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(2:30 pm - 5:30 pm)

Question Paper

Solutions

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PHYSICS

1. **The number density of molecules of a gas depends on their distance r from the origin as, $n(r)=n_0e^{-\alpha r^4}$. Then the total number of molecules is proportional to**

1) $n(r)=n_0-\alpha^{-3/4}$ **2)** $\sqrt{n_0}\alpha^{1/2}$ **3)** $n_0\alpha^{-3}$ **4)** $n_0\alpha^{1/4}$

Ans. **1**

Sol. $dN=n(r)dv\ dN=n_0e^{-\alpha r^4}\left(4\pi r^2dr\right)\ N=\int dN=4\pi N_0\int e^{-\alpha r^4}r^2dr$

2. **A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due t gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals**

1) 9 **2) 1/9** **3) 1/27** **4) 27**

Ans. **1**

Sol. $V_T=\frac{2}{9}\frac{r^2(p-\sigma)g}{\eta}\ V_T\alpha r^2\quad R_{big}=(27)^{1/3}\ r_{small}$

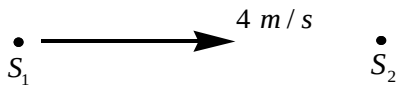
$R_{big}=3r_{small}\ \frac{v_1}{v_2}=\left(\frac{r_1}{r_2}\right)^2=\frac{R^2}{(R/3)^2}=9$

3. **Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards v with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals**

1) 2.5 m/s **2) 10.0 m/s** **3) 5.5 m/s** **4) 15.0 m/s**

Ans. **1**

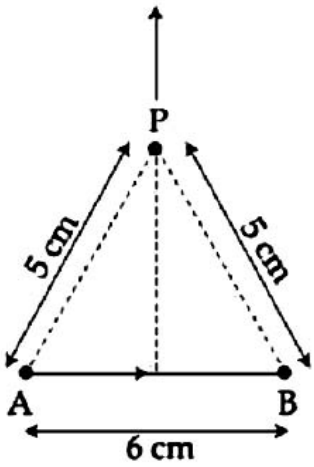
Sol.



$f^1=\left(\frac{V-u}{V}\right)f\quad f^x=\left(\frac{V+u}{V}\right)f\quad f^x-f^1=\left(\frac{V+u-V+u}{V}\right)f=\frac{2uf}{V}$

$10=\frac{2u\times660}{330}\qquad u=\frac{10}{4}=2.5\ \ m/s$

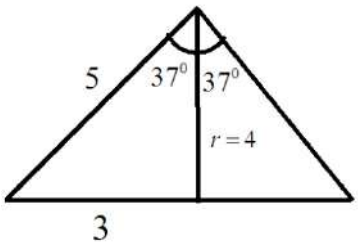
4. **Find the magnetic field at point P due to a straight line segment AB of length 6 cm carring a current of 5 A.(See figure)** $\left(\mu_0 = 4\pi \times 10^{-7}\, N - A^{-2}\right)$



1) $3.0\times10^{-5}T$ **2)** $2.0\times10^{-5}T$ **3)** $2.5\times10^{-5}T$ **4)** $1.5\times10^{-5}T$

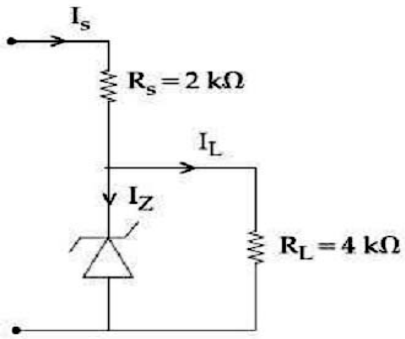
Ans. **4**

Sol. $B = \dfrac{\mu_0 i}{4\pi r}\left(\sin\theta_1 + \sin\theta_2\right)$



$$B = \frac{10^{-7}\times 5}{4\times 10^{-2}} \times 2 \times \frac{3}{5} = 1.5\times 10^{-5} T$$

5. **Figure shows a DC voltage regulator circuit, with a zener diode down voltage** $= 6V$**. If the unregulated input voltage varies between** $10V$ **to** $16V$**, then what is the maximum zener current?**



1) $3.5mA$ **2)** $7.5mA$ **3)** $1.5mA$ **4)** $2.5mA$

Ans. **1**

Sol. $I_{\max} = \dfrac{16-6}{2} = 5mA\; I_L = \dfrac{6}{4} = 1.5mA\; I_{zmax} = I_{\max} - I_L = 3.5mA$

6. **A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an anometer of range 0 to $I_0\left(I_0 < I_g\right)$ by connecting a shunt resistance R_A to it and connecting a shunt resistance R_A to is and (ii) into a voltmeter of range 0 to $V\left(V = GI_0\right)$ by connecting a series of resistance R_V to it. Then,**

1) $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ **and** $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$ **2)** $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ **and** $\frac{R_A}{R_V} = \left(\frac{I_g}{\left(I_0 - I_g\right)}\right)^2$

3) $R_A R_V = G^2$ **and** $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$ **4)** $R_A R_V = G^2$ **and** $\frac{R_A}{R_V} = \frac{I_g}{\left(I_0 - I_g\right)}$

Ans. **3**

Sol. $$S = \frac{G}{\frac{i}{ig} - 1} \Rightarrow R_A = \frac{G}{\frac{I_0}{I_g} - 1} \quad R_V = G\left[\frac{V}{V_g} - 1\right] \Rightarrow R_V = G\left[\frac{V}{i_g G} - 1\right]$$

$$R_V = G\left[\frac{i_0}{i_g}\right] - 1 \qquad R_V = G\left[\frac{i_0}{i_g} - 1\right] = G\left[\frac{G}{R_A}\right] \; R_A R_V = G^2$$

$$\frac{R_A}{R_V} = \frac{G / \frac{i_0}{i_g} - 1}{G / \left[\frac{i_0}{i_g} - 1\right]} = \frac{1}{\left[\frac{i_0}{i_g} - 1\right]^2} \quad \frac{R_A}{R_V} = \frac{i_g^2}{\left(i_0 - i_g\right)^2}$$

7. **Half lives of two radio active nuclei A and B are 10 minutes and 20 minutes, respectively. If , initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed, the ratio of decayed numbers of nuclei A and B will be:**

1) 1:8 **2) 9:8** **3) 8:1** **4)3:8**

Ans. **2**

Sol.

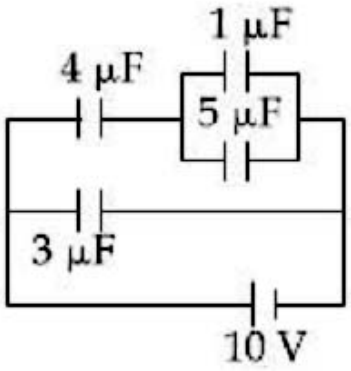
Initial	A	B
No		No
$T_{1/2}$	10 min	20 min
t	60 min	60min
n=	$6T_{1/2}$	$3T_{1/2}$

$$N = N_0 e^{-\lambda t}$$

$$N_{decayed} = N_0 - N = No\left(1 - e^{\lambda t}\right)$$

$$\frac{N_1}{N_2}=\frac{1-e^{-\frac{\ln 2(60)}{10}}}{1-e^{-\frac{\ln 2(60)}{20}}}=\frac{0.984375}{0.825}\ \frac{N_1}{N_2}=1.125\ \frac{N_1}{N_2}=9/8$$

8. **In the given circuit, the charge on $4\mu F$ capacitor will be:**



1) $9.6\,\mu C$ **2)** $24\,\mu C$ **3)** $5.4\,\mu C$ **4)** $13.4\,\mu C$

Ans. **2**

Sol. $C_{eqa}=\frac{6\times 4}{6+4}+2=5.4\ Q=C_aV=5.4\mu C\ \text{OR}\ V_4:V_6=6:4=3:2$

$$V_4=\frac{3}{5}\times 10=6V\ Q_4=4V_4=24\mu C$$

9. **A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect $120\,dB$ in intensity sound? [Given reference intensity of sound as $10^{-12}W/\text{m}^1$]**

1) $10\,\text{cm}$ **2)** $20\,\text{cm}$ **3)** $40\,\text{cm}$ **4)** $30\,\text{cm}$

Ans. **3**

Sol. $\beta=10\log\left(\frac{I}{I_0}\right)\ 120=10\log\left(\frac{I}{I_0}\right)\ 10^{12}=\frac{I}{I_0}$

$$I=1=\frac{\rho}{4\pi d^2}\ d^2=\frac{2}{4\pi}\ d=\frac{1}{\sqrt{2\pi}}=0.3989\,m\ d\approx 40\,cm$$

10. **A diatomic gas with rigid molecules does $10\,J$ of work when expanded at constant pressure. When would be the heat energy absorbed by the gas, in this process?**

1) $40\,J$ **2)** $30\,J$ **3)** $35\,J$ **4)** $25\,J$

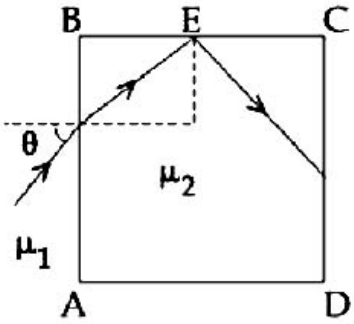
Ans. **3**

Sol. $dQ_P=nC_PdT,\ \gamma=7/5$

$$\frac{dw}{dQ_p}=1-\frac{1}{\gamma}=1-\frac{5}{7}=\frac{2}{7}$$

$$dQ_P = \frac{7}{2} \times 10 = 35\,J$$

11. **A transport cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1 (\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC.**

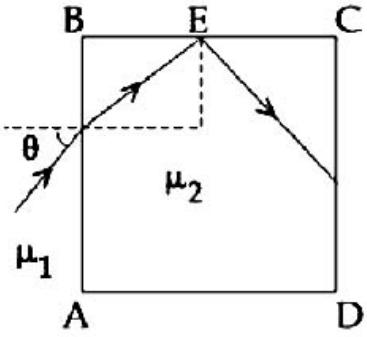


Then θ must satisfy:

1) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$ **2)** $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$ **3)** $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ **4)** $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

Ans. **2**

Sol.



$$\sin C = \frac{\mu_1}{\mu_2} \quad r = 90 - C \quad \mu_1 \sin\theta = \mu_2 \sin C = \mu_2\sqrt{1 - \sin^2 C}$$

$$\mu_1 \sin\theta = \sqrt{\mu_2^2 - \mu_1^2} \quad \sin\theta = \sqrt{\left(\frac{\mu_2}{\mu_1}\right)^2 - 1} \quad \theta \le \sin^{-1}\sqrt{\left(\frac{\mu_0}{\mu_1}\right)^2 - 1}$$

12. **The ratio of the weight of a body on the Earth's surface to that on the surface of a planet is $9:4$, the mass of the planet is $\frac{1}{9}th$ of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet? (Take the planets to have the same mass density)**

1) $\frac{R}{2}$ **2)** $\frac{R}{9}$ **3)** $\frac{R}{3}$ **4)** $\frac{R}{4}$

Ans. **1**

Sol. $w = ms = \frac{GMm}{R^2}\frac{W_E}{W_P} = \frac{m_E}{m_P} \times \left(\frac{R_P}{R_E}\right)^2 \frac{R_P}{R_E} = \frac{1}{2} \Rightarrow R_P = \frac{R}{2}$

13. **A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equal Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The co-efficient of volume expansion, of the material of the rod, is (nearly) equal to:**

1) $F/\left(3\pi r^2 YT\right)\frac{R}{2}$ **2)** $6F/\left(\pi r^2 YT\right)$ **3)** $9F/\left(\pi r^2 YT\right)$ **4)** $3F/\left(\pi r^2 YT\right)$

Ans. **4**

Sol. $F = YA\alpha\Delta T \;\; \alpha = \frac{F}{YA\Delta T} \;\; Y = 3\alpha = \frac{3F}{YA\Delta T} = \frac{3F}{Y\pi r^2 T}$

14. **Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights,** h_1 **and** h_2 **which of the following is correct?**

1) $R^2 = 4h_1h_2$ **2)** $R^2 = 2h_1h_2$ **3)** $R^2 = h_1h_2$ **4)** $R^2 = 16h_1h_2$

Ans. **4**

Sol. $R = \frac{u^2 \sin 2\theta}{g} \qquad h_1 = \frac{u^2 \sin^2\theta}{2g}, h_2 = \frac{u^2 \cos^2\theta}{2g}$

$h_1h_2 = \frac{u^4 \sin^2\theta\cos^2\theta}{4g} \times \frac{4}{4} \; h_1h_2 = \frac{R^2}{16}$

15. **In an amplitude modulator circuit, the carrier wave is given by,** $C(t) = 4\sin\left(2000\pi t\right)$ **while modulating signal is given by** $m(t) = 2\sin\left(2000\pi t\right)$**. The values of modulation index and index and lower side band frequency are:**

1) 0.5 and $9kHz$ **2)** 0.2 and $9\,kHz$ **3)** 0.5 and $10\,kHz$ **4)** 0.4 and $10\,kHz$

Ans. **1**

Sol. $M = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{Am}{A_c} = \frac{2}{4} = \frac{1}{2} = 0.5 \text{LSB} = f_c - f_m = 9KHz$

16. **A particle is moving with speed** $\upsilon = b\sqrt{x}$ **along positive** $x-axis$**. Calculate the speed of the particle at time** $t-\tau$ **(assume that the particle is origin at** $t=0$**)**

1) $\frac{b^2\tau}{\sqrt{2}}$ **2)** $\frac{b^2\tau}{4}$ **3)** $b^2\tau$ **4)** $\frac{b^2\tau}{2}$

Ans. **4**

Sol. $V = b\sqrt{x} \; \frac{dx}{dt} = b\sqrt{x}\int_0^x \frac{dx}{\sqrt{x}} = b\int_0^\tau dt \; 2\sqrt{x} = b\tau \; \sqrt{x} = b\tau/2 \; V = \frac{b^2\tau}{2}$

17. **The electron in a hydrogen atom first jumps from the third excited state to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2 of the photons emitted in this process is:**

1) $20/7$ **2)** $9/7$ **3)** $7/5$ **4)** $27/5$

Ans. **1**

Sol. $\frac{1}{\lambda}=R(1)^2\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)\ \frac{1}{\lambda_1}=R(1)^2\left(\frac{1}{3^2}-\frac{1}{9^2}\right)\ \frac{1}{\lambda_2}=R(1)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$

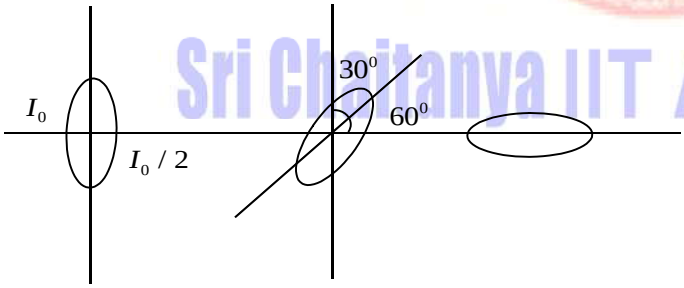
$$\frac{\lambda_1}{\lambda_2}=\frac{\frac{1}{4}-\frac{1}{9}}{\frac{1}{9}-\frac{1}{16}}=\frac{5/36}{7/9\times16}=\frac{5}{7}\times\frac{9\times16}{30}=\frac{20}{7}$$

18. **A system of three polarizers P_1,P_2,P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1, The pass axis of P_2, when a beam of unpolarized light of intensity I_O is incident on P_1, the intensity of light transmitted by the three polarizers of light transmitted by the three polarizers is 1. The ratio (I_O/I) equal (nearly):**

1)5.33 **2) 16.00** **3) 10.67** **4) 1.80**

Ans. **3**

Sol.



$$I=\frac{I_0}{2}\cos^2 30^0\cos^2 60\ I=\frac{I_0}{2}\times\frac{3}{4}\times\frac{1}{4}\quad\frac{I_0}{I}=\frac{32}{3}$$

19. **consider an electron in hyfrogen atom, revolving in its second excited state (havingradius $4.65A^0$). The de-Broglie wavelength of this electrons is:**

1) $9.7A^0$ **2)** $6.6A^0$ **3)** $3.5A^0$ **4)** $12.9A^0$

Ans. **1**

Sol. $n\lambda=2\pi r_n\ 3\lambda=2\pi\times4.65A^0\ \lambda=9.7A^0$

20. **Let a total charges 2 Q be distributed in a sphere of radius R, with the charge density given by $p(r)=kr$, where r is the distance from the centre. Two charges A and B, of $-Q$ each are placed on diametrically opposite points, at equal distance, a from the centre. If A and B not experience any force, then:**

1) $a=\frac{3R}{2^{1/4}}$ **2)** $a=R/\sqrt{3}$ **3)** $a=2^{-1/4}R$ **4)** $a=8^{-1/4}R$

Ans. **4**

Sol. $\rho(r)=Kr\ Q_{total}=2Q=\int_0^R\rho(r)\left(4\pi r^2dr\right)\ 2Q=4\pi k\int_0^R r^3dr\ 2Q=4\pi k\left(\frac{R^4}{4}\right)$

$Q=\frac{4kR^2}{2}\Rightarrow K=\frac{2Q}{\pi R^4}$ Force between charges –Q and Q $F=\frac{1}{4\pi\in_0}\ \frac{Q^2}{(2a)^2}=\frac{1}{4\pi\in_0}\frac{Q^2}{4a^2}$

$E\int ds=\frac{q_{in}}{\in_0}\ E\left(4\pi a^2\right)=\int_0^a\frac{\rho dv}{\in_0}\ E=\frac{Qa^2}{2\pi\in_0 R^4}\ F=QE\ \frac{Q^2}{16\pi\in_0 a^2}=\frac{Q^2a^2}{2\pi\in_0 R^n}\ a=8^{-1/4}R$.

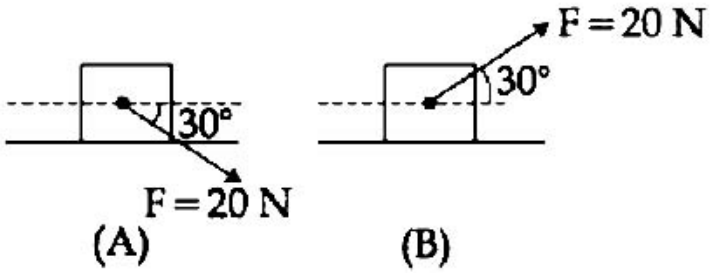
21. **A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (υ) in air by resonance tube method. Resonance is observed to occur at two successive length of the air column, $l_1=30\,cm$ and $l_2=70\,cm$. Then, υ is equal to:**

1) $332\,ms^{-1}$ **2)** $379\,ms^{-1}$ **3)** $338\ ms^{-1}$ **4)** $384\ ms^{-1}$

Ans. **4**

Sol. $V=2n\left(l_2-l_1\right)=2\times480\left(40\times10^{-2}\right)\ V=384\,m/s$

22. **A block of mass $5Kg$ is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F=20\,N$, making an angle of 30^0 with the horizontal, as shown in the figures. The co-efficient of friction between the block and floor is $\mu=0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be: $\left(g=10\,ms^{-2}\right)$**



1) $0.8\,ms^{-2}$ **2)** $0\,ms^{-2}$ **3)** $3.2\ ms^{-2}$ **4)** $0.4\ ms^{-2}$

Ans. **1**

Sol. CASE:-(I) $F_{net}=F\cos30^0-f_{K_1}\ ma_1=20\times\frac{\sqrt{3}}{2}-0.2N_1\ 5a_1=10\sqrt{3}-0.2\left[50+20\sin30^0\right]$

$a_1=2\sqrt{3}-\frac{12}{5}=1.06\,m/s^2$ CASE:-(II) $F_{net}=F\cos30^0-f_{K_2}$

$5a_2=10\sqrt{3}-0.2\left[50-20\sin30^0\right]\ a_2-a_1=1.86-1.06=0.8m/s^2$

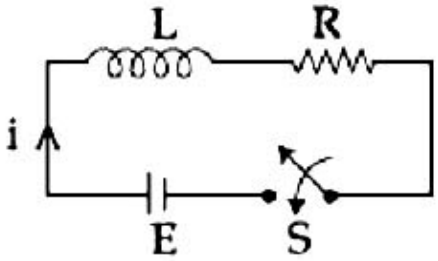
23. **A spring whose unstretched length is *l* has a force constant *k*. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be:**

1) n **2)** n^2 **3)** $\frac{1}{n^2}$ **4)** $\frac{1}{n}$

Ans. **4**

Sol. $Kl = \cos t \; \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{1}{n}$

24. **Consider the LR circuit shown in the figure. If the switch S is closed at $t = 0$ then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is:**



1) $\frac{2.7\,EL}{R^2}$ **2)** $\frac{EL}{7.3R^2}$ **3)** $\frac{EL}{2.7R^2}$ **4)** $\frac{7.3\,EL}{R^2}$

Ans. **1**

Sol. $i = \frac{\varepsilon}{R}\left[1 - e^{-Rt/L}\right] \; q = \int_0^{L/R} i\,dt = \left[\frac{\varepsilon}{R}t - \frac{\varepsilon}{R}\left(\frac{e^{-Rt/L}}{-R/L}\right)\right]^{L/R}$

$= \frac{\varepsilon}{R} \times \frac{L}{R} + \frac{EL}{R^2}e^{-1} + \frac{\varepsilon L}{R^2} = \frac{\varepsilon L}{R^2}\left[2 + \frac{1}{e}\right] = \frac{\varepsilon L}{R^2}[2 + 0.37] \; q = \frac{2.37\varepsilon L}{R^2}$

25. **A plane electromagnetic wave having a frequency $v = 23.9\,GHz$ propagates along the positive *z*-direction in free space. The peak value of the electric field is $60\,V/m$. which among the following is the acceptable magnetic field component in the electromagnetic wave ?**

1) $\vec{B} = 2\times10^{-7}\sin\left(0.5\times10^3_z - 1.5\times10^{11}t\right)\hat{i}$

2) $\vec{B} = 2\times10^{-7}\sin\left(0.5\times10^2 x + 1.5\times10^{11}t\right)\hat{i}$

3) $\vec{B} = 2\times10^{-7}\sin\left(0.5\times10^3 z + 1.5\times10^{11}t\right)\hat{i}$

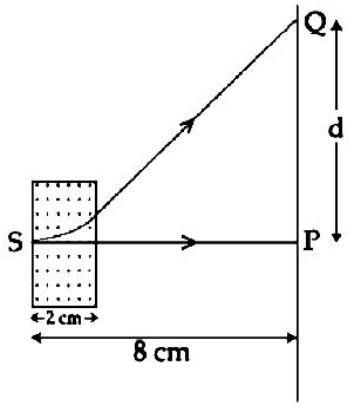
4) $\vec{B} = 60\sin\left(0.5\times10^3 z + 1.5\times10^{11}t\right)\hat{i}$

Ans. **1**

Sol. $E_0 = 60V/M \;\; B_0 = \frac{E_0}{C} = \frac{60}{3\times10^8} = 2\times10^{-7}T \;\; B = B_0\sin\left(Kx - \omega t\right) \; K = \frac{2\pi}{\lambda} = \frac{2\pi f}{C}$

$= \frac{2\pi \times 23.9 \times 10^9}{3 \times 10^8} = 500\, m^{-1}\ K = 0.5 \times 10^3\, m^{-1}\ w = 2\pi f = 150 \times 10^9 = 1.5 \times 10^{11}\, rad/\sec$

26. **An electron, moving along the $x-axis$ with an initial energy of $100\,eV$, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3} T)\hat{k}$ at S (See figure). The field extends between $x = 0$ and $x = 2\,cm$. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is: (electron's charge $= 1.6 \times 10^{-19} C$, mass of electron $= 9.1 \times 10^{-31} kg$)**



1) $12.87\,cm$ **2)** $2.25\,cm$ **3)** $11.65\,cm$ **4)** $1.22\,cm$

Ans. **1**

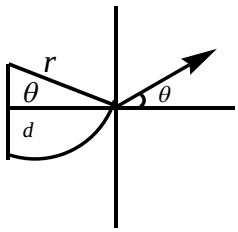
Sol. $\frac{1}{2}mv^2 = K = 100\,ev\ \frac{1}{2} \times 9.1 \times 10^{-31} v^2 = 100 \times 1.6 \times 10^{-19}\ v^2 = \frac{200 \times 1.6}{9.1 \times 10^{-12}}\ v = 0.59 \times 10^7$

$v = 5.9 \times 10^6\, m/s\ x_1 = vt_1\ y_1 = \frac{1}{2}at_1^2 = \frac{1}{2}\left(\frac{Bqv}{m}\right)\left(\frac{x}{v}\right)^2$

$y_1 = \frac{1}{2} \times \left(\frac{1.5 \times 10^{-3} \times 1.6 \times 10^{-19} \times 5.9 \times 10^6}{9.1 \times 15^{31}}\right)\left(\frac{2 \times 10^{-2}}{5.9 \times 10^6}\right)^2 = 0.00894\,m = 0.894\,cm$

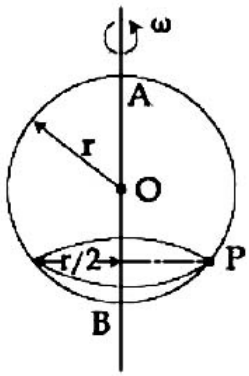
$r = \frac{mv}{Bq} = \frac{9.1 \times 10^{-31} \times 5.9 \times 10^6}{1.5 \times 10^{-3} \times 1.6 \times 10^{-19}} = 2.24\ cm\ \sin\theta = \frac{d}{r} = \frac{2}{2.24} \Rightarrow \theta = 63.23^0$

$y_2 = u\sin\theta t_1 - \frac{1}{2}gt_1^2\ y_2 = Tan\theta x_2 - \frac{g}{2x\cos^2\theta}x_2^2\ x_2 = 6\,cm = 0.119\,m = 11.9\,cm$



$d = y_1 + y_2 = 12.87\,cm$

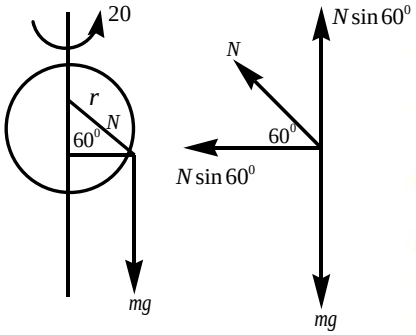
27. **A smooth wire of the length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB , as shown in figure, the bead is rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to:**



1) $2g/\left(r\sqrt{3}\right)$ **2)** $\frac{\sqrt{3}g}{2r}$ **3)** $\left(g\sqrt{3}\right)/r$ **4)** $2g/r$

Ans. **1**

Sol.



$$N\cos 60^0 = \frac{mr\omega^2}{2} \quad N\cos 60^0 = mg \quad Tan60^0 = \frac{g}{\frac{r}{2}w^2} = \frac{2g}{rw^2} \quad w^2 = \frac{2g}{\sqrt{3}r}$$

28. **One kg of water at 20^0C, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of $20\,\Omega$. The rms voltage in the mains is $200\,V$. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to:**

1) 22 minutes **2) 16 minutes** **3) 3 minutes** **4) 10 minutes**

Ans. **4**

Sol. $Q = Pt\ msat + mL = \frac{V_0^2}{R}t\left(1\times 4200\times 80\right) + \left(1\times 2260\times 10^3\right) = \frac{\left(\sqrt{2}\times 200\right)^2}{20}t\ t = 649\text{sec}$

$t = 10.8\text{min}\ t \approx 10\text{min}$

29. **A Carnot engine has an efficiency of 1/6. When by 62^0C, its efficiency is doubled. The temperature of the source and the sink are, respectively,**

1) $99^0C, 37^0C$ **2)** $124^0C, 62^0C$ **3)** $37^0C, 99^0C$ **4)** $62^0C, 124^0C$

Ans. **1**

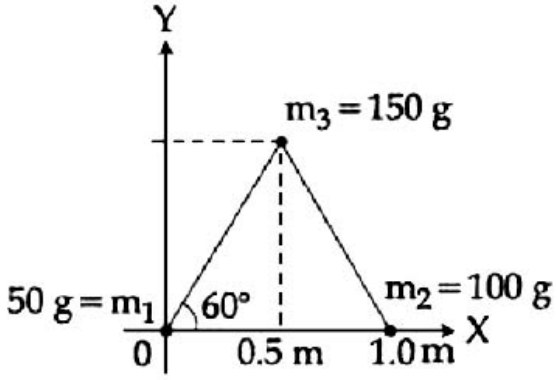
Sol. $\eta = 1 - \frac{T_2}{T_1}$

$$\frac{1}{6} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6} \quad 2\eta = 1 - \left(\frac{T_2 - 62}{T_1}\right)$$

$$2 \times \frac{1}{6} + \frac{62}{T_1} \Rightarrow T_1 = 62 \times 6 \quad T_1 = 372K$$

$$T_1 = 372 - 273 = 99^0C \quad T_2 = \frac{5}{6} \times 372 = 310K \quad T_2 = 310 - 273 = 37^0C$$

30. **Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1m (as shown in the figure). The (x, y) co-ordinates of the centre of mass will be:**



1) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$ **2)** $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$ **3)** $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$ **4)** $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$

Ans. **3**

Sol. $$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + 0 + 150\left(\sqrt{3}/2\right)}{300} = \sqrt{3}/4 = \frac{175}{300} = \frac{35}{60} = 7/12M$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + 0 + 150\left(\sqrt{3}/2\right)}{300} = \sqrt{3}/4$$

CHEMISTRY

31. **The ratio of number of atoms present in a simple cubic ,body centered cubic and face centered cubic structure are respectively**

1) 8 : 1: 6 **2) 1: 2 : 4** **3) 4 : 2 : 3** **4)4: 2 : 1**

Ans. **2**

Sol. simple cubic $Z_1 = 1$ Body centered cubic $Z_2 = 2$

Face centered cubic $Z_3 = 4$ $Z_1 : Z_2 : Z_3 = 1:2:4$.

32. **The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y.X and Y respectively, are**

1) $Ca(HCO_3)_2$ *and* CaO **2)** $Ca(HCO_3)_2$ *and* $Ca(OH)_2$

3) $Mg(HCO_3)_2$ *and* $MgCO_3$ **4)** $Mg(HCO_3)_2$ *and* $Mg(OH)_2$

Ans. **4**

Sol. $Mg(HCO_3)_2 \xrightarrow{\Delta} Mg(OH)_2 + 2CO_2$

33. **The molar solubility of** $Cd(OH)_2$ **is** $1.84\times10^{-15}\,M$ **in water. The expected solubility of** $Cd(OH)_2$ **in a buffer solution of** $pH = 12\,is$**:**

1) $6.23\times10^{-11}\,M$ **2)** $\frac{2.49}{1.84}\times10^{-9}\,M$ **3)** $1.84\times10^{-9}\,M$ **4)** $2.49\times10^{-10}\,M$

Ans. **4**

Sol. $Ksp = [Cd^{2+}][OH^-]^2$ $\quad 24.9\times10^{-15} = s^1\times(10^{-2})^2$ $\quad S' = 2.49\times10^{-10}\,M$.

34. **Which one of the following is likely to give a precipitate with** $AgNO_3$ **solution?**

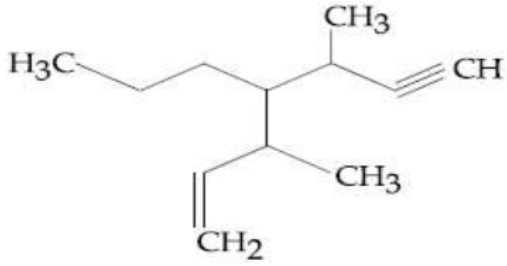
1) $(CH_3)_3CC\ell$ **2)** $CH_2 = CH - C\ell$ **3)** $CC\ell_4$ **4)** $CHC\ell_3$

Ans. **1**

Sol.

$$(CH_3)_3CC\ell + AgNO_3 \rightarrow AgC\ell + \underset{\text{(Stable Carbocation)}}{(CH_3)_3\overset{\oplus}{C}} + NO_3^-$$

35. **The IUPAC name for the following compound is:**



1) $3-\ methyl\ -4-(3-methylprop\ -1-enyl)-1-heptyne$

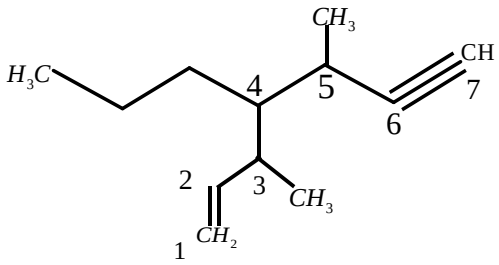
2) $3-methyl-4-(1-methylprop-2-yny1)-1-heptene$

3) $3,5\ -dimethyl-4-propylhept-1-en-6-yne$

4) $3,5\ -\dim ethyl-4-propylhept-6-en-1-yne$

Ans. **3**

Sol. $3,5-\text{dim}ethyl-4-propylhept-1-en-6-yne$



36. **The C-C bond length is maximum in**

1) C_{70} **2) diamond** **3)graphite** **4)** C_{60}

Ans. **2**

Sol. In diamond, carbon undergoes SP^3 hybridization and it has only $\sigma-bonds$.

Longest $C-C$ bond is present in diamond.

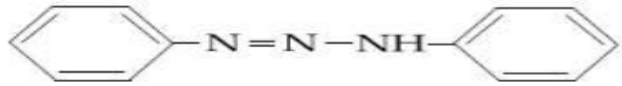
37. **The INCORRECT match in the following is**

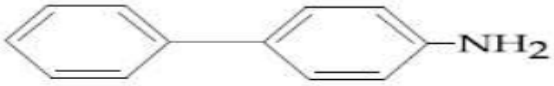
1) $\Delta G^0>0, K<1$ **2)** $\Delta G^0=0, K=1$ **3)** $\Delta G^0<0, K>1$ **4)** $\Delta G^0<0, K<1$

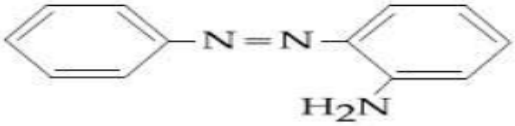
Ans. **4**

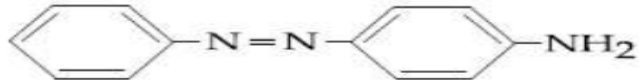
Sol. $\Delta G^0=-RT\ell n\ K$

38. **Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives**

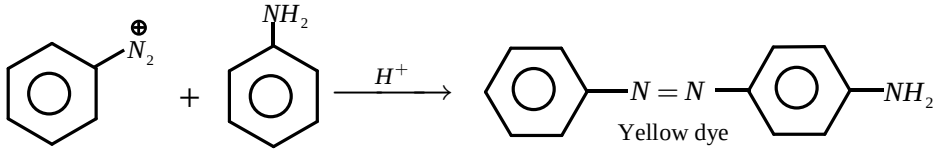
1. 

2. 

3. 

4. 

Ans. **4**

Sol. 

39. **In which one of the following equilibria , $K_p \neq K_C$?**

1) $NO_2(g)+SO_2(g) \rightleftarrows NO(g)+SO_3(g)$ 2) $2HI(g) \rightleftarrows H_2(g)+I_2(g)$

3) $2C(s)+O_2(g) \rightleftharpoons 2CO(g)$ 4) $2NO(g) \rightleftarrows N_2(g)+O_2(g)$

Ans. **3**

Sol. To get $Kp \neq Kc$ no. of gaseous reactants should not be equal to no. of gaseous products.

40. **The correct statement is :**

1) Pig iron is obtained from cast iron.

2) Leaching of bauxite using concentrated $NaOH$ solution gives sodium aluminateand sodium silicate.

3) the blistered appearance of copper during the metallurgical process is due to the evolution of CO_2

4) the Hall- Heroult process is used for the production of aluminium and iron.

Ans. **2**

Sol. $A\ell_2O_{3_{(s)}} + 2NaOH_{aq} + 3H_2O_{(\ell)} \rightarrow 2Na\left(A\ell(OH)_4\right)_{aq} + Na_2SiO_3$

41. **The primary pollutant that leads to photochemical smog is:**

1) nitrogen oxides **2) sulphur dioxide**

3) acrolein **4) ozone**

Ans. **1**

Sol. Primary pollutant are nitrogen oxides and hydrocarbons.

42. **Among the following , the INCORRECT statement about colloids is :**

1) They are larger than small molecules and have high molar mass.

2) The range of diameters of colloidal particles is between 1 and 1000nm.

3) They can scatter light.

4) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration.

Ans. **4**

Sol. Refer NCERT

Osmatic pressure of colloidal solution is less compared to true solution.

43. **Among the following , the energy of $2s$ orbital is lowest in :**

1) H **2) Na** **3) K** **4) Li**

Ans. **3**

Sol. more no. of protons lesser the energy.

44. **The pair that has similar atomic radii is :**

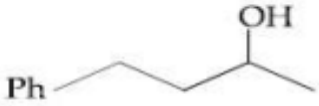
1) Mo and W **2) Mn and Re** **3) Ti and Hf** **4) Sc and Ni**

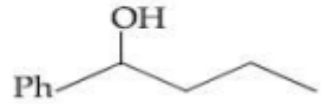
Ans. **1**

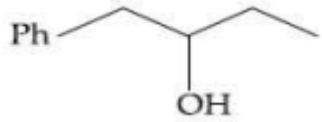
Sol. Due to lanthanoid contraction, Mo and W have near by atomic radii.

45. **Heating of 2-chloro -1-phenylbutane with EtOK/EtOH gives X as major product.Reaction of X with $Hg(OAc)_2 / H_2O$ followed by $NaBH_4$ gives Y as the major product.Y is :**

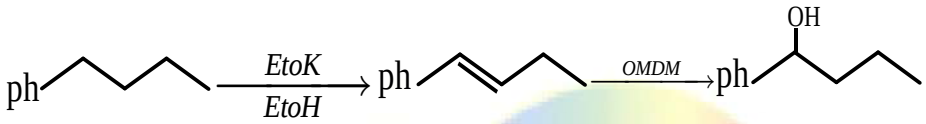
1. 

2. 

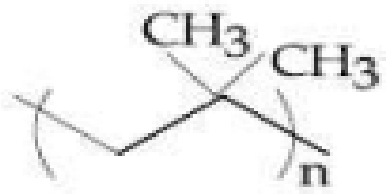
3. 

4. 

Ans. **3**

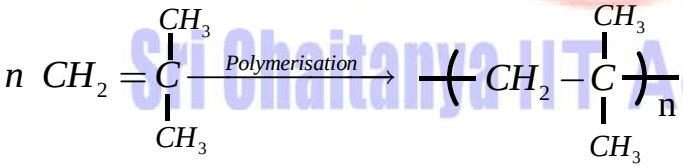
Sol. 

46. **The correct name of the following polymer is :**



1) polyisobutylene **2) polyisobutane** **3) polytert –butylenes** **4) polyisoprene**

Ans. **1**

Sol. 

47. **Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with $NaCl$ and concentrated H_2SO_4 to give a pungent gas Z. X, Y and Z. respectively are**

1) K_3MnO_4, K_2MnO_4 and Cl_2 **2)** $KMnO_4, K_2MnO_4$ and Cl_2

3) $K_2MnO_4, KMnO_4$ and SO_2 **4)** $K_2MnO_4, KMnO_4$ and Cl_2

Ans. **2**

Sol. $\underset{(X)}{2KMnO_4} \rightarrow \underset{(Y)}{K_2MnO_4} + MnO_2 + O_2$

$4NaC\ell + MnO_2 + 4H_2SO_4 \rightarrow MnC\ell_2 + 4NaHSO_4 + 2H_2O + \underset{(Z)}{C\ell_2}$

48. **The INCORRECT statement is :**

1) $Licl$ **crystallises from aqueous solution as** $Licl\,.\,2H_2O$

2) $LiNO_3$ **decomposes on heating to give** $LiNO_2$ **and** O_2

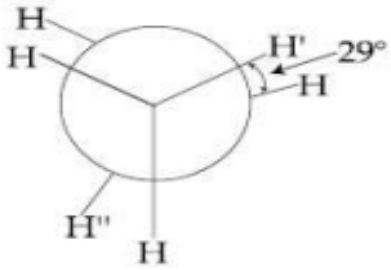
3) Lithium is least reactive with water among the alkali metals

4) Lithium is the strongest reducing agent among the alkali metals.

Ans. **2**

Sol. $LiNO_3 \xrightarrow{\Delta} Li_2O + NO_2 + O_2$

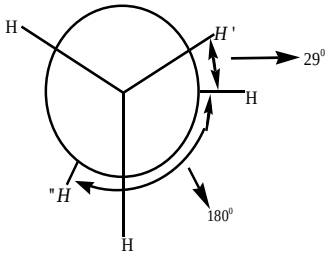
49. **In the following skew conformation of ethane** $H' - C - C - H''$ **dihedral angle is**



1) 151^0 **2)** 149^0 **3)** 58^0 **4)** 120^0

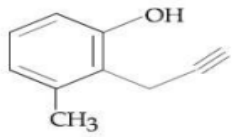
Ans. **1**

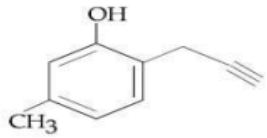
Sol.

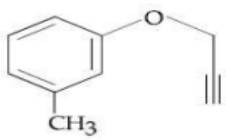


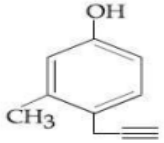
$H'-C-C-H''$ dihedral angle $=180^0-29^0=151^0$

50. **What will be the major product when m- cresol is reacted with propargyl bromide** $\left(HC\equiv C-CH_2Br\right)$ **in presence of** K_2CO_3 **in acetone?**

1.


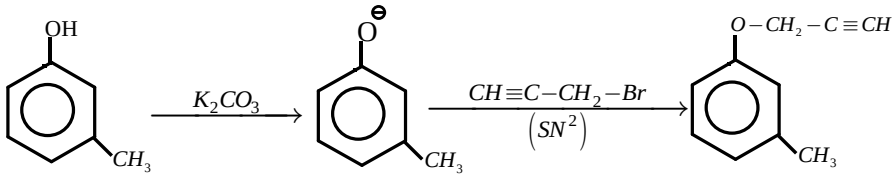
2.


3.


4.


Ans. 3

Sol.



51. **25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains.**

1) 22 g of carbon and 3 g of hydrogen

2) 18 g of carbon and 7 g of hydrogen

3) 20 g of carbon and 5 g of hydrogen

4) 24 g of carbon and 1 g of hydrogen

Ans. 4

Sol. 88g of $CO_2 \rightarrow 24g$ of carbon is present

9g of $H_2O \rightarrow 1g$ of hydrogen is present.

52. **A solution is prepared by dissolving 0.6 g of urea (molar mass =60 g mol^{-1}) and 1 .8 g of glucose (molar mass =180 g mol^{-1}) in 100 mL of water at 27 ^0C. The osmotic pressure of the solution is : $\left(R = 0.08206\ L\,atm\,K^{-1}\,mol^{-1}\right)$**

1) 2.46 atm **2) 1.64 atm** **3) 4.92 atm** **4) 8.2 atm**

Ans. 3

Sol. $\pi = (C_1 + C_2)RT = \left(\frac{0.01}{100} \times 1000 + \frac{0.01}{100} \times 1000\right) 0.08206 \times 300 = 4.92$ atm.

53. **The coordination numbers of Co and Al ion $\left[Co(Cl)(en)_2\right]Cl$ and $K_3\left[Al(C_2O_4)_3\right]$ respectively, are (en = ethane 1,2-diamine)**

1) 3 and 3 **2) 6 and 6** **3) 5 and 3** **4) 5 and 6**

Ans. 4

Sol. $\left[CO(C\ell)(en)_2\right]C\ell \Rightarrow$ co.ordination number of co is 5

$K_3\left[A\ell(C_2O_4)_3\right] \Rightarrow$ co.ordination number of co is 6.

54. **The decreasing order of electrical conductivity of the following aqueous solutions is**

0.1 M Formic acid (A) 0.1 M Acetic acid (B) 0.1 M Benzoic acid (C)

1) C > A >B **2) C > B > A** **3) A > C > B** **4) A > B > C**

Ans. 3

Sol. Ka is more conductivity is more

Formic acid $(Ka) > Benzoic\ acid\,(Ka) > Aceticacid\,(Ka)$

55. **which of the given statements is INCORRECT about glycogen?**

1) It is Present in animal cells.

2) It is a straight chain polymer similar to amylose.

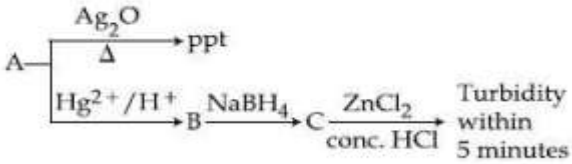
3) It is present in some yeast and fungi.

4) Only α -linkages are present in the molecule.

Ans. **2**

Sol. glycogen structure is similar to amylopectin and is more highly branched.

56. **Consider the following reactions**

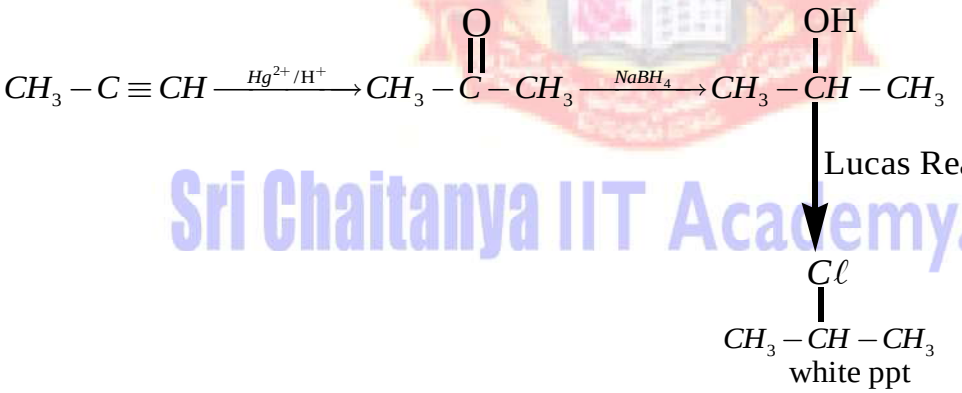


'A' is :

1) $CH_2 = CH_2$ **2)** $CH_3 - C \equiv CH$ **3)** $CH_3 - C \equiv C - CH_3$ **4)** $CH \equiv CH$

Ans. **2**

Sol.



57. **In comparison to boron, beryllium has**

1) greater nuclear charge and lesser first ionization enthalpy.

2) lesser nuclear charge and greater first ionization enthalpy

3) greater nuclear charge and greater first ionization enthalpy.

4)lesser nuclear charge and lesser first ionization enthalpy

Ans. **2**

Sol. Be has lesser nuclear charge, greater the IE.

58. **An 'Assertion' and a 'Reasaon' are given below Choose the correct answer from the following options :**

Assertion (A) : Vinyl halides do not undergo nucleophilic substitution easily.

Reasaon (R) : Even though the intermediate carbocation is stabilized by loosely held π -electrons the cleavage is difficult because of strong bonding.

1) Both (A) and (R) are correct statements and (R) is the correct explanation of (A)

2) Both (A) and (R) are wrong statements.

3) Both (A) and (R) are correct Statements but (R) is not the correct explanation of (A)

4) (A) is a correct statement but (R) is a Wrong statement.

Ans. **4**

Sol. Vinyl halides doesn't form stable carbocation.

59. **The compound use in the treatment of lead poisoning is**

1) EDTA **2) desferrioxime B**

3) D- penicillamine **4) Cis –platin**

Ans. **1**

Sol. EDTA used in the treatment of lead poisoning.

60. **NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation $2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$ The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes the rate of formation of NO_2 is**

1) 2.083 X10^{-3} mol $L^{-1}\min^{-1}$ **2) 8.333 X10^{-3} mol $L^{-1}\min^{-1}$**

3) 4.167X10^{-3} mol $L^{-1}\min^{-1}$ **4) 1.667 X10^{-2} mol $L^{-1}\min^{-1}$**

Ans. **4**

Sol. $-\frac{1}{2}\frac{d(N_2O_5)}{dt} = \frac{1}{4}\frac{d(NO_2)}{dt}$ $\frac{d(N_2O_5)}{dt} = \left(\frac{2.75-3.00}{30}\right) = \frac{-0.25}{30}$ $\frac{d(NO_2)}{dt} = -2\times\left(\frac{-0.25}{30}\right)$

$=1.667$ X10^{-2} mol $L^{-1}\min^{-1}$.

MATHEMATICS

61. **The derivative of** $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$ **with respect to** $\frac{x}{2}$**, where** $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ **is**

1) $\frac{1}{2}$ **2) 1** **3) 2** **4)** $\frac{2}{3}$

Ans. 3

Sol. $Let\ y=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)=\tan^{-1}\left(\tan\left(x-\frac{\pi}{4}\right)\right)=\left(x-\frac{\pi}{4}\right)$

$Let\ z=x/2\ \frac{dy}{dz}=\frac{\left(dy/dx\right)}{\left(dz/dx\right)}=\frac{(1)}{\left(1/2\right)}=2.$

62. **The equation of a common tangent to the curves,** $y^2=16x$ **and** $xy=-4$**, is**

1) $x+y+4=0$ **2)** $2x-y+2=0$ **3)** $x-2y+16=0$ **4)** $x-y+4=0$

Ans. 4

Sol. Tangent to parabola with slope m is $y=mx+\frac{4}{m}\quad\rightarrow 1$

This is tangent to $xy=-4\quad\rightarrow 2$

Elemenating y from 1, $2\ x\left(mx+\frac{4}{m}\right)=-4$

$\Rightarrow mx^2+4x+4m=0$ Now $\Delta=0\quad\Rightarrow m=1$

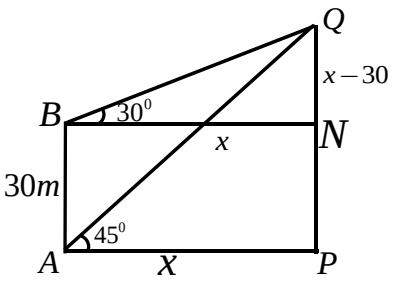
$\therefore$ The common tangent is $y=x+4.$

63. **The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be** 45^0 **from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be** 30^0**, then the distance (in m) of the foot of the tower from the point A is**

1) $15\left(1+\sqrt{3}\right)$ **2)** $15\left(5-\sqrt{3}\right)$ **3)** $15\left(3-\sqrt{3}\right)$ **4)** $15\left(3+\sqrt{3}\right)$

Ans. 4

Sol.



From $\Delta QBN\ Tan30^0=\frac{x-30}{x}$

$\Rightarrow x=\left(\frac{30\sqrt{3}}{\sqrt{3}-1}\right)=15\sqrt{3}\left(\sqrt{3}+1\right)=15\left(3+\sqrt{3}\right).$

64. **An ellipse, with foci at (0, 2) and (0, -2) and minor axis of length 4, passes through which of the following points ?**

1) $(1, 2\sqrt{2})$ **2)** $(\sqrt{2}, 2)$ **3)** $(2, 2\sqrt{2})$ **4)** $(2, \sqrt{2})$

Ans. **2**

Sol. $2be = 4, 2a = 4\ b^2e^2 = b^2 - a^2\ 4 = b^2 - 4 \Rightarrow b^2 = 8$

Equation of ellipse: $\frac{x^2}{4} + \frac{y^2}{8} = 1$ This is satisfied by $(\sqrt{2}, 2)$.

65. **If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to**

1) $\beta\gamma$ **2)** $\alpha\gamma$ **3) 0** **4)** $\alpha\beta$

Ans. **1**

Sol. *let* $\alpha,\ \beta = \alpha r\ \gamma = \alpha r^2\ \alpha x^2 + 2\beta x + \gamma = 0\ (x + r)^2 = 0$

$x = -r$ *is a root of* $x^2 + x - 1 = 0\ \gamma + 1 = r^2$

$\alpha(\beta + \gamma) = \alpha(\alpha r + \alpha r^2) = (\alpha r)(\alpha r^2) = \beta\gamma$

66. **Let $\alpha \in R$ and the three vectors $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set $S = \{\alpha : \vec{a}, \vec{b}$ *and* $\vec{c}$ *are coplanar*$\}$**

1) contains exactly two positive numbers

2) is singleton

3) is empty

4) contains exactly two numbers only one of which is positive

Ans. **3**

Sol. $\begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0$

$\alpha(3\ -2\ \alpha) - (6 + \alpha^2) + 3(-4 - \alpha) = 0 \Rightarrow 3\alpha^2 = -18\ \alpha$ has no real value.

67. **If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is**

1) 150 **2) 200** **3) 120** **4) 280**

Ans. **2**

Sol. $a_1 + (a_1 + 6d) + (a_1 + 15d) = 40\ 3a_1 + 21d = 40 \quad \rightarrow 1\ S_{15} = \frac{15}{2}[2a_1 + (14d)]$

$= 15 \times (a_1 + 7d) = 15 \times \frac{40}{3} = 200$ *units* .

68. **Let $z \in C$ with $\text{Im}(z)=10$ and it satisfies $\frac{2z-n}{2z+n}=2i-1$ for some natural number n. Then**

1) $n=20$ *and* $\text{Re}(z)=-10$ **2)** $n=40$ *and* $\text{Re}(z)=-10$

3) $n=40$ *and* $\text{Re}(z)=10$ **4)** $n=20$ *and* $\text{Re}(z)=10$

Ans. **2**

Sol. $Let\ z=x+10i$ Then $\frac{2(x+10i)-n}{2(x+10i)+n}=2i-1$

$\Rightarrow (2x-n)+20i=((2x+n)+20i)(2i-1)$

$\Rightarrow 2x-n=-(2x+n)+40 \therefore 4x=-40\ x=-10$

Comparing im. parts $\therefore \text{Re}(z)=-10\ 20=2(2x+n)-20 \Rightarrow n=40$.

69. **Let $\alpha \in (0,\pi/2)$ be fixed. If the integral**

$\int \frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx = A(x)\ \cos 2\alpha + B(x)\sin 2\alpha + C$,

where c is a constant of integration, then the functions $A(x)$ and $B(x)$ are respectively

1) $x-\alpha$ *and* $\log_e|\cos(x-\alpha)|$ **2)** $x-\alpha$ *and* $\log_e|\sin(x-\alpha)|$

3) $x+\alpha$ *and* $\log_e|\sin(x+\alpha)|$ **4)** $x+\alpha$ *and* $\log_e|\sin(x-\alpha)|$

Ans. **2**

Sol. $\int \frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx \int \frac{\sin(x-\alpha+2\alpha)}{\sin(x-\alpha)}dx,\quad x-\alpha=\theta$

$\cos\ 2\alpha \int d\theta + \sin 2\alpha \int \frac{\cos\theta}{\sin\theta}d\theta = \cos\ 2\alpha(x-\alpha)+\sin 2\alpha\ \log\sin(x-\alpha)+C$.

70. $\lim\limits_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$

1) 6 **2) 1** **3) 3** **4) 2**

Ans. **4**

Sol. $\underset{x\to 0}{Lt}\frac{3x\left(\sqrt{x^2+2x+1}+\sqrt{x^2-x+1}\right)}{3x}=2$

71. **The tangents to the curve $y=(x-2)^2-1$ at its points of intersection with the line $x-y=3$, intersect at the point**

1) $\left(-\frac{5}{2},1\right)$ **2)** $\left(\frac{5}{2},1\right)$ **3)** $\left(-\frac{5}{2},-1\right)$ **4)** $\left(\frac{5}{2},-1\right)$

Ans. **4**

Sol. $(x-3)=x^2-4x+3\ x=2,3\ P(2,-1)\ (3,0)\ \text{m} = 0,\ 2\ y=-1 \quad 2x-y=6\ x=5/2$.

72. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right).\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to

1) -72 **2)** -36 **3)** 36 **4)** -108

Ans. 2

Sol. $T_4=\frac{1}{60}\ {}^6C_3\ (2)^3(-3)^3=-72\ T_6=\frac{-1}{81}\ {}^6C_5\ (2)^3(-3)^3=36.$

73. If the area (in sq. units) bounded by the parabola $y^2=4\lambda x$ and the line $y=\lambda x$, $\lambda>0$, is $\frac{1}{9}$, then λ is equal to

1) 48 **2)** $4\sqrt{3}$ **3)** $2\sqrt{6}$ **4)** 24

Ans. 4

Sol. $A=\int_0^{4/\lambda}\left(\sqrt{4\lambda x}-\lambda x\right)dx\ \lambda=24.$

74. The Boolean expression $\sim(p\Rightarrow(\sim q))$ is equivalent to

1) $q\Rightarrow\sim p$ **2)** $p\wedge q$ **3)** $p\vee q$ **4)** $(\sim p)\Rightarrow q$

Ans. 2

Sol. $\equiv\sim(\sim p\ V\sim q)\equiv p\wedge q.$

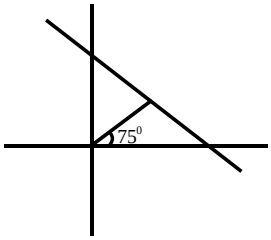
75. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60^0 with the line $x+y=0$. Then an equation of the line L is

1) $x+\sqrt{3}y=8$ **2)** $\sqrt{3}x+y=8$

3) $\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$ **4)** $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2}$

Ans. 3

Sol. $x\cos 75+y\sin 75=4$



76. A plane which bisects the angle between the two given planes $2x-y+2z-4=0$ and $x+2y+2z-2=0$, passes through the point

1) $(1,-4,1)$ **2)** $(1,4,-1)$ **3)** $(2,-4,1)$ **4)** $(2,4,1)$

Ans. 3

Sol. $\frac{2x-y+2z-4}{3}=\pm\frac{x+2y+2z-2}{3}\ 3x+y+4z-6=0\ x-3y-2=0.$

77. **A circle touching the x-axis at (3,0) and making an intercept of length 8 on the y-axis passes thorough the point**

1) $(3,10)$ **2)** $(1,5)$ **3)** $(3,5)$ **4)** $(2,3)$

Ans. **1**

Sol. $x^2+2gx+c=(x-3)^2\ g=-3\ c=9\ f=\pm 5\ x^2+y^2-6x\pm 10y+9=0$.

78. **For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is**

1) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$ **2)** $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$ **3)** $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ **4)** $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

Ans. **2**

Sol. $P(x=0)+P(x=1)\ {}^{50}C_{50}\left(\frac{4}{5}\right)^{50}+{}^{50}C_{49}\left(\frac{4}{5}\right)^{49}.\frac{1}{5}=\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

79. **Let A,B and C be sets such that $\phi\neq A\cap B\subseteq C$. Then which of the following statements is not true ?**

1) If $(A-C)\subseteq B$, *then* $A\subseteq B$ **2)** $(C\cup A)\cap(C\cup B)=C$

3) If $(A-B)\subseteq C$, *then* $A\subseteq C$ **4)** $B\cap C\neq\phi$

Ans. **1**

Sol. $A=\{2\}\ B=\{1,2,3,4\}$ *and* $C=\{2,5\}\ A\cap B=\{2\}\leq C$

$A-C=\{\ \}\leq B$ But $A\not\subseteq B$.

80. **Let $f(x)=5-|x-2|$ and $g(x)=|x+1|$, $x\in R$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim\limits_{x\to-\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to**

1) $-1/2$ **2)** $1/2$ **3)** $3/2$ **4)** $-3/2$

Ans. **2**

Sol. $\alpha=2,\beta=-1\Rightarrow\alpha\beta=2$ Required limit $=\frac{1}{2}$.

81. **If $[x]$ denotes the greatest integer $\leq x$, then the system of linear equations** $[\sin\theta]x+[-\cos\theta]y=0\ [\cot\theta]x+y=0$

1) have infinitely many solutions if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

2) have infinitely many solutions if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ **and has a unique solutions if** $\theta\in\left(\pi,\frac{7\pi}{6}\right)$

3) has a unique solution if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

4) has a unique solution if $\theta\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ **and have infinitely many solutions if** $\theta\in\left(\pi,\frac{7\pi}{6}\right)$

Ans. **2**

Sol. $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \Rightarrow [\sin\theta] = 0, [-\cos\theta] = 0$ *and* $[\cot\theta] = -1 \Rightarrow x = y$ infinitely many solutions.

And $\theta \in \left(\pi, \frac{7\pi}{6}\right) \Rightarrow [\sin\theta] = 7, [-\cos\theta] = 0$ and $[\cot\theta] = 0 \Rightarrow x = 0, y = 0$ unique solution.

82. **If** $^{20}C_1 + \left(2^2\right)^{20} C_2 + \left(3^2\right)^{20} C_3 + \ldots\ldots\ldots\ldots + \left(20^2\right)^{20} C_{20} = A\left(2^{\beta}\right)$**, then the ordered pair** (A, β) **is equal to**

1) $(420,18)$ **2)** $(380,18)$ **3)** $(380,19)$ **4)** $(420,19)$

Ans. **1**

Sol. $= \sum_{r=1}^{20} r^2 \ {}^{20}Cr = 20(20+1)2^{18} = 420\ \alpha^{18}$.

83. **Let S be the set of all** $\alpha \in R$ **such that the equation,** $\cos 2x + \alpha \sin x = 2\alpha - 7$ **has a solution. Then S is equal to**

1) $[1,4]$ **2) R** **3)** $[3,7]$ **4)** $[2,6]$

Ans. **4**

Sol. $2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0 \Rightarrow \sin x = \frac{\alpha - 4}{2}, 2 - 1 \le \frac{\alpha - 4}{2} \le 1 \Rightarrow 2 \le \alpha \le 6$

84. **The length of the perpendicular drawn from the point** $(2,1,4)$ **to the plane containing the lines** $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ **and** $\vec{r} = \left(\hat{i} + \hat{j}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ **is**

1) $\sqrt{3}$ **2)** $\frac{1}{\sqrt{3}}$ **3)** $\frac{1}{3}$ **4) 3**

Ans. **1**

Sol. $\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0 \ x - y - z = 0$ Distance from (2, 1, 4), $\frac{|2-1-4|}{\sqrt{3}} = \sqrt{3}$.

85. **A value of** α **such that** $\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right)$ **is**

1) -2 **2)** $-\frac{1}{2}$ **3) 2** **4)** $\frac{1}{2}$

Ans. **1**

Sol. $\left(\log\frac{x+\alpha}{x+\alpha+1}\right)_{\alpha}^{\alpha+1} = \log\left(\frac{9}{8}\right) \log\frac{(2\alpha+1)}{4\alpha(\alpha+1)} = \log\left(\frac{9}{8}\right) \Rightarrow \alpha = -2$.

86. **A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is**

1) $\frac{1}{2}$ **gain** **2)** $\frac{1}{2}$ **loss** **3)** $\frac{1}{4}$ **loss** **4) 2 gain**

Ans. **2**

Sol. Range of x: 15 12 -6 $P(x)$: $\frac{6}{36}$ $\frac{4}{36}$ $\frac{26}{36}$ Expection $= \frac{90+48-156}{36} = \frac{-18}{36} = -\frac{1}{2}$.

87. **A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to**

1) 27 **2) 24** **3) 28** **4) 25**

Ans. **4**

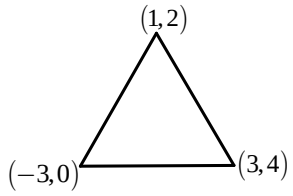
Sol. ${}^5C_2.{}^nC_1 + {}^5C_1.{}^nC_2 = 1750\ n^2 + 3n = 700\ n = 25$.

88. **A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (-1, 1) and (2, 3). Then the centroid of this triangle is**

1) $\left(1, \frac{7}{3}\right)$ **2)** $\left(\frac{1}{3}, \frac{5}{3}\right)$ **3)** $\left(\frac{1}{3}, 1\right)$ **4)** $\left(\frac{1}{3}, 2\right)$

Ans. **4**

Sol.



The Ans. Is centroid.

89. **The general solution of the differential equation** $\left(y^2 - x^3\right)dx - xydy = 0 (x \neq 0)$ **is (where c is a constant of integration)**

1) $y^2 - 2x^2 + cx^3 = 0$ **2)** $y^2 + 2x^3 + cx^2 = 0$

3) $y^2 + 2x^2 + cx^3 = 0$ **4)** $y^2 - 2x^3 + cx^2 = 0$

Ans. **2**

Sol. $\frac{dy}{dx} = \frac{y}{x} - \frac{x^2}{y}\ 2y\frac{dy}{dx} - \frac{2y^2}{x} = -x^2,\ y^2 = z\ \frac{dz}{dx} - \frac{2}{x}z = -2x^2$

90. **A value of** $\theta \in (0, \pi/3)$**, for which** $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$**, is**

1) $\frac{\pi}{18}$ **2)** $\frac{7\pi}{24}$ **3)** $\frac{7\pi}{36}$ **4)** $\frac{\pi}{9}$

Ans. **4**

Sol. $R_2 - R_1 \quad R_3 - R_2\ 2 + 4\ \cos 6\theta = 0\ \cos 6\theta = -\frac{1}{2}\ \theta = \frac{\pi}{9}$.

... Prepared by Sri Chaitanya Faculty