Sri Chaitanya

IIT Academy., India

JEE Main 2020

09 Jan 2020, Slot - 1
(9.30 PM - 12.30 PM)

Question Paper

Solutions

Corporate Office : Plot No-304, Kasetty Heights, Ayyappa Society Madhapur, Hyderabad-500081

www.srichaitanya.net
PHYSICS

1. The electric fields of two plane electromagnetic plane waves in vacuum are given by 
\[ E_1 = E_0 \hat{j} \cos(\omega t - kx) \] and \[ E_2 = E_0 \hat{k} \cos(\omega t - ky) \]

At \( t = 0 \), a particle of charge \( q \) is at origin with a velocity \( \vec{v} = 0.8 \hat{c} \) \( c \) is the speed of light in vacuum). The instantaneous force experienced by the particle is :

1) \( E_0 q (0.8 \hat{i} - \hat{j} + 0.4 \hat{k}) \)
2) \( E_0 q (0.8 \hat{i} + \hat{j} + 0.2 \hat{k}) \)
3) \( E_0 q (0.4 \hat{i} - 3 \hat{j} + 0.8 \hat{k}) \)
4) \( E_0 q (-0.8 \hat{i} + \hat{j} + \hat{k}) \)

KEY: 2

SOL: For \( E_1 \) corresponding \( B_1 = \frac{E_0}{c} \hat{k} \cos(\omega t - kx) \)
\( E_2 \) corresponding \( B_2 = \frac{E_0}{c} \hat{i} \cos(\omega t - kg) \)
\( \vec{F} = q\vec{E} + q(\vec{\alpha} \times \vec{\beta}) \)

2. A long, straight wire of radius \( a \) carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wires at distance \( \frac{a}{3} \) and \( 2a \) respectively from the axis of the wire is :

1) \( \frac{1}{2} \)
2) \( \frac{3}{2} \)
3) \( \frac{2}{3} \)
4) \( 2 \)

KEY: 3

SOL: \( \frac{B_p}{B_q} = \frac{\mu_0 \frac{i}{6\pi a}}{\mu_0 \frac{i}{2\pi (2a)}} = \frac{\frac{1}{3}}{2} \)
\( B_p = \frac{\mu_0 i}{6\pi a} \)
\( B_q = \frac{\mu_0 i}{2\pi (2a)} \)
\( \frac{B_p}{B_q} = \frac{2}{3} \)
3. A charged particle of mass ‘m’ and charge ‘q’ moving under the influence of uniform electric field \( E_i \) and a uniform magnetic field \( B_k \) follows a trajectory from point P to Q as shown in figure. The velocities at P and Q are respectively, \( v_i \) and \(-2v_j\). Then which of the following statements (A,B,C,D) are the correct? (Trajectory shown is schematic and not to scale)

![Diagram showing particle trajectory](image)

A) \( E = \frac{3}{4} \left( \frac{mv^2}{qa} \right) \)

B) Rate of work done by the electric field at P is \( \frac{3}{4} \left( \frac{mv^3}{a} \right) \)

C) Rate of work done by both the fields at Q is zero

D) The difference between the magnitude of angular momentum of the particle at P and Q is \( 2mav \)

1) (B), (C), (D)  
2) (A), (B), (C)  
3) (A), (C), (D)  
4) (A), (B), (C), (D)

**KEY:** 2

**SOL:**

Work done by magnetic field is zero

\[
W_{mag} + W_{ele} = \Delta kE - qE(2a) = \frac{1}{2} m \left[ 4v^2 - v'^2 \right]
\]

\[
E = \frac{3}{4} \left( \frac{mv^2}{qa} \right) \quad \text{Rate of work done at (p) is} \quad p = F.V = \frac{3}{4} \left( \frac{mv^3}{a} \right)
\]

Rate of work done at Q \( p = F.V = 0 \)

4. Two particles of equal mass \( m \) have respective initial velocities \( u \hat{i} \) and \( u \left( \hat{i} + \frac{\hat{j}}{2} \right) \). They collide completely inelastically. The energy lost in the process is :

1) \( \frac{3}{4} mu^2 \)  
2) \( \frac{1}{8} mu^2 \)  
3) \( \frac{1}{3} mu^2 \)  
4) \( \sqrt{\frac{2}{3}} mu^2 \)

**KEY:** 2

**SOL:**

\[
V_c = \frac{m_1n_1 + m_2n_2}{m_1 + m_2}
\]
5. A body A of mass \( m \) is moving in a circular orbit of radius \( R \) about a planet. Another body B of mass \( \frac{m}{2} \) collides with A with a velocity which is half \( \frac{\vec{v}}{2} \) the instantaneous velocity \( \vec{v} \) of A. The collision is completely inelastic. Then the combine body:

1) Falls vertically downwards towards the planet
2) Escapes from the Planet’s Gravitational field
3) Starts moving in an elliptical orbit around the planet
4) continuous to move in a circular orbit

KEY:3

SOL: There is a tangential velocity

6. Consider a sphere of radius \( R \) which carries a uniform charge density \( \rho \). If a sphere of radius \( \frac{R}{2} \) is carved out of it, as shown, the ratio \( \frac{E_A}{E_B} \) of magnitude of electric field \( E_A \) and \( E_B \) respectively, at points A and B due to the remaining portion is:

\[
\begin{align*}
1) & \quad \frac{18}{34} & 2) & \quad \frac{18}{54} & 3) & \quad \frac{17}{54} & 4) & \quad \frac{21}{34}
\end{align*}
\]

KEY:1

SOL:

\[
E_{in} = \frac{\rho r}{3 \varepsilon_0} \quad E_{out} = \left( \frac{\rho r^3}{3 \varepsilon_0} \right) \frac{1}{r^2}
\]

7. Water flows in a horizontal tube (see figure) The pressure of water change by 700 \( Nm^{-2} \) between A and B where the area of cross section are 40 cm\(^2\) and 20 cm\(^2\). Respectively. Find the rate of flows of water through the tube.

(density of water = 1000 kgm\(^{-3}\))
1) 3020 cm$^3$/s  2) 2420 cm$^3$/s  3) 1810 cm$^3$/s  4) 2720 cm$^3$/s

**KEY:** 4

**SOL:**

$$P_A + \frac{1}{2} ev^2_a = P_B + \frac{1}{2} P_{v_{13}}$$  
$$P_A - P_B = \frac{1}{2} \rho \left( v_B^2 - v_{13}^2 \right)$$

$$\Delta \rho = \frac{1}{2} \rho \left( v_B^2 - v_A^2 \right)$$

Rate of flow \( \times \) velocity at B \( = 2720 \text{ cm}^3/\text{s} \)

8.

Three solid spheres each of mass \( m \) and diameter \( d \) are stuck together such that the lines connecting the centre form an equilateral triangle of side of length \( d \). The ratio \( I_o / I_A \) of moment of inertia \( I_o \) of the system about an axis passing the centroid and about centre of any of the spheres \( I_A \) and perpendicular to the plane of the triangle is:

1) \( \frac{23}{13} \)  2) \( \frac{13}{23} \)  3) \( \frac{13}{15} \)  4) \( \frac{15}{13} \)

**KEY:** 2

**SOL:**

MI about centroid \( = 3 \left[ \frac{2}{5} m \left( \frac{d}{2} \right)^2 + m \left( \frac{d}{\sqrt{3}} \right)^2 \right] = \frac{13}{10} m d^2 \)

MI about axis passing through ‘r’ and \( \perp \) to place \( = 2 \left[ \frac{2}{5} m \left( \frac{d}{2} \right)^2 + m(d)^2 \right] + \frac{2}{5} m \left( \frac{d}{2} \right)^2 = \frac{23}{10} m d^2 \)  \( \therefore \text{Ratio} = \frac{13}{23} \)
9. Radiation, with wavelength \(6561 \text{ Å}\) falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of \(3 \times 10^{-4} \text{T}\). If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal close to:

1) 1.8 eV  
2) 0.8 eV  
3) 1.1 eV  
4) 1.6 eV

**KEY:** 3

**SOL:**

\[
E_i = \frac{hc}{\lambda} = 1.89 \text{ eV}
\]

\[
r = \frac{mv}{Be} = \frac{\sqrt{2mk}}{Be}
\]

\[
K = \frac{B^2e^2}{2m}
\]

Work function = \(E_i - k = 1.1 \text{ eV}\)

10. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass \(m\). Molecules of the gas B has an additional vibrational mode, and have a mass \(\frac{m}{4}\). The ratio of the specific heats \(C_v^A\) and \(C_v^B\) of gas A and B, respectively is:

1) 5:9  
2) 5:7  
3) 3:5  
4) 7:9

**KEY:** 2

**SOL:**

Specific heat of \(A = \frac{5}{2}RT\)

Specific heat of \(B = \frac{7}{2}RT\) \(\therefore \text{Ratio} = \frac{5}{7}\)

11. An electric dipole of moment \(p = (-\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29} \text{ Cm}\) is at the origin \((0,0,0)\). The electric field due to the dipole at \(r = +\hat{i} + 3\hat{j} + 5\hat{k}\)

(no equipments are used to find \(r \cdot p = 0\) ) is parallel to:

1) \((+\hat{i} - 3\hat{j} - 2\hat{k})\)  
2) \((-\hat{i} - 3\hat{j} + 2\hat{k})\)  
3) \((-\hat{i} + 3\hat{j} - 2\hat{k})\)  
4) \((+\hat{i} + 3\hat{j} - 2\hat{k})\)

**KEY:** 4
SOL:

\[ \vec{r} \cdot \vec{p} = 0 \] 
so 
\[ \vec{p} = \sqrt{\vec{r}} \]

\( \vec{E} \) is anti parallel to \( \vec{P} \)

12. Three harmonic waves having equal frequency \( v \) and same intensity \( I_0 \), have phase angles \( 0, \frac{\pi}{4} \) and \( -\frac{\pi}{4} \) respectively. When they are superimposed the intensity of the resultant wave is close to:

1) \( 0.2 \ I_0 \)  
2) \( 3 \ I_0 \)  
3) \( 5.8 \ I_0 \)  
4) \( I_0 \)

KEY: 3

SOL:

\[ \Delta r = (1 + \sqrt{2}) Ab \]

\[ I_r = KA^2_0 = (1 + \sqrt{2})^2 A^2 k \]

\[ I_0 = KA^2 \implies I_r = 5.8 \ I_0 \]

13. A particle moving with kinetic energy \( E \) has de Broglie wavelength \( \lambda \). If energy \( \Delta E \) is added to its energy, the wavelength become \( \lambda / 2 \). Value of \( \Delta E \) is:

1) \( E \)  
2) \( 4E \)  
3) \( 3E \)  
4) \( 2E \)

KEY: 3

SOL:

\[ \lambda = \frac{h}{\sqrt{2km}} \implies \lambda or \frac{1}{\sqrt{k}} \]

\[ K_f = 4 Ki \implies \Delta E = 3E \]

14. Consider a force \( \vec{F} = -x \hat{i} + y \hat{j} \). The work done by this force in moving a particle from point \( A(1,0) \) to \( B(0,1) \) along the line segment is (all quantities are in SI units)

\[ \begin{align*}
1) & \quad \frac{1}{2} \\
2) & \quad \frac{3}{2} \\
3) & \quad 1 \\
4) & \quad 2
\end{align*} \]
KEY: 3
SOL:

Let \( \vec{ds} = dx \hat{i} + dy \hat{j} + dz \hat{k} \)  
\( \vec{F}_w \)  
\( (0.1) = -xdx + ydy \)  
\( \text{IN} = \int dW = 1 J \)  
(1, 0)

15. In the given circuit diagram, a wire is joining points B and D. The current in this wire is:

1) 2A  
2) 0.4A  
3) 4A  
4) Zero

KEY: 1
SOL:

\( R_{eff} = 2 \Omega \quad i = \frac{v}{R} = 10 A \)

Apply law of junctions  
For B & D

16. If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is:

1) 0.001 mm  
2) 0.001 cm  
3) 0.01 cm  
4) 0.02 mm

KEY: 2
SOL:

\[ \text{Pitch} = \frac{3 \text{mm}}{6} \]

\[ \text{LC} = \frac{\text{pitch}}{\text{no.of HSD's}} = \frac{0.5 \text{ mm}}{50} = 0.001 \text{ cm} \]

17. A vessel of depth 2h is half filled with a liquid of refractive index \( 2 \sqrt{2} \) and the upper half with another liquid of refractive index \( \sqrt{2} \). The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be:

1) \( \frac{h}{\sqrt{2}} \)  
2) \( \frac{h}{3 \sqrt{2}} \)  
3) \( \frac{h}{2(\sqrt{2} + 1)} \)  
4) \( \frac{3}{4} h \sqrt{2} \)
KEY: 4
SOL:

Total shift(s) = \[ h \left( \frac{1}{\mu_1} - 1 + \frac{1}{\mu_2} \right) \]

Apparent depth = \( 2h - s \)  
Solving, \( d_{app} = \frac{3}{4} h \sqrt{2} \)

18. A quantity \( f \) is given by \( f = \sqrt{\frac{hc^5}{G}} \) where \( c \) is speed of light, \( G \) universal gravitational constant and \( h \) is the Planck’s constant. Dimensions of \( f \) is that of:

1) energy  
2) momentum  
3) area  
4) volume

KEY: 1
SOL:

\( f = \sqrt{\frac{nc^5}{G}} \) can be re-write as \( f = \sqrt{\frac{h\nu}{Gm^2 \times \frac{m^2}{r} \cdot c^5}} \)

\( f \) having dimensions \( ML^2T^{-2} \)

Represents energy

19. Which of the following is an equivalent cycle process corresponding to the thermodynamic cyclic given in the figure?

Where, \( 1 \rightarrow 2 \) is adiabatic (Graphs are schematic and are not to scale)

KEY: 4
SOL:

For the process \( 1 \rightarrow 2 \)

\( TV^{\gamma-1} = \text{const} \) is not a straight line
20. The aperture diameter of a telescope is 5 m. The separation between the moon and the earth is \(4 \times 10^5 \text{km}\). With light of wavelength of \(5500 \text{A}\), the minimum separation between objects on the surface of moon, so that they are just resolved, is close to:

1) 20 m  
2) 200 m  
3) 60 m  
4) 600 m

**KEY:** 3

**SOL:**

\[ x = \frac{1.22 \lambda}{d} \]

\[ = \frac{1.22 \times 5500 \times 10^{-10} \times 4 \times 10^5}{5} = 60 \text{ m} \]

21. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in Volts) at point A is ___ -

**KEY:** 12

**SOL:**

\[ V_A = 12 \text{ V} \]

22. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle \(30^0\) from the horizontal (see figure). Its angular speed when it hits the table it given as \(\sqrt{n} \text{s}^{-1}\), where \(n\) is an integer. The value of \(n\) is

**KEY:** 15
SOL:

According to conservation of energy

\[ mg \left(\frac{l}{2}\right) \sin \theta = \frac{1}{2} I_i \omega^2 \]
\[ \omega = \sqrt{\frac{3g}{2l}} \]

23. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be____

KEY: 10

SOL:

\[ \mathcal{E} = L \left| \frac{di}{dt} \right| \]
\[ L = \frac{100}{(0.25-0)/0.025 \times 10^{-3}} \]
\[ L = 10 \text{ mH} \]

24. The distance \( x \) covered by a particle in one dimensional motion varies with time \( t \) as \( x^2 = at^2 + 2bt + c \). If the acceleration of the particle depends on \( x \) as \( x^{-n} \), where \( n \) is an integer, the value of \( n \) is_____

KEY: 3

SOL:

Let acceleration be \( 'f' \)

Differentiating \( \text{w.r.t} \ 't' \)

\[ \nu x = at + b \]

Again differentiating \( \text{w.r.t} \ 't' \)

\[ b^2 + xf = a \]

\[ f = \frac{a}{x} - \left( \frac{at+b}{x} \right)^2 \frac{1}{x} \]

Solving, \[ f = \frac{ac-b^2}{x^3} \]

25. A body of mass \( m = 10 \text{ kg} \) is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s\(^{-1}\)) with which it can be rotated about its other end in space station is (Breaking stress of wire = \( 4.8 \times 10^7 \text{ Nm}^{-2} \) and area of cross section of the wire = \( 10^{-2} \text{ cm}^2 \)) is:

KEY: 4

SOL:

\[ ml \omega^2 = fA \]

\[ \omega = \sqrt{\frac{fA}{ml}} = \sqrt{\frac{(4.8 \times 10^7)10^{-6}}{10 \times 0.3}} \]

\[ \omega = 4 \text{ rad} / \text{sec} \]
CHEMISTRY

1. \([ Pd(F)(Cl)(Br)(I)]^{2-}\) has n number of geometrical isomers. Then, the spin-only magnetic moment and crystal field stabilization energy [CFSE] of \([Fe(CN)_6]^{3-}\), respectively, are : (Note : ignore the pairing energy)

1) 0 BM and \(-2.4\Delta_0\)  
2) 2.84 BM and \(-1.6\Delta_0\)  
3) 1.73 BM and \(-2.0\Delta_0\)  
4) 5.92 BM and 0

KEY: 3

SOL: The given complex is \([Mabcd]\) type complex ⇒ No. of G.I n=3

The cyanide complex is \([Fe(CN)_6]^{3-}\) \(Fe^{3+} \rightarrow d^5\) Config +SFL

\(CFT\) config : \(-t_2^g \quad e_g^{00}\)

\(no\) of unpaired \(e^- = 1\)

\(\Rightarrow \mu\sqrt{1+2} = \sqrt{3}\)

\(= 173\) BM

CSFE = \(-0.4\Delta_0\times n_{t_2g} + 0.6\Delta_0\times n_{e_g} = -0.4\Delta_0 \times 5 + 0.6\Delta_0 \times 0 = -2.0\Delta_0\)

2. For following reactions

\(A \xrightarrow{700K} \text{Product}\)

\(A \xrightarrow{500K \text{catalyst}} \text{Product}\)

It was found that the \(E_a\) is decreased by 30 kJ/mol in the presence of catalyst. If the rate remains unchanged, the activation energy for catalysed reaction is (Assume pre exponential factor is same):

1) 198 kJ/mol  
2) 135 kJ/mol  
3) 75 kJ/mol  
4) 105 kJ/mol

KEY: 3

SOL: \(K_{cat_1} = K_{uncat_2} \Rightarrow Ae^{-E_{a_1}/RT_1} = Ae^{-E_{a_2}/RT_2}\)

Let : \(E_{a_1} = x\) kJ/mol

\(E_{a_2} = x + 30 \Rightarrow \frac{E_{a_1}}{T_1} = \frac{E_{a_2}}{T_2} \Rightarrow \frac{x}{500} = \frac{x + 30}{700}\)

\(7x = 5x + 150 \Rightarrow x = 75\) kJ/mol
3. Identify (A) in the following reaction sequence:

\[ \begin{align*}
A &\xrightarrow{(i) \text{CH}_3\text{MgBr}} \text{B} \\
&\xrightarrow{(iii) \text{Conc.H}_2\text{SO}_4 / \Delta} \text{C}
\end{align*} \]

![Chemical structures](image)

KEY: 4

SOL: (b)

4. According to the following diagram, A reduces \( BO_2 \) when the temperature is:

![Graph](image)

\[ \begin{align*}
1) &> 1400^\circ C \\
2) &< 1200^\circ C \\
3) &> 1200^\circ C \text{ but } < 1400^\circ C \\
4) &< 1400^\circ C
\end{align*} \]
KEY: 4
SOL: The Curves Intersect at T=1400°C

T<1400°C
B+AO₂ → A+BO₂ will be spontaneous

T>1400°C
A+BO₂ → B+AO₂ will be spontaneous

5. The major product Z obtained in the following reaction scheme is:

![Reaction Scheme]

1) 2) 3) 4)

KEY: 3
SOL:

6. Which of these will produce the highest yield in Friedel Crafts reaction?

1) 2) 3) 4)
KEY: 1
SOL: Chlurobenzene will be most reactive as -NH₂ & -OH lp reacts with AlCl₃ and it forms cationic complex which deactivates the ring towards fried craft alkylation.

7. ‘X’ melts at low temperature and is a bad conductor of electricity in both liquid and solid state. X is:
   1) Carbon tetrachloride
   2) Silicon carbide
   3) Mercury
   4) Zinc sulphide

KEY: 4
SOL: Ncl₄ is Convalent compound & hence doesn’t give ions in molten liquid state and hence is a non conductor of electricity.

8. The compound that cannot act both as oxidising and reducing agent is:
   1) H₂O₂
   2) HNO₂
   3) H₃PO₄
   4) H₂SO₃

KEY: 3
SOL: In H₃PO₄, P is in its highest O.S of +5. So it can only act as O.A and get reduced.

9. The de Broglie wavelength of an electron in the 4th Bohr orbit is:
   1) 2πa₀
   2) 8πa₀
   3) 4πa₀
   4) 6πa₀

KEY: 2
SOL: For 4th orbit \( r_n = \frac{n^2}{z} \times a_0 = \frac{4^2}{1} \times a_0 = 16a_0 \)

For deBroglie theory,

\( n\lambda = 2\pi r_n \Rightarrow 4 \times \lambda = 2\pi \times 16a_0 \)

\( \lambda = 8\pi a_0 \)

10. The correct order of heat of combustion for following alkadienes is:

   a) \[
   \begin{array}{c}
   \text{CH₃-CH=CH-CH=CH-CH₃}
   \end{array}
   \]
   b) \[
   \begin{array}{c}
   \text{CH₃-CH=CH-CH=CH-CH₃}
   \end{array}
   \]
   c) \[
   \begin{array}{c}
   \text{CH₃-CH=CH-CH=CH-CH₃}
   \end{array}
   \]
   1) (c) < (b) < (a)
   2) (b) < (c) < (a)
   3) (a) < (b) < (c)
   4) (a) < (c) < (b)

KEY: 3
SOL: Stability Order –

trans –trans > cis-trans > cis - cis
(a) (b) (c)
trans –trans < cis-trans < cis - cis
(a) (b) (a)

H.O.C & \frac{1}{\text{stability}}

11. If enthalpy of atomization for $Br_2(l)$ is $xkJ/mol$ and bond enthalpy for $Br_2$ is $ykJ/mol$, the relation between them:

1) is $x > y$  
2) is $x < y$  
3) does not exist  
4) is $x = y$

KEY: 1

SOL:

\[ \begin{align*}
\text{Br}_2(l) \quad & \quad \Delta H = x \text{ KJ/mol} \\
\text{Br}_2(g) \quad & \quad \Delta H_{\text{ap}} = Z \text{ KJ/mol} \\
\text{Br}_2(g) \quad & \quad \text{BE} = K/\text{mol}
\end{align*} \]

$x=y+z \Rightarrow x>y$

12. The electronic configurations of bivalent europium and trivalent cerium are:

(atomic number : $Xe = 54, Ce = 58, Eu = 63$)

1) $[Xe]4f^4$ and $[Xe]4f^9$  
2) $[Xe]4f^7$ and $[Xe]4f^1$  
3) $[Xe]4f^76s^2$ and $[Xe]4f^26s^2$  
4) $[Xe]4f^2$ and $[Xe]4f^7$

KEY: 2

SOL: $Eu^{2+} \rightarrow [Xe]4f^7$

$Ce^{3+} \rightarrow [Xe]4f^1$

13. The acidic, basic and amphoteric oxides, respectively, are:

1) $MgO, Cl_2O, Al_2O_3$  
2) $Cl_2O, CaO, P_4O_{10}$

3) $N_2O_3, Li_2O, Al_2O_3$  
4) $Na_2O, SO_3, Al_2O_3$
KEY: 2

SOL: Acidic Oxides: Cl₂O, P₄O₁₀, N₂O₃, SO₃ (non metal oxides)

Basic Oxides: Na₂O, MgO, Li₂O, CaO (s-block oxides except BeO)

Amphoteric oxide Al₂O₃

14. If the magnetic moment of a dioxygen species is 1.73 B.M, it may be:
   1) O₂ or O₂⁻  2) O₂ or O₂⁺  3) O₂⁻ or O₂⁺   4) O₂⁻ or O₂⁺

SOL: \( \mu = 1.73 \text{ B.M} = \sqrt{3} \text{ B.M} \Rightarrow n = 1 \Rightarrow \text{no of unpaired e} = 1 \\
\rightarrow \sigma^{1s^2}\sigma^{2s^2}\sigma^{2p_x^2}\left\{\pi^{2p_y^2}\left\{\pi^{*2p_y^*}\right\}\right\}\)

O₂⁺ or O₂⁻ \(\rightarrow 1\text{unpaired e}^-

15. The increasing order of basicity for the following intermediates is
   (from weak to strong)

SOL: basicity \(\alpha = \frac{1}{\text{stability}}\)

stability \(\text{ve charge on resonance spC stability}\)

\(\text{basicity} < (\text{iii}) < (\text{ii}) < (\text{iv}) < (\text{i})\)
16. Complex $X$ of composition $Cr(H_2O)_6Cl_2$ has a spin only magnetic moment of 3.83 BM. It reacts with $AgNO_3$ and shows geometrical isomerism. The IUPAC nomenclature of $X$ is:

1) Dichloridotetraaqua chromium (IV) chloride dihydrate
2) Tetraaquadichlorido chromium (IV) chloride dehydrate
3) Hezaaqua chromium (III) chloride
4) Tetraaquadichlorido chromium (III) chloride dihydrate

KEY: 4

SOL:

\[ \mu = 3.83 \text{B.M} = \sqrt[15]{5.15} \text{BM} \Rightarrow n = 3 \]

\[ \mu = \sqrt{n(n+2)} \]

\[ \rightarrow Cr^{3+} \rightarrow t_{1g}^{11}e_{g}^{0,0} \]

It reacts with $AgNO_3 \Rightarrow$ atleast 1 Cl is outside co-ordination sphere

It reacts with G.I. It should have atleast 2Cl inside co-ordination sphere

Complex $\left[ Cr(H_2O)_4Cl_2 \right]Cl_2H_2O$

17. The $K_{sp}$ for the following dissociation is $1.6 \times 10^{-5}$

$PbCl_{(aq)}^{2+} \rightleftharpoons Pb^{2+}_{(aq)} + 2Cl^{-}_{(aq)}$

Which of the following choices is correct for a mixture of 300 mL $0.134 \text{ M } Pb(NO_3)_2$ and 100 mL $0.4 \text{ M } NaCl$?

1) $Q < K_{sp}$
2) Not enough data provided
3) $Q > K_{sp}$
4) $Q = K_{sp}$

KEY: 3

SOL:

\[ [Pb^{2+}] = \frac{0.134 \times 300}{400} = 0.1M \]

\[ [Cl^-] = \frac{0.4 \times 100}{400} = 0.1M \]

\[ Q = [Pb^{2+}][Cl^-]^2 = 0.1 \times 0.1^2 = 10^{-3} \quad Q > K_{sp} \]
18. A chemist has 4 samples of artificial sweetener A, B, C and D. To identify these samples, he performed certain experiments and noted the following observations:

i) A and D both form blue-violet colour with ninhydrin

ii) Lassaigne extract of C gives positive $\text{AgNO}_3$ test and negative $\text{Fe}_4\left[\text{Fe(CN)}_6\right]_3$ test

iii) Lassaigne extract of B and D gives positive sodium nitroprusside test.

Based on these observations which option is correct?
1) A: Saccharin; B: Alitame; C: Sucralose; D: Aspartame
2) A: Aspartame; B: Saccharin; C: Sucralose; D: Alitame
3) A: Aspartame; B: Alitame; C: Saccharin; D: Sucralose
4) A: Alitame; B: Saccharin; C: Aspartame; D: Sucralose

KEY: 2

SOL:

19. The major product ($Y$) in the following reactions is:

$$\overset{\text{CH}_2}{\text{CH}}_3 - \text{CH} - \text{C} = \text{CH} \xrightarrow{\text{H}_2\text{SO}_4, \text{H}_2\text{O}} X \xrightarrow{(i) \text{C}_2\text{H}_5\text{MgBr}, \text{H}_2\text{O} \quad (ii) \text{Conc. H}_2\text{SO}_4/\Delta} Y$$

1) 2) 3) 4)

KEY: 3

SOL:
20. B has a smaller first ionization enthalpy than Be. Consider the following statements:

I) it is easier to remove 2p electron than 2s electron
II) 2p electron of B is more shielded from the nucleus by the inner core of electrons than the 2s electrons of Be
III) 2s electron has more penetration power than 2p electron
IV) atomic radius of B is more than Be (atomic number \( B = 5, Be = 4 \))

The correct statements are:

1) (II), (II) and (III)  
2) (I), (III) and (IV)  
3) (I), (II) and (IV)  
4) (II), (III) and (IV)

KEY: 1
SOL: At. radius ↓ from left to right across a period

21. The molarity of \( HNO_3 \) in a sample which has density 1.4 g/mL and mass percentage of 63% is ______ (Molecular weight of \( HNO_3 = 63 \))

KEY: 14.00
SOL:

\[
\text{Density} = 1.4 \text{g/ml} : \ %w/w = 63\%
\]

\[
\frac{63\text{g} \ HNO_3}{100\text{g} \ \text{solut}} = 1\text{mole} \ \text{of solut}
\]

\[
\frac{63}{63} = 1\text{mole} \ \text{of solut} \rightarrow \ \text{100ml solut}
\]

\[
\text{Molarity} = \frac{\text{m.moles}}{\text{V(in ml)}} = \frac{1000 \times 1.4}{100} = 14.00
\]

22. The mass percentage of nitrogen in histamine is ______

KEY: 37.84
SOL:

\[
\text{Structure of Histamine is } \text{N} \]

\[
\text{Molecular formula of Histamine is } C_2H_5N_2 \]

\[
\text{Molecular mass of Histamine is } 111
\]

Percentage nitrogen by mass in Histamine = \( \frac{42}{111} \times 100 = 37.84 \)

23. 108 g of silver (molar mass 108 g \( mol^{-1} \)) is deposited at cathode from \( \text{AgNO}_3(aq) \) solution by a certain quantity of electricity. The volume (in L) of oxygen gas produced at 273 K and 1 bar pressure from water by the same quantity of electricity is

KEY: 5.67
SOL:

\[ \text{Ag}^+ + e^- \rightarrow \text{Ag} \]

\[ 2H_2O \rightarrow 4H^+ + O_2 + 4e^- \quad \text{4 moles of } e^- \rightarrow 1 \text{Mole of } O_2 \]

\[ 1 \text{mole of } e^- \rightarrow \frac{1}{4} \text{Mole of } O_2 \]

\[ V = \frac{nRT}{p} = \frac{1 \times 8.314 \times 273}{100} = 22.697 L \]

For \( \frac{1}{4} \) mol of \( O_2 \), \( V = 5.67 L \)

24. The hardness of a water sample containing \( 10^{-3} M \) \( MgSO_4 \) expressed as \( CaCO_3 \) equivalents (in ppm) is _______

(molar mass of \( MgSO_4 \) is 120.37 g/mol)

KEY: 100

SOL: \( 1 \) \( l \rightarrow 10^{-3} \) moles of \( Mg^{2+} \)

\( 10^{-3} l \rightarrow 1 \) mole of \( Mg^{2+} = 1 \) mole of \( Ca^{2+} = 100 \) g of \( CaCO_3 \)

ppm hardness = 100 ppm

25. How much amount of \( NaCl \) should be added to 600 g of water \( (\rho = 1.00 \ g / mL) \) to decrease the freezing point of water to \(-0.2^0C\)?

_____ (The freezing point depression constant for water = 2 K kg \( mol^{-1} \))

KEY: 1.76

SOL: \( \Delta T_f = 0.2 K; m = \frac{\Delta T_f}{i * \Delta K_f} = \frac{0.2}{2 \times 2} = 0.05 m \)

For \( NaCl, i = 2 \)

\( 0.05 \) moles \( NaCl \rightarrow 1000 \) g of \( H_2O \)

\( 600 \) g of \( H_2O \)

\( \frac{0.05 \times 600}{1000} = 0.03 \) moles of \( NaCl \)

\( = 0.03 \times 58.5 \) g of \( NaCl \)

\( = 1.76 \) g of \( NaCl \)
MATHEMATICS

1. In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is:

1) \(\frac{15}{16}\)  
2) \(\frac{11}{16}\)  
3) \(\frac{13}{16}\)  
4) \(\frac{9}{16}\)

Key: 2

Sol: 

The possibilities are AA, (1A,1B)A, (1A,2B)A 

So the required probability will be 

\[
\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} + \frac{3!}{1!2!} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{11}{16}
\]

2. Let \(f(x)\) be any function continuous and twice differentiable function on \((a,b)\). If for all \(x \in (a,b)\), \(f'(x) > 0\) and \(f''(x) < 0\), then for any \(c \in (a,b)\), \(\frac{f(c) - f(a)}{f(b) - f(c)}\) is greater than

1) 1  
2) \(\frac{c-a}{b-c}\)  
3) \(\frac{b+a}{b-c}\)  
4) \(\frac{b-c}{c-a}\)

Key: 2

Sol: 

\(f\) is increasing and downward facing

\[
\begin{align*}
A & \quad C & \quad B \\
a & \quad c & \quad b
\end{align*}
\]

So, slope of AC will be greater than slope of BC

\[
\frac{f(c) - f(a)}{c-a} > \frac{f(b) - f(c)}{b-c} \Rightarrow \frac{f(c) - f(a)}{f(b) - f(c)} > \frac{c-a}{b-c}
\]

3. Let the observations \(x_i (1 \leq i \leq 10)\) satisfy the equations, \(\sum_{i=1}^{10} (x_i - 5) = 10\) and \(\sum_{i=1}^{10} (x_i - 5)^2 = 40\). If \(\mu\) and \(\lambda\) are the mean and variance of the observations, \(x_1 - 3, x_2 - 3, \ldots, x_{10} - 3\), then the ordered pair \((\mu, \lambda)\) is equal to:

1) \((3,6)\)  
2) \((6,3)\)  
3) \((3,3)\)  
4) \((6,6)\)
Key: 3
Sol:
\[ \text{Given } \sum_{i=1}^{10} x_i = 60 \text{ and } \sum_{i=1}^{10} x_i^3 - 10 \sum_{i=1}^{10} x_i + 250 = 40 \Rightarrow \sum_{i=1}^{10} x_i^2 = 390 \]

For the observations \( x_1 - 3, x_2 - 3, \ldots, x_{10} - 3, \)

\[ \text{Mean } = \frac{\sum_{i=1}^{10} (x_i - 3)}{10} = 3 \]

\[ \text{Variance } = \frac{\sum_{i=1}^{10} (x_i - 3)^2}{10} = \frac{\sum_{i=1}^{10} (x_i - 6)^2}{10} = \frac{\sum_{i=1}^{10} x_i^2 - 12(60) + 360}{10} = 3 \]

4. Negation of the statement: ‘\( \sqrt{5} \) is an integer or 5 is an irrational’ is

1) \( \sqrt{5} \) is irrational or 5 is an integer
2) \( \sqrt{5} \) is an integer and 5 is irrational
3) \( \sqrt{5} \) is not an integer and 5 is not irrational
4) \( \sqrt{5} \) is not an integer or 5 is not irrational

Key: 2
Sol: \( \sim (p \lor q) = (\sim p) \land (\sim q) \)

Where \( p \) is \( \sqrt{5} \) is an integer and \( q \) is 5 is an irrational

5. The value of \( I = \int_{0}^{\pi} \left( \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} \right) dx \) is equal to

Key: 2
Sol:
\[ 2I = \int_{0}^{\pi} \left( \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} + \frac{(2\pi - x) \sin^8 x}{\sin^8 x + \cos^8 x} \right) dx \]
\[ I = \int_{0}^{\pi} 2\pi \sin^8 x \cos^8 x dx = \int_{0}^{\pi/2} 4\pi \sin^8 x \cos^8 x dx \]
\[ 2I = 4\pi \int_{0}^{\pi/2} \left( \frac{\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{\cos^8 x}{\sin^8 x + \cos^8 x} \right) dx \]
\[ \Rightarrow I = 2\pi \int_{0}^{\pi/2} 1dx = 2\pi \times \frac{\pi}{2} = \pi^2 \]

6. Let C be the centroid of the triangle with vertices (3, -1), (1, 3) and (2, 4). Let P be the point of intersection of the lines \( x + 3y - 1 = 0 \) and \( 3x - y + 1 = 0 \). Then the line passing through the points C and P also passes through the point:

1) (7, 6) 2) (-9, -6) 3) (9, 7) 4) (-9, -7)

Key: 2
Sol: Given \( C = \left( \frac{3+1+2}{3}, \frac{-1+3+4}{3} \right) = (2,2) \)

The line passing through P and C can be written as \((x+3y-1)+\lambda(3x-y+1)=0\)

C lies on it \(\Rightarrow 7+5\lambda = 0 \Rightarrow \lambda = -\frac{7}{5}\)

So, the equation will be \(5(x+3y-1)-7(3x-y+1)=0\) or \(-2(8x-11y+6)=0\)

\((-9,-6)\) will lie on this line

7. If

\[
 f(x) = \begin{cases} 
 \frac{\sin(a+2)x+\sin x}{x}; & x < 0 \\
 b; & x = 0 \\
 \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{x^{\frac{2}{3}}}; & x > 0 
\end{cases}
\]

Is continuous at \(x=0\), then \(a+2b\) is equal to:

1) 2  
2) 0  
3) -1  
4) 1

Key: 2

Sol: We have \(LHL = a + 3, f(0) = b\) and \(RHL = \lim_{h \to 0} \left( \frac{1 + 3h^{\frac{1}{3}} - 1}{h} \right) = 1\)

For it to be continuous, \(LHL = f(0) = RHL\)

So, we get \(a = -2, b = 1\) and \(a + 2b = 0\)

8. If the matrices \(A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}\), \(B=\text{adj} \ A\) and \(C=3A\), then \(\frac{\text{adj} \ B}{|C|}\) is equal to:

1) 16  
2) 2  
3) 8  
4) 72

Key: 3

Sol: We have \(|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = (9+4) - 1(3-4) + 2(-1-3) = 13 + 1 - 8 = 6\)

\(|\text{adj} \ B| = |\text{adj} (\text{adj} A)| = |\text{adj} A|^{n-1} = |A|^{n-1} = |A|^4 = 6^4\)

\(|C| = |3A| = 3^3 \times 6\) \(\frac{\text{adj} B}{|C|} = \frac{6^4}{3^3 \times 6} = 8\)
9. If \( f'(x) = \tan^{-1}(\sec x + \tan x), -\frac{\pi}{2} < x < \frac{\pi}{2}, \) and \( f(0) = 0, \) then \( f(1) \) is equal to:

1) \( \frac{1}{4} \)  
2) \( \frac{\pi + 2}{4} \)  
3) \( \frac{\pi - 1}{4} \)  
4) \( \frac{\pi + 1}{4} \)

Key: 4

Sol:  
\[
 f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right) 
\]

\[
 = \tan^{-1}\left(\frac{1 - \cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)}\right) = \tan^{-1}\left(2\sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) 
\]

\[
 = \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2} 
\]

\[
\int f'(x)\,dx = \int\left(\frac{\pi}{4} + \frac{x}{2}\right)\,dx 
\]

\[
f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c 
\]

\[
f(0) = c = 0 \Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4} 
\]

So, \( f(1) = \frac{\pi + 1}{4} \)

10. The integral \( \int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}} \) is equal to: (where C is a constant of integration)

1) \( -\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{\frac{13}{7}} + C \)  
2) \( \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C \)  
3) \( -\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C \)  
4) \( \frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C \)

Key: 2

Sol:  
\[
 I = \int \left(\frac{x-3}{x+4}\right)^{\frac{7}{2}} \frac{1}{(x+4)^{\frac{2}{7}}} \,dx 
\]

Put \( \frac{x-3}{x+4} = t \)  \( \Rightarrow \frac{7}{(x+4)^{\frac{2}{7}}} \,dx = 7t^6 \,dt \)

So, \( I = \int t^6 \,dt = t + c = \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c \)
11. The value of \( \cos^3\left(\frac{\pi}{8}\right) \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) \sin\left(\frac{3\pi}{8}\right) \) is:

1) \( \frac{1}{4} \)  
2) \( \frac{1}{\sqrt{2}} \)  
3) \( \frac{1}{2} \)  
4) \( \frac{1}{2\sqrt{2}} \)

Key: 4

Sol: 
\[
\cos^3\left(\frac{\pi}{8}\right) \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) \sin\left(\frac{3\pi}{8}\right) = 4\cos^6\left(\frac{\pi}{8}\right) - 4\sin^6\left(\frac{\pi}{8}\right) \cos^4\left(\frac{\pi}{8}\right) + 3\sin^4\left(\frac{\pi}{8}\right)
\]

12. If the number of five digit numbers with distinct digits and 2 at the 10th place is 336 \( k \), then \( k \) is equal to:

1) 4  
2) 8  
3) 6  
4) 7

Key: 2

Sol: 
Number of numbers = \( 8 \times 8 \times 7 \times 6 = 2688 = 336k \Rightarrow k = 8 \)

13. The product \( 2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32} \ldots \) to \( \infty \) is equal to:

1) 1  
2) \( \frac{1}{2} \)  
3) \( \frac{1}{4} \)  
4) 2

Key: 3

Sol: 
\[
2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32} \ldots \infty = 2^{2^{1/8}} \cdot 2^{2^{1/16}} \cdot 2^{2^{1/32}} \ldots \infty = 2^{1-1/(2^n)} = \sqrt{2}
\]

14. Let \( z \) be a complex number such that \( \left| \frac{z-i}{z+2i} \right| = 1 \) and \( |z| = \frac{3}{2} \), then the value of \( |z+3i| \) is:

1) \( \sqrt{10} \)  
2) \( 2\sqrt{3} \)  
3) \( \frac{15}{4} \)  
4) \( \frac{7}{2} \)

Key: 4

Sol: 
\[
|z-i|^2 = |z+2i|^2 \Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2 \Rightarrow -2y+1 = 4y+4 \Rightarrow y = -\frac{1}{2}
\]
\[
|z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4} \Rightarrow x^2 = \frac{24}{4} = 6 \Rightarrow x = \pm\sqrt{6}
\]
\[ z = \pm \sqrt{6} - \frac{i}{2} \]
\[ |z + 3i| = \left| \pm \sqrt{6} + \frac{5i}{2} \right| = \sqrt{6 + \frac{25}{4}} = \frac{7}{2} \]

15. If \( e_1 \) and \( e_2 \) are the eccentricities of the ellipse, \( \frac{x^2}{18} + \frac{y^2}{4} = 1 \) and the hyperbola, \( \frac{x^2}{9} - \frac{y^2}{4} = 1 \) respectively and \( (e_1, e_2) \) is a point on the ellipse, \( 15x^2 + 3y^2 = k \), then \( k \) is equal to:

1) 17  
2) 16  
3) 15  
4) 14

Key: 2

Sol: We have \( e_1 = \sqrt{1 - \frac{4}{18}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{21}}{3} \) and \( e_2 = \sqrt{1 + \frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3} \)

Given \( (e_1, e_2) \) lie on \( 15x^2 + 3y^2 = k \) \( \Rightarrow 15e_1^2 + 3e_2^2 = k \) \( \Rightarrow k = 15 \left( \frac{7}{9} \right) + 3 \left( \frac{13}{9} \right) \)

\( \therefore k = 16 \)

16. A circle touches the y-axis at the point \( (0, 4) \) and passes through the point \( (2, 0) \). Which of the following lines is not a tangent to this circle?

1. \( 3x-4y-24=0 \)
2. \( 3x+4y-6=0 \)
3. \( 4x+3y-8=0 \)
4. \( 4x-3y+17=0 \)

Key: 3

Sol: Using family of circles, the required circle will be of the form \( (x-0)^2 + (y-4)^2 + \lambda x = 0 \)

Given, it passes through \( (2,0) \) \( \Rightarrow 4 + 16 + 2\lambda = 0 \) \( \Rightarrow \lambda = -10 \)

So, the circle will be \( x^2 + y^2 - 10x - 8y + 16 = 0 \) having centre at \( (5,4) \) and radius as \( \sqrt{5^2 + 4^2 - 16} = 5 \). For a line to be tangent, perpendicular distance from centre should be equal to radius. So, check the options and we get \( 3x + 4y - 6 = 0 \) as tangent

So, check the options and we get \( 4x + 3y - 8 = 0 \) is not tangent

17. If for some \( \alpha \) and \( \beta \) in \( R \), the intersection of the following three planes

\( x + 4y - 2z = 1 \)
\( x + 7y - 5z = \beta \)
\( x + 5y + \alpha z = 5 \)

is a line in \( R^3 \), then \( \alpha + \beta \) is equal to:

1) 0  
2) 10  
3) 2  
4) -10
20. Given 3 planes has line of intersection which means infinite solutions

\[ \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \]

\[ \Rightarrow (7\alpha + 25) - (4\alpha + 10) + (-20 + 14) = 0 \Rightarrow \alpha = -3 \]

Also \( D_z = 0 \Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} \]

\[ \Rightarrow 1(35 - 5\beta) - (15) + 1(4\beta - 7) = 0 \Rightarrow \beta = 13 \]

\( \alpha + \beta = -3 + 13 = 10 \)

18. The number of real roots of the equation \( e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0 \) is:

1. 3  
2. 2  
3. 4  
4. 1

Key: 4

Sol:

Let \( e^x = t, t \in (0, \infty) \)

Given equation will be \( t^4 + t^3 - 4t^2 + t + 1 = 0 \)

Divide by \( t^2 \) to get \( t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0 \Rightarrow \left(t^2 + \frac{1}{t}\right) + \left(t + \frac{1}{t}\right) - 4 = 0 \)

Let \( t + \frac{1}{t} = \alpha, \alpha \in [2, \infty) \Rightarrow t^2 + \frac{1}{t^2} = \alpha^2 - 2 \)

So, the equation will be \( (\alpha^2 - 2) + \alpha - 4 = 0 \)

\( \alpha^2 + \alpha - 6 = 0 \Rightarrow \alpha = -3, 2 \)

\( \therefore \alpha \in [2, \infty), \text{we get} \alpha = 2 \Rightarrow e^x + e^{-x} = 2 \)

So, \( x = 0 \) only solution

19. If for all real triplets \((a, b, c)\), \( f(x) = ax + bx + cx^2 \); then \( \int_0^1 f(x)dx \) is equal to:

1) \( 2\left\{3f(1) + 2f\left(\frac{1}{2}\right)\right\} \)
2) \( \frac{1}{2}\left\{f(1) + 3f\left(\frac{1}{2}\right)\right\} \)
3) \( \frac{1}{3}\left\{f(0) + f\left(\frac{1}{2}\right)\right\} \)
4) \( \frac{1}{6}\left\{f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right\} \)

Key: 4

Sol:

\( \int_0^1 (a + bx + cx^2)dx = \left[ ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1 = a + \frac{b}{2} + \frac{c}{3} \)
We have \( f(1) = a + b + c, \ f(0) = a \) and \( f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4} \)

Now \( \frac{1}{6}\left(f(1) + f(0) + 4f\left(\frac{1}{2}\right)\right) = \frac{1}{6}\left(a + b + c + 4\left(a + \frac{b}{2} + \frac{c}{4}\right)\right) \)

\[= \frac{1}{6}(6a + 3b + 2c) = a + \frac{b}{2} + \frac{c}{3} \]

20. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 \( \text{cm}^3/\text{min} \). When the thickness of ice is 5 cm, then the rate (in \( \text{cm/min.} \)) at which the thickness of ice decreases, is:

1) \( \frac{1}{18\pi} \)  
2) \( \frac{1}{36\pi} \)  
3) \( \frac{5}{6\pi} \)  
4) \( \frac{1}{54\pi} \)

Key: 1

Sol: Let thickness = \( x \ \text{cm} \)

Total volume \( V = \frac{4}{3}\pi(10 + x)^3 \)

\[\frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}\]

Given \( \frac{dV}{dt} = -50 \ \text{cm}^3/\text{min} \)

At \( x = 5 \ \text{cm} \), we get \( -50 = 4\pi(10 + 5)^2 \frac{dx}{dt} \)

\[\frac{dx}{dt} = -\frac{1}{18\pi} \ \text{cm/min} \]

21. The coefficient of \( x^4 \) in the expansion of \((1 + x + x^2)^{10}\) is__________

Key: 615

Sol: General term = \( \frac{10!}{\alpha!\beta!\gamma!} x^{\beta+2\gamma} \), where \( \alpha + \beta + \gamma = 10 \)

For coefficient of \( x^4 \), \( \beta + 2\gamma = 4 \)

So, the possible sets will be

\[\gamma = 0, \beta = 4, \alpha = 6 \quad \Rightarrow \quad \frac{10!}{6!4!0!} = 210\]

\[\gamma = 1, \beta = 2, \alpha = 7 \quad \Rightarrow \quad \frac{10!}{7!2!1!} = 360\]

\[\gamma = 2, \beta = 0, \alpha = 8 \quad \Rightarrow \quad \frac{10!}{8!0!2!} = 45\]

Total = 615
22. The projection of the line segment joining the points (1, -1, 3) and (2, -4, 11) on the line joining the points (-1, 2, 3) and (3, -2, 10) is ________
Key: 8
Sol: $\overline{AB} = (i) - (3j) + 8k$
$\overline{CD} = 4i - 4j + 7k$
Projection of $\overline{AB}$ on $\overline{CD}$ will be $\frac{\overline{AB}.\overline{CD}}{|\overline{CD}|} = \frac{4+12+56}{\sqrt{16+16+49}} = \frac{72}{9} = 8$

23. The number of distant solutions of the equation, $\frac{\log \sin x}{\pi} = 2 - \frac{\log \cos x}{\pi}$ in the interval $[0, 2\pi]$, is________
Key: 8
Sol: $\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}} |\cos x|$
$\log_{\frac{1}{2}} |\sin x| \cos x| = 2$
$|\sin x \cos x| = \frac{1}{4}$
$\sin 2x = \pm \frac{1}{2}, 2x \in [0, 4\pi]$
Number of solutions = 8

24. If for $x \geq 0, y = y(x)$ is the solution of the differential equation,
$(x+1)dy = ((x+1)^2 + y-3)dx, y(2) = 0$, then $y(3)$ is equal to________
Key: 3
Sol: $\frac{dy}{dx} = (1+x) + \left(\frac{y-3}{1+x}\right)$
$\frac{dy}{dx} - \frac{1}{1+x}y = (1+x) - \frac{3}{1+x}$
I.F. = $e^{-\int \frac{1}{1+x} dx} = \frac{1}{1+x}$
So, the solution will be $\frac{y}{1+x} = \int \left[1 - \frac{3}{(1+x)^2}\right] dx$
\[
\frac{y}{1+x} = x + 3(1+x)^{-1} + c
\]
\[
y = (1+x) \left[ x + \frac{3}{(1+x)} + c \right]
\]
At \( x = 2, y = 0 \)
\[
0 = 3(2 + 1 + c) \Rightarrow c = -3 \quad \text{At} \ x = 3, y = 3
\]
25. If the vectors, \( \vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}, \vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k} \) and \( \vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k} \) \( (a \in R) \) are coplanar and \( 3(\vec{p} \cdot \vec{q})^2 - \lambda |\vec{r} \times \vec{q}|^2 = 0 \), then the value of \( \lambda \) is ____________

**Key:** 1

**Sol:** 3 vectors \( \vec{p}, \vec{q}, \vec{r} \) are coplanar \( \Rightarrow \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0 \Rightarrow a + 1 + a + a = 0 \Rightarrow a = -\frac{1}{3} \)

So, \( \vec{p} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k}, \vec{q} = \frac{1}{3}(\hat{i} + 2\hat{j} - \hat{k}), \vec{r} = \frac{1}{3}(-\hat{i} - \hat{j} + 2\hat{k}) \)

\[
\vec{p} \cdot \vec{q} = \frac{2}{9}(-2 - 2 + 1) = -\frac{1}{3}
\]

\[
\vec{r} \times \vec{q} = \frac{1}{9} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -1 & -1 & 2 \end{vmatrix} = \frac{1}{9}((\hat{i}(-1) - \hat{j}(2 - 1) + \hat{k}(1 + 2))
\]
\[
= \frac{1}{9}(3\hat{i} + 3\hat{j} + 3\hat{k}) = \frac{\hat{i} + \hat{j} + \hat{k}}{3}
\]

\[
\lambda = \frac{3(\vec{p} \cdot \vec{q})^2}{|\vec{r} \times \vec{q}|^2} = 1
\]

Prepared by

**Sri Chaitanya Faculty**