Cognitate

Question 1:

A. Centre (0, 0)

B. Gradient of the line from the centre of the circle to the point $(-4, 2)$ is $\frac{2}{-4} = -\frac{1}{2}$

(1 mark)

The gradient of the tangent is the negative reciprocal of $-\frac{1}{2}$ which is + 2 (1 mark)

Using $y = mx + c$

$2 = 2(-4) + c$

$c = 10$

$y = 2x + 10$ (2 marks)

C. z is equal to the radius squared.

$z = 4^2 + 2^2 = 20$ (1 mark)

Question 2:

A. Solve for y first so you can eliminate the need for negative square roots.

$x = y^2$
$y = 4 - 3y^2$
$-3y^2 - y + 4 = 0$ (1 mark)

Therefore $y = -\frac{4}{3}$ or 1 (1 mark)

Subbing values of y into the original equation gives $x = (-\frac{4}{3})^2 = \frac{16}{9}$ and 1 (2 marks)

Award full marks of any alternative method

Question 3:

A. Odd integers are $(2n + 5)$ and $(2n + 1)$ (1 mark for selecting 2 odd integers)

Doubling any number and adding 1 will always be odd and adding 4 to that number will always be odd.

The difference between the squares of the integers will be $(2n + 5)^2 - (2n + 1)^2$ (1 mark)

$= 4n^2 + 20n + 25 - 4n^2 - 4n - 1 \quad = 16n + 24$ (1 mark)

The mean of the original integers is the integers added together divided by the number of integers:

$\frac{2n+1+2n+5}{2} = \frac{4n+6}{2} = 2n+3$ (1 mark)

$16n + 24 = 8(2n+3)$ (1 mark)

Therefore, the difference between the squares of the integers is 8 times the mean.

Award full marks for any alternative method including using different odd integers in terms of n.

Question 4:

A. $\frac{(x^2+2x-15)(2x+4)}{(x^2-4)(x^2-7x+12)} = \frac{(x+5)(x-3)(2x+4)}{(x+2)(x-2)(x-3)(x-4)}$ (1 mark)

$(2x+4) = (x+2)(2), (x^2-4) = (x+2)(x-2)$

$= \frac{(x+5)(x-3)(x+2)(2)}{(x+2)(x-2)(x-3)(x-4)}$ (1 mark)

$\frac{(x+5)(x-3)(2)}{(x-2)(x-3)(x-4)}$ (1 mark)

$= \frac{2(x+5)}{(x-2)(x-4)}$ (1 mark)

Question 5:

A. As n is a positive integer. $(2n+2)^2$ is also positive.

The right side can simply be referred to as m, where m is a positive even integer. (1 mark for recognising that $(2n+2)^2$ is even)

$(x+1)(x+2y+1) = m$

since (x + 1) and (x + 2y +1) must multiply to make an even number, this can only be true when either both terms are even, or 1 term is odd, and the other term is even.

Scenario 1: Both terms are even.

x + 1 is even, therefore x is odd. Then x + 2y + 1 is even, so this scenario is possible. (1 mark)

Scenario 2: First term is odd and the second term is even.

x + 1 is odd, therefore x is even. Then x + 2y + 1 is odd. This is impossible as both x and y are even, thus x + 2y + 1 must be odd. This scenario is not possible as both terms are odd. (1 mark)

Scenario 3: the second term is odd and the first term is even.

If x +1 is even, x is odd. This will give the same answer as scenario 1 where x + 2y + 1 is even, so this scenario doesn't exist. (1 mark)

Therefore, x must be odd.

Award full marks for any alternative method that concludes x is odd.