Yet Another Proof of LEADERSHIP.
Sri Chaitanya Secures Top Ranks in All India JEE Main 2020

Below 100 All category 129 Ranks
Below 1000 All category 723 Ranks
Students Qualified for JEE Advanced All category 15252 Students

OUR REGULAR CLASSROOM PROGRAMME

- Aiming for below AIR 1000 in JEE Advanced and JEE Main
- Aiming for below AIR 5000 in JEE Advanced and JEE Main
- Aiming for below AIR 1000 in JEE Main and Below 10000 in Advanced
- Aiming for below AIR 10000 in JEE Main and Qualifying for Advanced

ADMISSIONS OPEN (2020-21)

Website: www.srichaitanya.net Email: webmaster@srichaitanya.net
Phone: 040-66 06 06 06 | www.srichaitanya.net
# Physics

**Max Marks: 100**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

**Marking scheme:** +4 for correct answer, 0 if not attempted and -1 in all other cases.

### Question 1

If one mole of the polyatomic gas is having two vibrational modes and \( \beta \) is the ratio of molar specific heats for polyatomic gas \( \left( \beta = \frac{C_p}{C_v} \right) \) then the value of \( \beta \) is:

1) 1.35  
2) 1.2  
3) 1.02  
4) 1.25

---

**Key:** 2

**Sol:**

\[
\frac{C_p}{C_v} = \gamma = 1 + \frac{2}{n} = 1 + \frac{2}{10} = 1.2 \quad (\text{for non linear})
\]

### Question 2

The atomic hydrogen emits a line spectrum consisting of various series. Which series of hydrogen atomic spectra is lying in the visible region?

1) Brackett series  
2) Balmer series  
3) Paschen series  
4) Lyman series

---

**Key:** 2

**Sol:** Conceptual  *Balmer series*

### Question 3

A carrier signal \( C(t) = 25 \sin \left(2.512 \times 10^6 t\right) \) is amplitude modulated by a message signal \( m(t) = 5 \sin \left(1.57 \times 10^6 t\right) \) and transmitted through an antenna. What will be the bandwidth of the modulated signal?

1) 1987.5 MHz  
2) 50 MHz  
3) 2.01 GHz  
4) 8 GHz

---

**Key:** 2

**Sol:**

Band width = \( f_m + f_m = 2f_m \)

\( w_m = 1.57 \times 10^6 = 2 \pi f_m \)

Band width = 50Hz

### Question 4

A geostationary satellite is orbiting around an arbitrary planet ‘P’ at a height of 11 R above the surface of ‘P’, R being the radius of ‘P’ is _____ ‘P’ has the time period of 24 hours.

1) 3  
2) \( \frac{6}{\sqrt{2}} \)  
3) \( 6\sqrt{2} \)  
4) 5

---

**Key:** 1
5. The velocity of a particle is \( v = v_0 + gt + Ft^2 \). Its position is \( x = 0 \) at \( t = 0 \); then its displacement after time \( t = 1 \) is:

1) \( v_0 + \frac{g}{2} + \frac{F}{3} \)  
2) \( v_0 + g + F \)  
3) \( v_0 + \frac{g}{2} + F \)  
4) \( v_0 + 2g + 3F \)

Key :1

Sol:
\[
V = V_0 + gt + Ft^2 \Rightarrow \int ds = \int (V_0 + gt + Ft^2)dt \Rightarrow S = V_0 + \frac{g}{2} + \frac{F}{3}
\]

6. Two identical photocathodes receive the light of frequencies \( f_1 \) and \( f_2 \) respectively. If the velocities of the photo-electrons coming out are \( v_1 \) and \( v_2 \) respectively, then

1) \( v_1^2 + v_2^2 = \frac{2h}{m} [f_1 + f_2] \)  
2) \( v_1^2 - v_2^2 = \frac{2h}{m} [f_1 - f_2] \)  
3) \( v_1 - v_2 = \left[ \frac{2h}{m} (f_1 - f_2) \right]^{1/2} \)  
4) \( v_1 + v_2 = \left[ \frac{2h}{m} (f_1 + f_2) \right]^{1/2} \)

Key :2

Sol:
\[
E = KE + w_0 \Rightarrow KE = E - w_0 \quad \frac{1}{2}mv_1^2 = hv_1 - w_0 \quad \frac{1}{2}mv_2^2 = hv_2 - w_0 \quad v_1^2 - v_2^2 = \frac{2h}{m} (v_1 - v_2)
\]

7. The four arms of a wheatstone bridge have resistances as shown in the figure. A galvanometer of 15Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.

1) 2.44mA  
2) 2.44μA  
3) 4.87μA  
4) 4.87mA

Key :4
Sol: by applying Kirchhoff law

\[ \begin{align*}
-100i_i - i_x 15 + (i - i_x)60 &= 0 \quad \text{...(1)} \\
-10(i - i_x) + (i - i_x + i_x)5 + i_x 15 &= 0 \quad \text{...(2)} \\
V_A - 100i_i - 10(i_i - i_x) &= V_c \quad \text{.....(3)} \quad (V_a - V_c \text{ is given})
\end{align*} \]

On solving, \( i_x = 4.87 \text{mA} \)

8. A sphere of mass 2 kg and radius 0.5 m is rolling with an initial speed of 1\( \text{ms}^{-1} \) goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A?

1) 0.60 s  
2) 0.80 s  
3) 0.57 s  
4) 0.52 s

Key :3

Sol: \( a = g \sin \theta = \frac{25}{7} \quad \text{t = } 2t_i = \frac{2V}{a} = 0.58 \quad \text{V = 0} \quad \text{at,} \)

9. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle?

1) \( \frac{\mu_0 I}{4\pi r} (2 + \pi) \)  
2) \( \frac{\mu_0 I}{4\pi r} (2 - \pi) \)  
3) \( \frac{\mu_0 I}{2\pi r} (2 - \pi) \)  
4) \( \frac{\mu_0 I}{2\pi r} (2 + \pi) \)
Key :1
Sol: \[ B_1 = \frac{\mu_0 i}{4\pi r} \quad B_2 = \frac{\mu_0 i}{2r}\left(\frac{\pi}{2\pi}\right) \]
\[ B_3 = \frac{\mu_0 i}{4\pi r} \quad (B_1, B_2 & B_3 \text{ are inside}) \]
\[ B = B_1 + B_2 + B_3 \]

10. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved?
1) Inductive reactance will be doubled and current will be halved
2) Both, inductive reactance and current will be halved
3) Both, inducting reactance and current will be doubled
4) Inductive reactance and current will be doubled

Key :4
Sol: \[ X_L = L\omega \]

11. Two particles A and B of equal masses are suspended from two massless spring of spring constants \( K_1 \) and \( K_2 \) respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is
1) \( \sqrt{\frac{K_1}{K_2}} \)
2) \( \frac{K_1}{K_2} \)
3) \( \frac{K_2}{K_1} \)
4) \( \sqrt{\frac{K_2}{K_1}} \)

Key :4
Sol: \[ V_1 - V_2 \Rightarrow A_1 w_1 = A_2 w_2 \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \omega = \sqrt{\frac{K}{m}} \]

12. An object is located at 2 km beneath the surface of the water. If the fractional compression \( \frac{\Delta V}{V} \) is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be \( \ldots \); \{Given: density of water is \( 1000 \text{kgm}^{-3} \) and \( g = 9.8 \text{ms}^{-2} \) \}
1) \( 2.26 \times 10^9 \text{Nm}^{-2} \) 2) \( 1.44 \times 10^9 \text{Nm}^{-2} \) 3) \( 1.96 \times 10^7 \text{Nm}^{-2} \) 4) \( 1.44 \times 10^7 \text{Nm}^{-2} \)

Key :2
\[ P = hdg \]
Sol: \[ B = \frac{P}{\Delta V} = 1.44 \times 10^9 \]
13. Two cells of emf 2E and E with internal resistance \( r_1 \) and \( r_2 \) respectively are connected in series to an external resistor \( R \) (see figure). The value of \( R \), at which the potential difference across the terminals of the first cell becomes zero is

\[
\text{1) } r_1 + r_2 \quad \text{2) } r_1 - r_2 \quad \text{3) } \frac{r_1}{2} - r_2 \quad \text{4) } \frac{r_1}{2} + r_2
\]

Key: 3

Sol:

Applying Kirchhoff law

\[2E + E - i(2r_1 + 2r_2) = 0 \quad \text{... (1)}\]

\[V_B - i_r + 2E = V_A\]

\[V_B - V_A = 0\]

And \( 2E - i_r = 0 \quad \text{... (2)}\)

On solving (1) and (2) \( R = \frac{r_1}{2} - r_2 \)

14. Two identical blocks A and B each of mass \( m \) resting on the smooth horizontal floor are connected by a light spring of natural length \( L \) and spring constant \( K \). A third block C of mass \( m \) moving with a speed \( v \) along the line joining A and B collides with A. The maximum compression in the spring is

\[
\text{1) } \sqrt{\frac{m}{2K}} \quad \text{2) } \sqrt{\frac{mv}{2K}} \quad \text{3) } \frac{mv}{K} \quad \text{4) } v \sqrt{\frac{m}{2K}}
\]

Key: 3
Sol: According Law of conservation of linear momentum, And according to Law of conservation of energy \( \frac{1}{2} Kx^2 = \frac{1}{2} \mu V^2 \) (where \( \mu \) is reduced mass) \( x = \sqrt{\frac{mV}{2K}} \)

15. Match List-I with List-II

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Phase difference between current and voltage in a purely resistive AC circuit</td>
<td>i) ( \frac{\pi}{2} ); current leads voltage</td>
</tr>
<tr>
<td>B) Phase difference between current and voltage in a pure inductive AC circuit</td>
<td>ii) zero</td>
</tr>
<tr>
<td>C) Phase difference between current and voltage in a pure capacitive AC circuit</td>
<td>iii) ( \frac{\pi}{2} ); current lags voltage</td>
</tr>
<tr>
<td>D) Phase difference between current and voltage in an LCR series circuit</td>
<td>iv) ( \tan^{-1} \left( \frac{X_C - X_L}{R} \right) )</td>
</tr>
</tbody>
</table>

Choose the most appropriate answer from the options given below:

1) A-iii; B-iv; C-iii; D-I  
2) A-ii; B-iii; C-iv; D-I
3) A-I; B-iii; C-iv; D-ii  
4) A-ii; B-iii; C-I’; d-iv

Key : 4

Sol: In “C” circuit, current leads by \( \frac{\pi}{2} \) with emf

In “L” circuit, current lags \( \frac{\pi}{2} \) with emf

\[ \tan \phi = \frac{X_L - X_C}{R} \]

16. A sound wave of frequency 245 Hz travels with speed of 300 ms\(^{-1}\) along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave?

1) \( Y(x,t) = 0.03 \left[ \sin 51.1x - (0.2 \times 10^3)t \right] \)  
2) \( Y(x,t) = 0.03 \left[ \sin 51.1x - (1.5 \times 10^3)t \right] \)
3) \( Y(x,t) = 0.06 \left[ \sin 0.8x - (0.5 \times 10^3)t \right] \)  
4) \( Y(x,t) = 0.06 \left[ \sin 51.1x - (1.5 \times 10^3)t \right] \)

Key : 2

Sol: \( w = 2\pi n, A = 3cm, n = 245Hz \)

\( y = 0.03 \left[ \sin 51.1x - 1.5 \times 10^3 \right] m \)

17. A rubber bull is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to \( \frac{81}{100} \) of the height through which it falls. Find the average speed of the ball. (Take \( g = 10ms^{-2} \))

1) 2.0 ms\(^{-1}\)  
2) 2.50 ms\(^{-1}\)  
3) 3.50 ms\(^{-1}\)  
4) 3.0 ms\(^{-1}\)
Key: 2
Sol:  \( u = \sqrt{2gh} \) \( V = e\sqrt{2gh} \Rightarrow e = 0.9 \)
\[
V_a = \frac{\text{Total distance}}{\text{Total time}} = \frac{s}{t} \\
V_a = 2.5 \text{ms}^{-1}
\]
18. Which one of the following will be the output of the given circuit?

1) NOR Gate  
2) AND Gate  
3) XOR Gate  
4) NAND Gate

Key: 3
Sol:  
for and \( \overline{AB} \)  
For or \( A + B \)
For AND gets \((\overline{A})B)(AB) = \overline{A}B + \overline{A}B \)
XOR gets

19. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take in \( 2 = 0.693 \))

1) 0.69 \times 10^2 \text{kgs}^{-1}  
2) 3.3 \times 10^2 \text{kgs}^{-1}  
3) 5.7 \times 10^{-3} \text{kgs}^{-1}  
4) 1.16 \times 10^2 \text{kgs}^{-1}

Key: 4
Sol:  \( A = A_0 e^{-bt/2m} \)  
\( A_0 = 12, A = 6, m = 1 \text{kg} \)

On solving \( b = 1.16 \times 10^{-2} \text{kg/sec} \)

20. Which one is the correct option for the two different thermodynamic processes?

1) a only  
2) c and d  
3) c and a  
4) b and c

Key: 2
Sol:  isothermal \( T=\text{constant} \)
This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. The disc of mass M with uniform surface mass density $\sigma$ as shown in the figure. The centre of mass of quarter disc (the shaded area) is at the position $\frac{x a}{3 \pi}, \frac{x a}{3 \pi}$ where $x$ is _____, (Round off to the Nearest integer)

[a is an area as shown in the figure]

Key :4

Sol: The centre of mass of certain symmetrical shape of disc from centre is

$$X = \frac{4R \sin \theta / 2}{3 \theta} \therefore x = 4$$

Where $\theta$ is the angle made by the shape at the center

$\therefore x(\text{coordinante}) = X \cos 45 \qquad \therefore y(\text{coordinante}) = X \sin 45$

22. A $2\mu F$ capacitor $C_1$ is first charged to a potential difference of 10 V using a battery. Then the battery is removed and the capacitor is connected to an uncharged capacitor $C_2$ of $8\mu F$. The charge in $C_2$ on the equilibrium condition is ______ $\mu C$. (Round off the Nearest integer)

Key :16

Sol: $q = q_i$ (Law conservation of charge)

$$20 \times 10^{-6} = (C_1 + C_2) \Rightarrow V = 2\text{volt}$$

$q_2 = VC_2 = 16\mu C$
23. The electric field in a region is given by \( \vec{E} = \frac{2}{3} E_0 \hat{i} + \frac{3}{5} E_0 \hat{j} \) with \( E_0 = 4.0 \times 10^3 \text{N/C} \). The flux of this field through a rectangular surface area 0.4m² parallel to the Y-Z plane is _______ Nm²C⁻¹

Key: 640

Sol: flux \((\phi) = \overline{E \cdot A} = 640 \)
\( A = 0.4 \hat{i} \)

24. Seawater at a frequency \( f = 9 \times 10^5 \text{Hz} \) has permittivity \( \varepsilon = 80\varepsilon_0 \) and resistivity \( \rho = 0.25 \Omega \cdot \text{m} \).

Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source \( V(t) = V_0 \sin 2\pi f t \). Then the conduction current density of becomes \( 10^4 \) times the displacement current density after time \( t = \frac{1}{800} \text{s} \). The value of \( x \) is ______.

(Given: \( \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{Nm²C⁻²} \))

Key: 6

Sol: \( E = J_1 \rho \)  
\( J_1 \rightarrow \text{conduction current density} \)
\( E = \frac{V}{d} = \frac{V_0 \sin 2\pi nt}{d} \)

And displacement current density \( J_2 = \frac{i}{A} = \frac{1}{A} \frac{dq}{dt} \)
\( q = CV \)
\( J_2 = \frac{80\varepsilon_0}{d} \frac{d(V_0 \sin 2\pi nt)}{dt} \)
\( V = V_0 \sin 2\pi nt \)

\( J_1 = 10^4 J_2 \) on solving \( x = 6 \)

25. The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3 m is \( E \). The electric field intensity produced by the radiation coming from 60 W at the same distance is \( \sqrt{x}E \). Where the value of \( x= \_____

Key: 3

Sol: \( I = C \varepsilon_0 E_1^2 \Rightarrow \frac{P}{A} = C \varepsilon_0 E_1^2 \)
\( E_1 = \sqrt{\frac{xE}{5}} \)
\( \frac{P}{4\pi r^2} = c \varepsilon_0 E_1^2 \)
\( x = 3 \)
26. A particle of mass $m$ moves in a circular orbit in a central potential field $U(t) = U_0 x^4$. If Bohr’s quantization conditions are applied, radii of possible orbitals $r_n$ vary with $n^\alpha$, where $\alpha$ is ____

Key: 3

Sol: 
$$F_c = -\frac{dU}{dr} \quad F_c = \frac{mv^2}{r} \quad \frac{mv^2}{r} = 4U_0 r^4 \quad v \propto r^2, r^3 \propto n \quad r \propto \hbar^{1/3}$$

27. A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is ______ N. (Round off to the Nearest integer)

(Take $g = 10 \text{ms}^{-2}$)

Key: 30

Sol: 
\[T = f_L = \mu N\]
\[N = N_1 + mg\]
\[= m_1 g - T + mg\]

FBD For man
\[N_1 = m_1 g - T \quad N_1 = m_1 g - T \quad \therefore T = \mu [(m_1 + m) g - T]\]
28. The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2}{3}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. The value of $'x'$ is ______

Key : 30

Sol: $\lambda_1 = \frac{\lambda_1}{\mu} \Rightarrow \mu = \frac{3}{2}$, applying $\frac{\mu - 1}{v} \frac{1}{u} = \frac{\mu - 1}{R}$

$u = -15, V = 30$

$R = \frac{30}{18}$

$\frac{\mu - 1}{v} \frac{1}{u} = \frac{\mu - 1}{R} \Rightarrow R = \frac{30}{18} = \frac{x}{13} \Rightarrow x = 30$

29. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm$^3$ of oleic acid per cm$^3$ of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm$^2$ by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}$ cm. Then the thickness of oleic layer will be $x \times 10^{-14}$ m where $x$ is ___.

Key : 25

Sol: $V_1 = V_2 \Rightarrow At = V_2$ (A $\rightarrow$ Area, t $\rightarrow$ thickness)

$4t = 100 \times \frac{4}{3} \pi r^3$

$t = 25 \times \frac{4}{3} \pi \left(\frac{3}{40\pi}\right) \times 10^{-9}$

30. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. If is desired to make the body move by applying the minimum possible force F.N. The value of F will be ______. (Round off to the Nearest integer) [Take $g = 10$ms$^{-2}$]

Key : 5

Sol: min. force = $\frac{\mu mg}{\sqrt{1 + \mu^2}} = 5$
CHEMISTRY

**SINGLE CORRECT ANSWER TYPE**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 in all other cases.

31. Which of the following statement(s) is(are) incorrect reason for eutrophication?
   a) excess usage of fertilisers  
   b) excess usage of detergents
   c) dense plant population in water bodies
   d) lack of nutrients in water bodies that prevent plant growth

1) a only  
2) c only  
3) b and d only  
4) d only

Key :4

Sol: Lack of nutrients in water bodies that prevent plant growth (ref. NCERT)

32. For the coagulation of a negative sol, the species below, that has the highest flocculating power is:
   1) $\text{Na}^+$  
   2) $\text{Ba}^{2+}$  
   3) $\text{PO}_4^{3-}$  
   4) $\text{SO}_4^{2-}$

Key :2

Sol: For $-ve$ sol, $+ve$ charge on ion increases $-ve$ flocculating power increases

33. During which of the following processes, does entropy decrease?
   a) Freezing of water to ice at $0^\circ C$
   b) Freezing of water to ice at $-10^\circ C$
   c) $\text{N}_2(g) + 3\text{H}_2(g) \rightarrow 2\text{NH}_3(g)$
   d) Adsorption of $\text{CO}(g)$ on lead surface
   e) Dissolution of $\text{NaCl}$ in water

Choose the correct answer from the options given below:

1) b and c only  
2) a and e only  
3) a,b,c and d only  
4) a,c and e only

Key :3

Sol: A) Liquid $\rightarrow$ solid, $\Delta s = -ve$
   B) Liquid $\rightarrow$ solid; $\Delta s = -ve$
   C) No. of moles of gases decreases, $\Delta s = -ve$
   D) For adsorption, $\Delta s = -ve$
   E) No. of ions increases, $\Delta s = +ve$

34. Primary, secondary and tertiary amines can be separated using:
   1) Benzene sulphuric acid  
   2) Chloroform and KOH
   3) para-Toluene sulphonyl chloride  
   4) Acetyl amide
35. The set that represents the pair of neutral oxides of nitrogen is:
   1) N₂O and N₂O₃  2) NO and N₂O  3) N₂O and NO₂  4) NO and NO₂

Key :2
In the above reaction, the structural formula of (A), “X” and “Y” respectively are:

Key: 3

Sol:

$$N_2\text{O}$$
37. MATCH LIST-I WITH LIST-II.

<table>
<thead>
<tr>
<th>List-I (chemical Compound)</th>
<th>List-II (Used as)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Sucralose</td>
<td>i) synthetic detergent</td>
</tr>
<tr>
<td>B) Glyceryl ester of stearic acid</td>
<td>ii) Artificial sweetener</td>
</tr>
<tr>
<td>C) Sodium benzoate</td>
<td>iii) Antiseptic</td>
</tr>
<tr>
<td>D) Bithional</td>
<td>iv) Food preservative</td>
</tr>
</tbody>
</table>

Choose the correct match:
1) A-ii; B-I; C-iv; D-iii
2) A-I; B-ii; C-iv; D-iii
3) A-iii; B-ii; C-iv; D-i
4) A-iv; B-iii; C-ii; D-i

Key: 1

Sol: Sucrose – Artificial sugar
Glyceryl ester of stearic acid – synthetic detergent
Sodium benzoate – Food preservative
Bithional – Antiseptic

38. One of the by-products is formed during the recovery of NH$_3$ from solvary process is:
1) NH$_4$Cl
2) NaHCO$_3$
3) Ca(OH)$_2$
4) CaCl$_2$

Key: 4

Sol: CaCl$_2$

39. Nitrogen can be estimated by Kjeldahl’s method for which of the following compound?

![Chemical structures](image)

Key: 3

Sol: Kjeldahl’s method is not applicable to nitro group, nitrogen present in the ring and azo groups

40. The common positive oxidation states sides for an element with atomic number 24, are:
1) +1 and +3 to +6
2) +2 to +6
3) +1 to +6
4) +1 and +3

Key: 2

Sol: Cr common oxidation states are +2 and +6

41. The set of elements that differ in mutual relationship from those of the other sets is:
1) Li-Mg
2) Be-Al
3) B-Si
4) Li-Na

Key: 4

Sol: Li-Na both belongs to same group so do not have diagonal relationship

42. Amongst the following, the linear species is:
1) Cl$_2$O
2) NO$_2$
3) N$_3^-$
4) O$_3$

Key: 3

Sol: $N_3^-$; $\overline{N} = \overline{N} = \overline{N}$
43. The functional groups that are responsible for the ion-exchange property of cation and anion exchange resins, respectively, are:

1) $SO_3H$ and $COOH$
2) $NH_2$ and $SO_3H$
3) $SO_3H$ and $NH_2$
4) $NH_2$ and $COOH$

Key: 3
Sol: $SO_3H$ and $NH_2$

44. $C_{12}H_{22}O_{11} + H_2O \xrightarrow{Enzyme A} C_6H_{12}O_6 + C_6H_{12}O_6$

$C_6H_{12}O_6 \xrightarrow{Enzyme B} 2C_2H_2OH + 2CO_2$

In the above reactions, the enzymes A and enzyme B respectively are:

1) Amylase and Invertase
2) Zymase and Invertase
3) Invertase and Zymase
4) Invertase and Amylase

Key: 3
Sol: Enzyme A – Invertase
    Enzyme B – Zymase

45. Choose the correct statement regarding the formation of carbocations A and B given

1) Carbocation A is more stable and formed relatively at slow rate
2) Carbocation B is more stable and formed relatively at faster rate
3) Carbocation B is more stable and formed relatively at slow rate
4) Carbocation A is more stable and formed relatively at faster rate

Key: 2
Sol: Order of stability of carbocations $3^o > 2^o > 1^o$

46. Fructose is an example of:

1) Aldohexose
2) Ketohexose
3) Pyranose
4) Heptose

Key: 2
Sol: Fructose is a keto functional poly hydroxyl compound
47. Match List-I with List-II.

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) ([Co(NH_3)_6][Cr(CN)_6])</td>
<td>i) Linkage isomerism</td>
</tr>
<tr>
<td>B) ([Co(NH_3)_3(NO_2)_3])</td>
<td>ii) Solvate isomerism</td>
</tr>
<tr>
<td>C) ([Cr(H_2O)_6]Cl_3)</td>
<td>iii) Coordination isomerism</td>
</tr>
<tr>
<td>D) (cis-[CrCl_4(ox)_2]^{2-})</td>
<td>iv) Optical isomerism</td>
</tr>
</tbody>
</table>

Choose the correct answer from the options given below:

1) A-ii; B-I; C-iii; D-iv  
2) A-iii; B-I; C-ii; D-iv  
3) A-i; B-ii; C-iii; D-iv  
4) A-iv; B-ii; C-iii; D-i

Key: 2

Sol: a-iii, b-i, c-ii, d-iv (Ref. NCERT)

48. Match List-I with List-II:

<table>
<thead>
<tr>
<th>LIST-I</th>
<th>LIST-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Haemalite</td>
<td>i) (Al_2O_3\cdot xH_2O)</td>
</tr>
<tr>
<td>B) Bauxite</td>
<td>ii) (Fe_2O_3)</td>
</tr>
<tr>
<td>C) Magnetite</td>
<td>iii) (CuCO_3\cdot Cu(OH)_2)</td>
</tr>
<tr>
<td>D) Malachite</td>
<td>iv) (Fe_3O_4)</td>
</tr>
</tbody>
</table>

Choose the correct answer from the options given below:

1) A-iv; B-I; C-ii; D-iii  
2) A-I; B-iii; C-ii; D-iv  
3) A-ii; B-I; C-iv; D-iii  
4) A-ii; B-iii; C-I’d-iv

Key: 3

Sol: Ref NCERT (a-ii, b-i, c-iv, d-iii)

49. The correct pair(s) of the ambient nucleophiles is(are):

a) \(AgCN\) / \(KCN\)  
b) \(RCOOAg\) / \(RCOOK\)  
c) \(AgNO_3\) / \(KNO_3\)  
d) \(AgI\) / \(KI\)

1) a and c only  
2) a only  
3) b only  
4) b and c only

Key: 1

Sol: \(-CN; -NC\ and \-NO_2; -ONO\)

Only tertiary hydrogen can oxidize to alcohol with \(KMnO_4\)
50. Given below are two statements:

Statement-I: 2-methylbutane on oxidation with $KMnO_4$ gives 2-methylbutan-2-ol

Statement-II: n-alkanes can be easily oxidized to corresponding alcohols with $KMnO_4$

1) Statement I is correct but statement II is incorrect
2) Statement I is incorrect but statement II is correct
3) Both statement I and statement II are incorrect
4) Both statement I and statement II are correct

Key :1

Sol:

Given the reaction $2$-methylbutane + $4KMnO_4 \rightarrow 2$-methylbutan-2-ol

51. Consider the reaction $N_2O_4(aq) \rightarrow 2NO_2(g)$. The temperature at which $K_c = 20.4$ and $K_p = 600.1$ is ___ K. (Round off to the Nearest integer)

(Assume all gases are ideal and $R = 0.0831 L bar K^{-1} mol^{-1}$)

Key :354

Sol:

\[
\frac{K_p}{K_c R} = \frac{600.1}{20.4 \times 0.0831} = 353.99 \approx 354 \text{ K}
\]

52. The number of chlorine atoms in 20 mL of chlorine gas at STP is ___. (Round off to the Nearest Integer)

(Assume chlorine is an ideal gas at STP $R = 0.0831 L bar mol^{-1} K^{-1}, N_A = -6.023 \times 10^{23}$)

Key :1

Sol:

Total no. of atoms = $n N_A \times \text{Atomicity}$

\[
\frac{PV}{RT} = 6.023 \times 10^{23} \times 2 = \frac{1 \times 0.02}{0.083 \times 273} \times 6.023 \times 10^{23} \times 2 = 1 \times 10^{24}
\]

53. A 1 molal $K_4Fe(CN)_6$ solution has a degree of dissociation at 0.4. Its boiling point is equal to that of another solution which contains 18.1 weight percent of a non electrolytic solute A. The molar mass of A is ___ u.

(Density of water = 1.0 g cm$^{-3}$)

Key :85
54. In the ground state of atomic \( Fe(Z = 36) \), the spin-only magnetic moment is ______ \( \times 10^{-1} \) \( BM \).

(Given: \( \sqrt{3} = 1.73, \sqrt{5} = 1.41 \))

Key : 49

Sol: \( Fe \rightarrow [Ar]3d^64s^2 \)

\( n = 4; \mu = \sqrt{n(n+2)}BM = 4.89 = 48.9 \times 10^{-1} \)

55. KBr is doped with \( 10^{-5} \) mole percent of \( SrBr_2 \). The number of cationic vacancies in 1 g of KBr crystal is __\(10^{14}\).

(Atomic Mass: \( K:39.1u, Br:79.9u \) \( N_A = 6.023 \times 10^{23} \))

Key : 5

Sol: \( Sr^{2+} \) will result one cation vacancy

\( n_{SrBr_2} = n_{Sr^{2+}} = \frac{10^{-5}}{100} = 10^{-7} \)

Per mole cationic vacancies = \( 10^{-2} \times 6.023 \times 10^{23} \)

i.e \( \frac{119g}{1g} \rightarrow \frac{6.023 \times 10^{16}}{119} = 5.06 \times 10^{14} \)

56. A KCl solution of conductivity \( 0.14 \text{Sm}^{-1} \) shows a resistance of \( 4.19 \Omega \) in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops to \( 1.03 \Omega \). The conductivity of the HCl solution is __\(10^{-2} \text{Sm}^{-1}\).

Key : 57

Sol: \( K = GG' ; G' = KR = 0.14 \times 4.19 \)

\( K = \frac{1}{R} G' = \frac{1}{1.03} \times 0.14 \times 4.19 \approx 57 \times 10^{-2} \text{Sm}^{-1} \)

57. Consider the above reaction. The percentage yield of amide product is _____

(Given: Atomic mass: \( C:120u, H:1.0u, N:14.0u, O:16.0u, Cl:35.5u \))

Key : 77
58. On complete reaction of FeCl$_3$ with oxalic acid in aqueous solution containing KOH, resulted in the formation of product A. The secondary valency of Fe in the product A is __

Key: 6

Sol: \[
\text{FeCl}_3 + 3\text{KOH} + 3\text{H}_2\text{C}_2\text{O}_4 \rightarrow \text{K} \left[ \text{Fe(O}_4\right] \right] + 3\text{HCl} + 3\text{H}_2\text{O} \]

Secondary valency = Co-ordination number = 6

59. The total number of C-C sigma bond/s in mesityl oxide (C$_6$H$_{10}$O) is __

Key: 5

Sol: \[
\begin{array}{c}
\text{H} \\
\text{\_\_\_} \\
\text{\_\_\_} \\
\text{\_\_\_} \\
\text{\_\_\_} \\
\text{\_\_\_} \\
\text{H}
\end{array}
\]

60. The reaction \[ 2A + B_2 \rightarrow 2AB \] is an elementary reaction.

For a certain quantity of reactants, if the volume of reaction vessel is reduced by a factor of 3, the rate of the reaction increases by a factor of __

Key: 27

Sol: \[
\text{rate} = K [A]^2 [B_2]
\]

Volume of vessel decreases 3 times i.e concentration increases 3 times

\[
\therefore \text{rate} = K [3A]^2 [3B_2] = 27 K [A]^2 [B_2]
\]
61. The value of \( \sum_{r=0}^{6} \binom{6}{r} \binom{6}{6-r} \) is equal to:

1) 924  
2) 1324  
3) 1124  
4) 1024

Key :1

Sol: 
\[ \sum_{r=0}^{6} \binom{6}{r} \binom{6}{6-r} = \sum_{r=0}^{6} \binom{6}{r} \binom{6}{6-r} \]

w.k.T \((1+x)^6(1+x)^6 = \left[ \binom{6}{0} + \binom{6}{1}x + \binom{6}{2}x^2 + \binom{6}{3}x^3 + \binom{6}{4}x^4 + \binom{6}{5}x^5 + \binom{6}{6}x^6 \right] \left[ \binom{6}{0} + \binom{6}{1}x + \binom{6}{2}x^2 + \binom{6}{3}x^3 + \binom{6}{4}x^4 + \binom{6}{5}x^5 + \binom{6}{6}x^6 \right] \]

= \binom{6}{0}^2 + \binom{6}{1} \cdot \binom{6}{1} + \binom{6}{2} \cdot \binom{6}{2} + \binom{6}{3} \cdot \binom{6}{3} + \binom{6}{4} \cdot \binom{6}{4} + \binom{6}{5} \cdot \binom{6}{5} + \binom{6}{6}^2

G.E = \text{Coefficient of } x^6 \text{ in } (1+x)^{12} = \binom{12}{6} = 924

62. If the equation of plane passing through the mirror image of a point \((2,3,1)\) with respect to line 
\[ \frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1} \]
and containing the line 
\[ \frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1} \]
is \(ax + by + cz = 24\)
then \(a + b + c\) is equal to:

1) 21  
2) 18  
3) 19  
4) 20

Key :3

Sol: 
Given line \( L_1 = \frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1} \)

Any pt on line \( L_1 = R = (2r-1, r+3, -r-2) \)

D.R of PR \( PR = (2r-3, r, -r-3) \)

\( PR \perp \text{line} \Rightarrow 2(2r-3)+1(r)-1(-r-b) = 0 \Rightarrow r = \frac{1}{2} \)

\( R = (0,7/2, -5/2) \Rightarrow Q(-2,4, -6) \)

Plane passing through Q and line \( L_2 \)
\[
\begin{vmatrix}
 x-2 & y-1 & z+1 \\
 3 & -2 & 1 \\
 4 & -3 & 5 \\
\end{vmatrix} = 0 \Rightarrow 7x+11y+z = 24
\]

\( \alpha = 7, \beta = 11, r = 1 \Rightarrow a + b + c = 19 \)

63. If the Boolean expression \((p \land q) \oplus (p \otimes q)\) is a tautology, then \(\oplus\) and \(\otimes\) are respectively given by:

1) \(\land, \rightarrow\)  
2) \(\land, \lor\)  
3) \(\rightarrow, \rightarrow\)  
4) \(\lor, \rightarrow\)

Key :3

Sol: 
\((p \land q) \times (p \lor q) \equiv (p \land q) \rightarrow (p \rightarrow q) \)

\[ \equiv \sim (p \land q) \lor (\sim p \lor q) \]

\[ \equiv (\sim p \lor q) \lor (\sim p \lor q) \]

\[ \equiv \sim p \lor (q \lor q) \]

\[ \equiv \sim p \lor t \]

\[ \equiv t \]
64. If \( x, y, z \) are in arithmetic progression with common difference \( d \), \( x \neq 3d \), and the determinant of the matrix
\[
\begin{vmatrix}
3 & 4\sqrt{2} & x \\
4 & 5\sqrt{2} & y \\
5 & k & z
\end{vmatrix}
\]
is zero, then the value of \( k^2 \) is:

1) 12  
2) 36  
3) 6  
4) 72  

Key : 4

Sol: \( x, y, z \) are AP \( \Rightarrow 2y = x + z \Rightarrow x - 2y + z = 0 \)
\[
\begin{vmatrix}
3 & 4\sqrt{2} & x \\
4 & 5\sqrt{2} & y \\
5 & k & z
\end{vmatrix} = 0
\]
\( \Rightarrow 4 \begin{vmatrix}
5\sqrt{2} & y \\
k & z
\end{vmatrix} = 0 \Rightarrow (k - 6\sqrt{2})(4z - 5y) = 0 \Rightarrow k = 6\sqrt{2} \Rightarrow k^2 = 72 \]

65. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be \( \frac{1}{2} \) and probability of occurrence of 0 at the odd place be \( \frac{1}{3} \). Then the probability that ‘10’ is followed by ‘01’ is equal to:

1) \( \frac{1}{6} \)  
2) \( \frac{1}{3} \)  
3) \( \frac{1}{18} \)  
4) \( \frac{1}{9} \)  

Key : 4

Sol: Probability of 1 covers in odd p caca is \( \frac{2}{3} \)

<table>
<thead>
<tr>
<th>Case (1)</th>
<th>Even</th>
<th>Odd</th>
<th>Even</th>
<th>Odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{3} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{1}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{Prob} = \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{9}
\]

66. If the curve \( y = y(x) \) is the solution of the differential equation
\[
2\left(x^2 + x^{5/4}\right)dy - y\left(x + x^{1/4}\right)dx = 2x^{9/4}dx, x > 0
\]
which passes through the point \( (1, 1 - \frac{4}{3}\log_2 2) \), then the value of \( y(16) \) is equal to:

1) \( 4\left(\frac{31}{3} + \frac{8}{3}\log_3 3\right) \)  
2) \( \left(\frac{31}{3} - \frac{8}{3}\log_3 3\right) \)  
3) \( \left(\frac{31}{3} + \frac{8}{3}\log_3 3\right) \)  
4) \( \left(\frac{31}{3} - \frac{8}{3}\log_3 3\right) \)

Key : 4
Sol: Given equal \( \frac{dy}{dx} - \frac{y}{2x} = \frac{x^{5/4}}{x + x^{3/4}} \)

\( I.F = e^{-\int \frac{1}{2x} \, dx} = e^{\int \frac{1}{2} \log_m \, dx} = \frac{1}{\sqrt{x}} \)

\( G.S : \frac{y}{\sqrt{x}} = \int \frac{1}{\sqrt{x}} \cdot \frac{x^{5/4}}{x + 4^{1/4}} \, dm \)

\( \frac{y}{\sqrt{x}} = \frac{4}{3} x^{3/4} - \frac{4}{3} \log(x^{3/4} + 1) + C \)

\( x = 1 \Rightarrow y = 1 - \frac{4}{3} \log 2 \Rightarrow C = -\frac{1}{3} \)

When \( x = 16 \) \( \Rightarrow y = 4 \left( \frac{31}{3} - \frac{8}{3} \log e \cdot 3 \right) \)

67. Let the tangent to the circle \( x^2 + y^2 = 25 \) at the point \( R(3,4) \) meet x-axis and y-axis at points P and Q respectively. If \( r \) is the radius of the circle of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then \( r^2 \) is equal to:

1) 585/66
2) 625/72
3) 125/72
4) 529/64

Key : 2

Sol: Equal of tangent : \( 3x + 4y - 25 = 0 \) \( \Rightarrow P \left( \frac{25}{3}, 0 \right), Q \left( 0, \frac{25}{4} \right) \)

Incentre \( I = \left( \frac{25}{12}, \frac{25}{12} \right) \)

\( OP = \frac{25}{3} \), \( OQ = \frac{25}{4} \), \( PQ = \frac{125}{12} \)

Radius \( = OI = \frac{25\sqrt{2}}{12} = r \)

\( r^2 = 72 \)

68. Let L be the tangent line to the parabola \( y^2 = 4x - 20 \) at \( (6,2) \). If L is also a tangent to the ellipse \( \frac{x^2}{2} + \frac{y^2}{b} = 1 \), then the value of b is equal to:

1) 11
2) 14
3) 20
4) 16

Key : 2

Sol: Equal tangent at \( (6,2) \) \( \Rightarrow y = x - 4 \)

it touches ellipse \( 16 = 2(1) + b \Rightarrow b = 14 \)

69. Let \( S_1, S_2 \) and \( S_3 \) be three sets defined as

\( S_1 = \{ x \in C : |z - 1| \leq \sqrt{2} \} \)

\( S_2 = \{ z \in C : \Re((1-i)z) \geq 1 \} \)

\( S_3 = \{ z \in C : \Im(z) \leq 1 \} \)

Then the set \( S_1 \cap S_2 \cap S_3 \)

1) has exactly two elements
2) is a singleton
3) has exactly three elements 4) has infinitely many elements

Key :4

Sol:

\[ S_1 : (x-1)^2 + y^2 \leq 2 \quad S_2 : x + y \geq 1 \quad S_3 : y \leq 1 \]

\[ S_1 \bigcap S_2 \bigcap S_3 \]

Has infinitely many points

70. If the sides AB, BC and CS of a triangle ABC have 3, 5 and 6 integer points respectively, then the total number of triangles that can be considered using these points as vertices, is equal to:

1) 240 2) 333 3) 360 4) 364

Key :2

Sol:

Total \( P_\Delta = 3 + 5 + 6 = 14 \)

No of \( \Delta^{\text{ inters}} \)

\[ = 14C_3 - (3C_3 + 5C_3 + 6C_3) = 333 \]

71. The value of \( \lim_{n \to \infty} \frac{\lfloor r \rfloor + [2r] + \ldots + [nr]}{n^2} \)

Where \( r \) is a non-zero real number and \( \lfloor r \rfloor \) denotes the greatest less than or equal to \( r \), is equal to:
Key :1
Sol: \[ r - 1 < [r] \leq r \]
\[ 2r - 1 < [2r] \leq 2r \]
\[ nr - 1 < [nr] \leq nr \]
\[ \Rightarrow \frac{r + 2r + \ldots + nr}{n^2} < \frac{[r] + \ldots + [nr]}{n^2} \leq \frac{r + 2r + \ldots + nr}{n^2} \]
\[ \Rightarrow \lim_{n \to \infty} \frac{r(n(n+1))}{2} - \frac{n}{2} < \lim_{n \to \infty} \frac{[r] + \ldots + [nr]}{n^2} \leq \lim_{n \to \infty} \frac{r(n(n+1))}{2} \]
\[ < \frac{r}{2} \leq \frac{r}{2} \]
By sandwich theorem given limit = \[ \frac{r}{2} \]

72. The number of solutions of the equation \[ \sin^{-1}\left(x^2 + \frac{1}{3}\right) + \cos^{-1}\left(x^2 - \frac{2}{3}\right) = x^2 \text{, for } x \in [-1,1] \text{, and } [x] \text{ denotes the greatest integer less than or equal to } x \text{, is:} \]
1) 4 2) 2 3) Infinite 4) 0
Key :4
Sol: \[ \sin^{-1}\left(x^2 + \frac{1}{3}\right) + \cos^{-1}\left(x^2 - \frac{2}{3}\right) = x^2 \text{, where } t = \left(x^2 + \frac{1}{3}\right) x \in [-1,1] \Rightarrow t = 0.1 \]
\[ t = 0 \Rightarrow x^2 \pi \text{ & } x < \frac{2}{3} \text{ & } t = 1 \Rightarrow x^2 \pi \text{ & } \frac{2}{3} < x^2 < \frac{5}{3} \Rightarrow \text{ No solution} \]

73. Let O be the origin. Let \[ \overrightarrow{OP} = x \hat{i} + y \hat{j} - k \text{ and } \overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k} \text{, } x, y \in R, x > 0 \text{, be such that } |\overrightarrow{PQ}| = \sqrt{20} \text{ and the vector } \overrightarrow{OP} \text{ and } \overrightarrow{OQ} \text{, then the value of } x^2 + y^2 + z^2 \text{ is equal to:} \]
1) 1 2) 7 3) 2 4) 9
Key :4
Sol: \[ \overrightarrow{OP} \perp \overrightarrow{OQ} \Rightarrow -x + 2y - 3x = 0 \Rightarrow y = 2x \text{..............}(1) \]
\[ |\overrightarrow{PQ}| = \sqrt{20} \Rightarrow (x + 1)^2 (2 - y)^2 + (3x + 1)^2 = 20 \]
\[ \Rightarrow 14x^2 - 14 = 0 \Rightarrow x = \pm 1, x > 0 \Rightarrow x = 1, y = 2 \]
OP, OQ, OR are collinear
\[
\begin{vmatrix}
  x & y & -1 \\
  -1 & 2 & 3x \\
  3 & z & -7 \\
\end{vmatrix} = 0 \Rightarrow z = -2
\]
\[ x^2 + y^2 + z^2 = 1 + 4 + 4 = 9 \]
74. Let \( y = y(x) \) be the solution of the differential equation
\[
\cos x (3 \sin x + \cos x + 3) \, dy = \left(1 + y \sin x (3 \sin x + \cos x + 3)\right) \, dx, 0 \leq x \leq \frac{\pi}{2}, y(0) = 0.
\]
Then, \( y \left(\frac{\pi}{3}\right) \) is equal to:
1) \( 2 \log_2 \left(\frac{3\sqrt{3} - 8}{4}\right) \)
2) \( 2 \log_2 \left(\frac{2\sqrt{3} + 10}{11}\right) \)
3) \( 2 \log_2 \left(\frac{2\sqrt{3} + 9}{6}\right) \)
4) \( 2 \log_2 \left(\frac{\sqrt{3} + 7}{2}\right) \)

**Key : 2**

**Sol:**
\[
\frac{dy}{dx} - (\tan x) y = \frac{1}{\cos x (3 \sin x + \cos x + 3)}
\]

\[
I.F = \cos x
\]

\[
y(\cos x) = \int \frac{1}{3 \sin x + \cos x + 3} \, dx
\]

\[
t = \tan \frac{x}{2} \quad ............
\]

\[
y \cos x = \log \left(\frac{1 + \tan \frac{x}{2}}{2 + \tan \frac{x}{2}}\right) + c
\]

\[
x = 0, y = 0 \Rightarrow c = \lim 2
\]

Where \( x = \frac{\pi}{3} \), \( y = 2 \log_2 \left(\frac{10 + 2\sqrt{3}}{11}\right) \)

75. The value of the limit \( \lim_{\theta \to 0} \frac{\tan (\pi \cos^2 \theta)}{\sin (2 \pi \sin^2 \theta)} \) is equal to:
1) \(-\frac{1}{2}\)
2) \(\frac{1}{4}\)
3) \(-\frac{1}{4}\)
4) 0

**Key : 1**

**Sol:**
\[
\lim_{\theta \to 0} \frac{\tan (\pi \cos^2 \theta)}{\sin (2 \pi \sin^2 \theta)} \quad .............\sin^2 \theta = 1 - \cos^2 \theta
\]

\[
= \lim_{\theta \to 0} \frac{\sin (\pi \cos^2 \theta)}{\cos (\pi \cos^2 \theta)}
\]

\[
= \lim_{\theta \to 0} -2 \sin (2 \pi \cos^2 \theta) \cos (\pi \cos^2 \theta)
\]

\[
= \lim_{\theta \to 0} -2 \cos^2 (\pi \cos^2 \theta) = \left(-\frac{1}{2}\right) \lim_{\theta \to 0} \sec^2 (\pi \cos^2 \theta) = \left(-\frac{1}{2}\right)
\]

76. Let \( f : R \to R \) be defined as \( f(x) = e^x \sin x \). If \( F : [0,1] \to R \) is a differentiable function such that \( F(x) = \int_0^x f(t) \, dt \), then the value of \( \int_0^1 (F'(x) + f(x)) e^x \, dx \) lies in the interval
1) \( \left[\frac{330}{360}, \frac{331}{360}\right] \)
2) \( \left[\frac{331}{360}, \frac{334}{360}\right] \)
3) \( \left[\frac{335}{360}, \frac{336}{360}\right] \)
4) \( \left[\frac{337}{360}, \frac{339}{360}\right] \)

**Key : 1**
77. The number of solutions of the equation $v$ in the interval $[0, 2\pi]$ is:

1) 5  
2) 3  
3) 2  
4) 4

Key: 2

Sol: $\sin \frac{\pi x}{4} - \frac{x}{2}$

From graph 3 solutions

78. If the integral $\int_{0}^{10} \frac{\sin 2\pi x}{e^{x} + 1} \, dx = \alpha e^{-1} + \beta e^{\frac{x}{2}} + \gamma$, where $\alpha, \beta, \gamma$ are integers and $[x]$ denotes the greatest integer less than or equal to $x$, then the value of $\alpha + \beta + \gamma$ is equal to:

1) 25  
2) 10  
3) 20  
4) 0

Key: 4

Sol: Integrand is periodic with period 1

$I = 10 \int_{0}^{1} \frac{\sin 2\pi x}{e^{x} + 1} \, dx$

$= 10 \left[ \int_{0}^{1/2} e^{x} \, dx + \int_{1/2}^{1} \frac{-1}{e^{x}} \, dx \right]$

$= 10e^{1} - 10e^{-1/2}$

$\alpha = 10, \beta = -10, \gamma = 0 \Rightarrow \alpha + \beta + \gamma = 0$
79. Consider the function \( f: \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = \begin{cases} 2 - \sin \left( \frac{1}{x} \right) & , x \neq 0 \\ 0 & , x = 0 \end{cases} \). Then \( f \) is:

1) not monotonic on \((-\infty, 0)\) and \((0, \infty)\)
2) monotonic on \((-\infty, 0) \cup (0, \infty)\)
3) monotonic on \((-\infty, 0)\) only
4) monotonic on \((0, \infty)\) only

Key :1

Sol: \( f(x) = \begin{cases} 2x - x \sin \frac{1}{x} & x > 0 \\ -2x + x \sin \frac{1}{x} & x < 0 \end{cases} \)

\( f'(x) = \begin{cases} 2 + \frac{1}{x} \cos \left( \frac{1}{x} \right) - \sin \left( \frac{1}{x} \right) & x > 0 \\ -2 - \frac{1}{x} \cos \left( \frac{1}{x} \right) - \sin \left( \frac{1}{x} \right) & x < 0 \end{cases} \)

\( f'(x) = \begin{cases} >0 & x > 0 \\ <0 & x < 0 \end{cases} \)

\( f(x) \) is non monotonic in \((-\infty, 0) \cup (0, \infty)\)

80. Two tangents are drawn from a point \( P \) to the circle \( x^2 + y^2 - 2x - 4y + 4 = 0 \), such that the angle between these tangents is \( \tan^{-1} \left( \frac{12}{5} \right) \), where \( \tan^{-1} \left( \frac{12}{5} \right) \in (0, \pi) \). If the centre of the areas of \( \triangle PAB \) and \( \triangle CAB \) is:

1) 9 : 4
2) 2 : 1
3) 3 : 1
4) 11 : 4

Key :1

Sol: \( x^2 + y^2 - 2x - 4y + 4 = 0 \) caste \( C = (1, 2), r = 1 \)

Angle between tangent \( 2\theta = \tan^{-1} \left( \frac{12}{5} \right) \) \( \Rightarrow \tan \theta = \frac{2}{3} \)

\( \tan \theta = \frac{r}{AP} = \frac{2}{3} \) \( \Rightarrow AP = \frac{3}{2} = BP \)

\( \therefore \frac{Ar(\triangle PAB)}{Ar(\triangle CAB)} = \frac{PA.PB \sin P}{AC.BC \sin C} = \frac{3 \cdot 3 \cdot \frac{12}{13}}{2 \cdot 2 \cdot \frac{12}{13}} = \frac{9}{4} = 9 : 4 \)
This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. Let \( I_n = \int_1^e x^n (\log |x|)^n \, dx \), where \( n \in \mathbb{N} \). If \( 20I_{10} = \alpha I_9 + \beta I_8 \), for natural numbers \( \alpha \) and \( \beta \), then \( \alpha - \beta \) is equal to ______

Key :1

Sol:
\[
I_n = \int_1^e x^n (\log |x|)^n \, dx \Rightarrow I_{10} = \int_1^e x^{10} (\log |x|)^{10} \, dx
\]

\[
I_{10} = (\log x)^{10} x^{20} \Bigg|_{1}^{e} - \frac{10}{20} \int_{1}^{e} x^9 (\log x)^9 \, dx
\]

Integration by parts

\[
I_{10} = e^{20} - \frac{10}{20} I_9 \Rightarrow 20I_{10} = 9I_8 + 10I_9
\]

\[
I_9 = e^{20} - \frac{9}{20} I_8
\]

\( \alpha = 10, \beta = 9, \alpha - \beta = 1 \)

82. If \( 1, \log_{10} (4^x - 2) \) and \( \log_{10} \left(4^x + \frac{18}{5}\right) \) are in arithmetic progression for a real number \( x \), then

the value of the determinant

\[
\begin{vmatrix}
2 \left(x - \frac{1}{2}\right) & x^2 & x \\
1 & 0 & x \\
x & 1 & 0
\end{vmatrix}
\]

is equal to:

Key :2

Sol:
\[
2\log_{10} (4^x - 2) = 1 + \log_{10} \left(4^x + \frac{18}{5}\right) \Rightarrow (4^x - 2)^2 = 10 \left(4^x + \frac{18}{5}\right)
\]

\[
(4^x)^2 - 14(4^x) - 32 = 0
\]

\( 4^x = 16 \) or \( 4^x = -2 \) \( x = 2 \)

\[
\begin{vmatrix}
2 \left(x - \frac{1}{2}\right) & x^2 & 3 & 1 & 4 \\
1 & 0 & x & 1 & 0 & 2 \\
x & 1 & 0 & 2 & 1 & 0
\end{vmatrix}
\]

83. Let \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) and \( B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) such that \( AB = B \) and \( a + d = 2021 \), then the value of \( ad - bc \)

is equal to ______

Key :2020
Sol: \[ AB = B \Rightarrow AB = B = 0 \Rightarrow (A - I)B = 0 \]
\[ \Rightarrow |A - I| = 0 \text{ since } |B| \neq 0 \]
\[ \Rightarrow \begin{vmatrix} a - 1 & b \\ c & d - 1 \end{vmatrix} = 0 \]
\[ \Rightarrow ad - bc - a - d + 1 = 0 \]
\[ ad - bc = a + d - 1 = 2021 - 1 = 2020 \]

84. Let \( f : [-3, 1] \rightarrow R \) be given as

\[
f(x) = \begin{cases} 
\min \{(x+6), x^2\}, & -3 \leq x \leq 0 \\
\max \{\sqrt{x}, \sqrt{x^2}\}, & 0 \leq x \leq 1 
\end{cases}
\]

If the area bonded by \( y = f(x) \) and x-axis is A, then the value of 6A is equal to ____

Key :41

Sol:

Area \( A = \int_{-3}^{0} (x+6)dx + \int_{0}^{1} x^2dx + \int_{1}^{6} \sqrt{x}dx \)
\[ \Rightarrow A = \frac{41}{6} \Rightarrow 6A = 41 \]

85. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ____

Key :68
Sol: \[
\frac{x_1 + x_2 + \ldots + x_{2n}}{2n} = 6 \Rightarrow x_1 + \ldots + x_{2n} = 12n
\]
\[
\frac{x_{2n+1} + \ldots + x_{3n}}{n} = 3 \Rightarrow x_{2n+1} + \ldots + x_{3n} = 3n \Rightarrow x_1, x_2, \ldots, x_{3n} = 15n
\]
Mean = \[\frac{15n}{3n} = 5\]
Variance = 4
\[
\sum_{i=1}^{3n} x_i^2 - (\bar{x})^2 = 4 \Rightarrow \sum_{i=1}^{3n} x_i^2 = 87n
\]
Now mean \[
\bar{x} = \frac{(x_1 + 1) + \ldots + (x_{2n} + 1) + (x_{2n+1} - 1) + \ldots + (x_{3n} - 1)}{3n} = \frac{16}{3}
\]
Now valance \[
\frac{\sum_{i=1}^{2n} (x_i + 1)^2 + \sum_{i=2n+1}^{3n} (x_i - 1)^2}{3n}(\bar{x})^2 = \frac{68}{9} = k \Rightarrow 9k = 68
\]
86. Let \(\bar{x}\) be a vector in the plane containing vectors \(\hat{a} = 2\hat{i} - \hat{j} + \hat{k}\) and \(\hat{b} = \hat{i} + 2\hat{j} - \hat{k}\). If the vector \(\bar{x}\) is perpendicular to \((3\hat{i} + 2\hat{j} - \hat{k})\) and its projection on \(\hat{a}\) is \(\frac{17\sqrt{6}}{2}\), then the value of \(|\bar{x}|^2\) is equal to blank.
Key: 786
Sol: \[
\bar{x} = \lambda \hat{a} + \mu \hat{b}
\]
\[
i(2\lambda + \mu) + j(-\lambda + 2\mu) + k(\lambda - \mu)
\]
\[
\bar{x}.(3\hat{i} + 2\hat{j} - \hat{k}) = 0 \Rightarrow 3\lambda + 8\mu = 0 \ldots \ldots \ldots \ldots \ldots (1)
\]
\[
\frac{\bar{x} \bar{a}}{|\bar{a}|} = \frac{17\sqrt{6}}{2} \Rightarrow 6\lambda - \mu = 51 \ldots \ldots \ldots \ldots \ldots (2)
\]
\[
\Rightarrow \lambda = 8, \mu = -3 \Rightarrow \bar{x} = 13\hat{i} - 14\hat{j} + 11\hat{k} \Rightarrow |\bar{x}|^2 = 486
\]
87. Let P be an arbitrary point having sum of the squares of the distances from the planes \(x + y + z = 0, lx - nz = 0\) and \(x - 2y + z = 0\), equal to 9. If the locus of the point P is \(x^2 + y^2 + z^2 = 9\), then the value of \(l - n\) is equal to blank.
Key: 0
Sol: let \( P(\alpha, \beta, \gamma) \) \( \Rightarrow d_1^2 + d_2^2 + d_3^2 = 9 \)

\[ \Rightarrow \left( \frac{\alpha + \beta + \gamma}{3} \right)^2 + \left( \frac{l\alpha - n\gamma}{l^2 + n^2} \right)^2 + \left( \frac{\alpha - 2\beta + r^2}{6} \right) = 9 \]

Locus \( \left( \frac{x + y + z}{3} \right)^2 + \left( \frac{l(x - y)}{l^2 + n^2} \right)^2 + \left( \frac{x - 2y + z}{6} \right)^2 = 9 \)

\[ \Rightarrow x^2 \left( \frac{1}{2} + \frac{1}{6} + \frac{\ell^2}{l^2 + x^2} \right) + y^2 \left( \frac{1}{3} + \frac{4}{6} \right) + z^2 \left( \frac{1}{3} + \frac{x^2}{l^2 + x^2} + \frac{1}{6} \right) + 2xz \left( \frac{1}{3} + \frac{1}{6} - \frac{\ell x}{l^2 + x^2} \right) + \ldots \ldots = 9 \]

Given \( x^2 + y^2 + z^2 = 9 \)

Coefficient of \( xz = 0 \) \( \Rightarrow \frac{1}{2} - \frac{ln}{l^2 + n^2} = 0 \Rightarrow (\ell - n)^2 = 0 \Rightarrow \ell - n = 0 \)

88. Let \( f : [-1,1] \rightarrow R \) be defined as \( f(x) = ax^2 + bx + c \) for all \( x \in [-1,1] \), then the least value of \( \alpha \) is equal to _____

Key :5

Sol: \( f(x) = ax^2 + bx + c, \quad f'(x) = 2ax + b, \quad f''(x) = 2a \)

\( f(-1) = 2 \Rightarrow a - b + c = 2 \ldots (1) \)

\( f'(-1) = 1 \Rightarrow -2a + b = 1 \ldots (2) \)

Max of \( f''(x) = \frac{1}{2} \)

\[ 2a = \frac{1}{2} \Rightarrow a = \frac{1}{4} \quad \Rightarrow b = \frac{3}{2}, \quad c = \frac{13}{4} \]

\( f(x) = \frac{1}{4}(x^2 + 6x + 13) \)

\( f'(x) = \frac{1}{4}(2x + 6) = 0 \Rightarrow x = -3 \; \text{and} \; [-1,1] \)

\( f(1) = 5, \quad f(-1) = 2 \)

\( f_{max} = 5 \; \text{since} \; x \in [-1,1] \)

89. Let the coefficients of third, fourth and fifth terms in the expansion of \( (x + \frac{a}{x^n})^n, x \neq 0 \), be in the ratio 12:8:3. Then the term independent of \( x \) in the expansion, is equal to _____.

Key :3
Sol:  

3rd team \( T_{r+1} = n_{c}a^{r}x^{-3r} \)

\( T_{1} = na, T_{2} = na^{3}x^{-6}, T_{3} = na^{6}x^{-9}, T_{4} = na^{9}x^{-12} \)

\[
\frac{\text{cof of } T_{3}}{\text{cof of } T_{4}} \Rightarrow a(n-2) = 2 \ldots \ldots (1)
\]

\[
\frac{\text{cof of } T_{4}}{\text{cof of } T_{5}} \Rightarrow a(n-3) = \frac{3}{2} \ldots \ldots (2)
\]

\( \Rightarrow n = 6, a = \frac{1}{2} \)

Independent of \( x : n - 3r = 0 \Rightarrow 3r = n = 6 \)

\( r = 2 \)

\( \therefore 3\text{rd tem} \)

90. Let \( \tan \alpha, \tan \beta : \alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}, n \in N \) be the slopes of three line segments OA, OB and OC respectively, where O is origin. If Circumcentre of \( \triangle ABC \) coincides with origin and its orthocentre lies on y-axis, the value of \( \cos 3\alpha \cos 3\beta \cos 3\gamma \) is equal to_____

Key :144

Sol: \( A = (\cos \alpha, \sin \alpha), B = (\cos \beta, \sin \beta), C = (\cos \gamma, \sin \gamma) \)

Circumcentre = \( (0,0) \)

Orthocentre \( H = (\cos \alpha + \cos \beta + \cos \gamma, \sin \alpha + \sin \beta + \sin \gamma) \) lies on y-axis

\( \Rightarrow \cos \alpha + \cos \beta + \cos \gamma = 0 \)

\( \Rightarrow \cos^{3} \alpha + \cos^{3} \beta + \cos^{3} \gamma = 3 \cos \alpha \cos \beta \cos \gamma \)

\[
\sum \cos 3\alpha = 4(\sum \cos^{3} \alpha) - 3(\sum \cos \alpha)
\]

\( = 12 \cos \alpha \cos \beta \cos \gamma \)

Given expression = \( \left( \frac{12 \cos \alpha \cos \beta \cos \gamma}{\cos \alpha \cos \beta \cos \gamma} \right)^{2} = 144 \)