 & 0 & 1 \\
  c & c & b \\
\end{vmatrix} = 0 \Rightarrow -ac - a(b - c) + c(c) = 0
\]

\[ \Rightarrow -ac - ab + ac + c^2 = 0 \Rightarrow c^2 = ab \]
65. The sum of all those terms which are rational numbers in the expansion of \( \left( \frac{1}{2^3} + \frac{1}{3^4} \right)^{12} \) is:

1) 35  
2) 43  
3) 27  
4) 89

Key: 2

SOL:

General term \( =^{12}C_r \left( 2^{\frac{3}{4}} \right)^{12-r} \left( 3^{\frac{1}{4}} \right)^r =^{12}C_r 2^{\frac{4r-9}{4}} 3^{\frac{r}{4}} \)

\( r = \) multiple of lcm \( \{3,4\} \)

Required sum \( =^{12}C_0 2^4 + ^{12}C_{12} 3^3 = 16 + 27 = 43 \)

66. The number of real solutions of the equation \( x^2 - |x| - 12 = 0 \)

1) 2  
2) 3  
3) 1  
4) 4

Key: 1

SOL:

\[ |x|^2 - |x| - 12 = 0 \]
\[ |x| = 4, -3 \text{ (not possible)} \implies |x| = 4, \implies x = \pm 4 \]
\[ \therefore \text{Number of real solutions} = 2 \]

67. The value of the integral \( \int_{-1}^{1} \log \left( x + \sqrt{x^2 + 1} \right) dx \) is:

1) 0  
2) -1  
3) 1  
4) 2

Key: 1

SOL:

\[ I = \int_{-1}^{1} \log x + \sqrt{x^2 + 1} \, dx \]

\[ f(x) = \log \left( \sqrt{x^2 + 1} + x \right) \]

\[ f(-x) = \log \left( \sqrt{x^2 + 1} - x \right) \]

\[ = - f(x), \quad \text{So } f(x) \text{ is an odd function} \implies I = 0 \]

68. Consider functions \( f : A \to B \) and \( g : B \to C (A, B, C \subseteq R) \) such that \( (gof)^{-1} \) exists, then:

1) \( f \) and \( g \) both are on to  
2) \( f \) and \( g \) both are one-one  
3) \( f \) is on to and \( g \) is one-one  
4) \( f \) is one-one and \( g \) is on to

Key: 4

SOL:

\( (gof)^{-1} \) exists only when \( gof \) is bijective

\[ \Rightarrow f \text{ is one-one, } g \text{ is on to} \]
69. The lowest integer which is greater than \( \left( 1 + \frac{1}{10^{100}} \right)^{10^{100}} \) is ____________

1) 1  
2) 4  
3) 2  
4) 3

Key: 4

SOL:

Let \( 10^{100} = n \)

So, \( \left( 1 + \frac{1}{n} \right)^n = \sum_{r=0}^{n} \frac{n^r}{r!} \)

\( = 1 + \frac{n(n-1)}{2n^2} + \frac{n(n-1)(n-2)}{6n^3} + \ldots \)

\( \Rightarrow \left( 1 + \frac{1}{n} \right)^n > 2 \quad \text{Also} \quad \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n = e < 3. \)

70. If \( |\vec{a}| = 2, |\vec{b}| = 5 \) and \( |\vec{a} \times \vec{b}| = 8 \), then \( |\vec{a} \cdot \vec{b}| \) is equal to:

1) 3  
2) 6  
3) 5  
4) 4

Key: 2

SOL:

\( |\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 \cdot |\vec{b}|^2 \)

\( 64 + (\vec{a} \cdot \vec{b})^2 = 4 \times 25 \Rightarrow (\vec{a} \cdot \vec{b})^2 = 100 - 64 = 36 \Rightarrow \vec{a} \cdot \vec{b} = 6 \)

71. Consider the statement “The match will be played only if the weather is good, and ground is not wet”. Select the correct negation from the following:

1) The match will not be played, and weather is not good, and ground is wet
2) The match will be played, and weather is not good, or ground is wet
3) If the match will not be played, then either weather is not good, or ground is wet
4) The match will not be played, or weather is good, and ground is not wet

Key: 2

SOL:

\( p: \text{ The match will be played} \)
\( q: \text{ Weather is good} \)
\( r: \text{ Ground is wet} \)

\( \sim (p \rightarrow q \land \sim r) \equiv \sim (p \lor (q \land r)) \)

\( s = 0 \equiv p \land (\sim q \lor r) \)

72. If \( [x] \) be the greatest integer less than or equal to \( x \), then \( \sum_{n=8}^{100} \left\lfloor \frac{(-1)^n \cdot n}{2} \right\rfloor \) is equal to:

1) 4  
2) 0  
3) -2  
4) 2

Key: 1
SOL:

\[
\sum_{n=8}^{100} \left( \frac{(-1)^n \cdot n}{2} \right) = \left[ \frac{8}{2} \right] + \left[ -\frac{9}{2} \right] + \left[ \frac{10}{2} \right] + \ldots + \left[ -\frac{99}{2} \right] + \left[ \frac{100}{2} \right]
\]

73. The first of the two samples in a group have 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation \(\sqrt{13.44}\), Then the standard deviation of the second sample is:

1) 5  
2) 6  
3) 4  
4) 8

Key: 3

SOL:

\[n_1 = 100 \quad \bar{x}_1 = 15 \quad \sigma_1 = 3\]
\n\[n_1 + n_2 = 250 \quad \bar{x} = 15.6 \quad \sigma = \sqrt{13.44}\]
\n\[d_1 = \bar{x} - \bar{x}_1 = 0.6\]
\n\[d_2 = \bar{x} - \bar{x}_2 = 15.6 - 16 = -0.4\]
\n\[\sigma^2 = \frac{1}{n_1 + n_2} \left[ n_1 \left( \sigma_1^2 + d_1^2 \right) + n_2 \left( \sigma_2^2 + d_2^2 \right) \right]\]
\n= \frac{1}{250} \left[ 100(9 + 0.36) + 150(\sigma_2^2 + 0.16) \right]
\n= \frac{1}{250} \left[ 2(9.36) + 3\sigma_2^2 + 0.48 \right]
\n\frac{3}{5} \sigma_2^2 = 9.6 \Rightarrow \sigma_2^2 = 16 \Rightarrow \sigma_2 = 4

74. If \(P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}\) then \(P^{50}\) is:

1) \(\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}\)  
2) \(\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}\)  
3) \(\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}\)  
4) \(\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}\)

Key: 1

SOL:

\[P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}, \quad P^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad P^3 = \begin{bmatrix} 1 & 0 \\ 3/2 & 1 \end{bmatrix}, \ldots, \quad P^{50} = \begin{bmatrix} 1 & 0 \\ 50/2 & 1 \end{bmatrix}\]

75. The number of distinct real roots of \(\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0\) in the interval \(-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}\) is:

1) 4  
2) 3  
3) 1  
4) 2

Key: 3
SOL:
\[
\begin{vmatrix}
\sin x & \cos x & \cos x \\
\cos x & \sin x & \cos x \\
\cos x & \cos x & \sin x
\end{vmatrix} = 0 \Rightarrow (s + 2c) (s - c)^2 = 0
\]

\( s = c \)

76. \( ^nP_r = ^nP_{r+1} \) and \( ^nC_r = ^nC_{r-1} \) then the value of \( r \) is equal to

1) 3 2) 2 3) 4 4) 1

Key: 2

SOL:
\[
^nP_r = ^nP_{r+1} \Rightarrow n! \frac{n!}{(n-r)!} = n! \frac{n!}{(n-r-1)!} \quad n-r = 1
\]

\[
^nC_r = ^nC_{r-1}, \quad ^{r+1}C_r = ^{r+1}C_{r-1}
\]

\[
r + 1 = \frac{(r+1)(r)}{2} \Rightarrow r = 2
\]

77. Let the equation of the pair of lines \( y = px \) and \( y = qx \), can be written as

\( (y - px)(y - qx) = 0 \). Then the equation of the pair of the angle bisectors of the lines

\( x^2 - 4xy - 5y^2 = 0 \) is:

1) \( x^2 - 3xy + y^2 = 0 \) 2) \( x^2 - 3xy - y^2 = 0 \)

3) \( x^2 + 3xy - y^2 = 0 \) 4) \( x^2 + 4xy - y^2 = 0 \)

Key: 3

SOL: Equation of angular bisector of pair of lines \( x^2 - 4xy - 5y^2 = 0 \)

Is \( h(x^2 - y^2) = (a-b)xy \),

\[-2(x^2 - y^2) = 6xy, \quad x^2 - y^2 + 3xy = 0\]

78. If \( f(x) = \begin{cases} \int_0^x (5+|t-t|)dt, & x > 2 \\ 5x+1 & x \leq 2 \end{cases} \), then

1) \( f(x) \) is not differentiable at \( x = 1 \)

2) \( f(x) \) is continuous but not differentiable at \( x = 2 \)

3) \( f(x) \) is everywhere differentiable

4) \( f(x) \) is not continuous at \( x = 2 \)

Key: 2
\[ f(x) = \int_{0}^{x} [5+|1-t|] dt \quad x > 2 \]

\[ 5x+1 \quad x \leq 2 \]

If \( x > 2 \), \( f(x) = \int_{0}^{1} (5+1-t) dt + \int_{1}^{x} (5+t-1) \]

\[ = \int_{0}^{1} (6-t) dt + \int_{1}^{x} (4+t) dt \]

\[ = \frac{x^2}{2} + 4x + 1 \]

\[ \lim_{x \to 2^-} f(x) = 11, \quad \lim_{x \to 2^+} f(x) = 11, \quad f(2) = 11 \]

\[ f'(2^-) = 5 \quad f'(2^+) = 6 \]

\( f \) is continuous everywhere but not differentiable at \( x = 2 \)

79. If the greatest value of the term independent of \( x \) in the expansion of

\[ \left( x \sin \alpha + a \cdot \frac{\cos \alpha}{x} \right)^{10} \]

is \( \frac{10!}{(5!)^2} \), then the value of \( \alpha \) is equal to:

1) -1 2) 1 3) 2 4) -2

Key: 3

**SOL:**

\[ \left( x \sin \alpha + a \cdot \frac{\cos \alpha}{x} \right)^{10} \]

\[ T_{r+1} = ^{10}C_r x^{10-2r} a^r (\sin \alpha)^{10-r} (\cos \alpha)^r \]

\( r = 5 \)

Independent term = \( ^{10}C_5 a^5 \sin^5 \alpha \cos^5 \alpha \)

Greatest independent term \( \frac{10!}{(5!)^2} = \frac{10!}{2^5} \cdot a^5 \), \( a = 2 \)

80. If a tangent to the ellipse \( x^2 + 4y^2 = 4 \) meets the tangents at the extremities of its major axis at \( B \) and \( C \), then the circle with \( BC \) as diameter passes through the point:

1) \( (\sqrt{3}, 0) \) 2) \( (1, 1) \) 3) \( (\sqrt{2}, 0) \) 4) \( (-1, 1) \)

Key: 1

**SOL:** Given ellipse is \( \frac{x^2}{4} + \frac{y^2}{1} = 1 \), we have a property Circle with \( BC \) as diameter

will pass through the foci. foci are \( (\sqrt{a^2-b^2},0),(-\sqrt{a^2-b^2},0) = (\sqrt{3},0),(-\sqrt{3},0) \)
81. The equation of a circle is \( \text{Re} \left( z^2 \right) + 2 \left( \text{Im} \left( z \right) \right)^2 + 2 \text{Re} \left( z \right) = 0 \). Where \( Z = x + iy \) A line which passes through the center of the given circle and the vertex of the parabola \( x^2 - 6x - y + 13 = 0 \), has \( y \) - intercept equal to ________

KEY: 1

SOL:

\[
\text{Re} \left( z^2 \right) + 2 \left( \text{Im} \left( z \right) \right)^2 + 2 \text{Re} \left( z \right) = 0.
\]

\[
\text{Re}(x + iy)^2 + 2(\text{Im}(x + iy))^2 + 2\text{Re}(Z = x + iy) = 0 \Rightarrow x^2 + y^2 + 2x = 0
\]

Then the center of the circle is \((-1,0)\)

\[
y = x^2 - 6x + 13 \Rightarrow (y - 4) = (x - 3)^2 \text{ having vertex is (3,4)}
\]

So, the equation of the straight-line joining \((-1,0), (3,4)\) is

\[
x - y = -1
\]

82. Consider the function \( f(x) = \frac{P(x)}{\sin(x - 2)} \) \((x \neq 2)\), and \( f(x) = 7, (x = 2) \)

Where \( P(x) \) is the polynomial such that \( P(x) \) is always constant and \( P(3) = 9 \). If \( f(x) \) is continuous at \( x = 2 \), then \( P(5) \) is equal to ________

KEY: 39

SOL:

Let \( P(x) = (ax + b)(x - 2) \)

Given \( f(x) \) is continuous at \( x = 2 \) \( \Rightarrow f(2^+) = f(2) \)

\( \Rightarrow 2a + b = 7 \)

Also given \( P(3) = 9 \) \( \Rightarrow (3 - 2)(3a + b) = 9 \)

On solving we get \( a = 2, b = 3 \) and \( P(5) = 39 \)

83. If the co-efficient of \( x^7 \) and \( x^8 \) in the expansion of \( \left( 2 + \frac{x}{3} \right)^n \) are equal, then the value of \( n \) is equal to ________

KEY: 55

SOL: coefficient of \( x^7 = \text{coefficient of } x^8 \)

\[
\Rightarrow C_r . 2^7 \left( \frac{1}{3} \right)^7 = C_s . 2^8 \left( \frac{1}{3} \right)^8
\]

\[
\Rightarrow C_r . 2.3 = C_s \Rightarrow n = 55
\]
84. If the lines \( \frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3} , \frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1} \) are co-planar, then the value of \( k \) is _________.

**KEY:** 1

**SOL:** Two lines coplanar means shortest distance is zero

\[
\begin{vmatrix}
1+k & 2+2 & 3+3 \\
1 & 2 & 3 \\
3 & 2 & 1
\end{vmatrix} = 0 \Rightarrow (k+1)(-4) = -8 \Rightarrow k = 1
\]

85. If the rectangular is inscribed in an equilateral triangle of side length \( 2\sqrt{2} \) as shown in the figure, then the square of the largest area of such a rectangle is ____________.

**KEY:** 3

**SOL:**

Let sides of the rectangle are \( a, b \) let area of the rectangle is \( A \) then \( A = ab \)

from the given triangle \( \tan 60^\circ = \frac{b}{CF} = \frac{2b}{2\sqrt{2} - a} \)

\[
\sqrt{3} = \frac{2b}{2\sqrt{2} - a} , \quad A = a(2\sqrt{2} - a) \frac{\sqrt{3}}{2}
\]

apply derivative is zero we get \( a = \sqrt{2}, \quad b = \frac{\sqrt{3}}{\sqrt{2}} \)

so maximum value of \( A = \sqrt{2}. \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{3} \)
86. If \((\vec{a} + 3\vec{b})\) is perpendicular to \((7\vec{a} - 5\vec{b})\) and \((\vec{a} - 4\vec{b})\) is perpendicular to \((7\vec{a} - 2\vec{b})\),
then the angle between \(\vec{a}\) and \(\vec{b}\) (in degrees) is_________

KEY: 60

SOL:
Given \((\vec{a} + 3\vec{b})\)(\(7\vec{a} - 5\vec{b}\)) = 0, \((\vec{a} - 4\vec{b})\)(\(7\vec{a} - 2\vec{b}\)) = 0,
let \(|\vec{a}| = a, |\vec{b}| = b, (\vec{a}, \vec{b}) = \theta\)
\[\Rightarrow 7a^2 + 16abc \cos \theta - 15b^2 = 0, 7a^2 - 30abc \cos \theta + 8b^2 = 0\]
\[\Rightarrow 7\left(\frac{a}{b}\right) + 16 \cos \theta - 15\left(\frac{b}{a}\right) = 0, 7\left(\frac{a}{b}\right) - 30 \cos \theta + 8\left(\frac{b}{a}\right) = 0\]
\[\Rightarrow 46 \cos \theta - 23\left(\frac{b}{a}\right) = 0, \text{ by eliminating } \left(\frac{b}{a}\right) \text{ we get } \cos \theta = \frac{1}{2}\]

87. A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of \(n\) is __________

KEY: 4

SOL:
\[P(x \geq 1) \geq 0.9\]
\[\Rightarrow 1 - p(0) \geq 0.9 \quad \Rightarrow 1 - \frac{1}{2^n} \geq \frac{9}{10}\]
\[\Rightarrow 2^n \geq 10 \Rightarrow n = 4, 5, 6, \ldots\]

88. Let \(n \in N\) and \([x]\) denote the greatest integer less than or equal to \(x\). If the sum of \((n + 1)\) terms "\(C_0, 3\cdot C_1, 5\cdot C_2, 7\cdot C_3, \ldots, C_n\) is equal to \(2^{100} - 101\),
then \(2\left[\frac{n-1}{2}\right]\) is equal to:

KEY: 98

SOL:
\[\Rightarrow s = c_0 + 3c_1 + 5c_2 + \ldots + (2n + 1)c_n\]
\[\Rightarrow s = (2n + 1)c_0 + (2n - 1)c_1 + (2n - 3)c_2 + \ldots + 1c_n (c_r = c_{r,r})\]
\[\Rightarrow 2s = (2n + 2)[c_0 + c_1 + \ldots + c_n] \Rightarrow s = (n + 1)(2^n) \Rightarrow n = 100\]

leads \(2\left[\frac{n-1}{2}\right] = 2\left[\frac{99}{2}\right] = 2(49) = 98\)
89. Let a curve \( y = f(x) \) pass through the point \( (2, \log_2 2^2) \) and have slope \( \frac{2y}{x \log_e x} \) for all positive real value of \( x \). Then the value of \( f(e) \) is equal to _________.

KEY: 1

SOL:

Let \( y = f(x) \)

\[
\frac{dy}{dx} = \frac{2y}{x \ln x}
\]

\[
\frac{dy}{2y} = \frac{dx}{x \ln x} \Rightarrow \frac{1}{2} \ln y = \ln \ln x + k \Rightarrow \sqrt{y} = k \ln x
\]

given point \((2, (\ln 2)^2)\) lies on \( y = f(x) \)

leads to \( \sqrt{y} = (\ln x)^2 \Rightarrow y(e) = 1 \)

90. If \( a + b + c = 1, ab + bc + ca = 2 \) and \( abc = 3 \), then the value of \( a^4 + b^4 + c^4 \) is equal to _________.

KEY: 13

SOL:

Given \( a + b + c = 1, (ab + bc + ca) = 2, abc = 3 \)

\[ \Rightarrow a, b, c \text{ are the roots of } x^3 - x^2 + 2x - 3 = 0 \]

\[ \Rightarrow x^3 = x^2 - 2x + 3 \Rightarrow x^3 = x^2 - 2x^2 + 3x \]

Now \( a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca) = -3 \)

\[
a^4 + b^4 + c^4 = (a^2 - 2a + 3) + (b^2 - 2b + 3) + (c^2 - 2c + 3)
\]

\[ = (a^2 + b^2 + c^2) - 2(a + b + c) + 9 = (-3) - 2(1) + 9 = 4 \]

\[ a^4 + b^4 + c^4 = (a^2 + b^2 + c^2) - 2(a^2 + b^2 + c^2) + 3(a + b + c) \]

\[ = (a^2 + b^2 + c^2) - 2(a^2 + b^2 + c^2) + 3(a + b + c) \]

\[ = (4) - 2(-3) + 3(1) = 13 \]