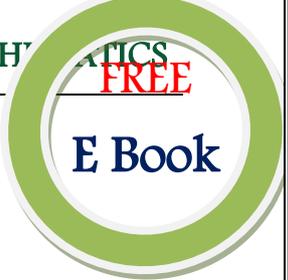




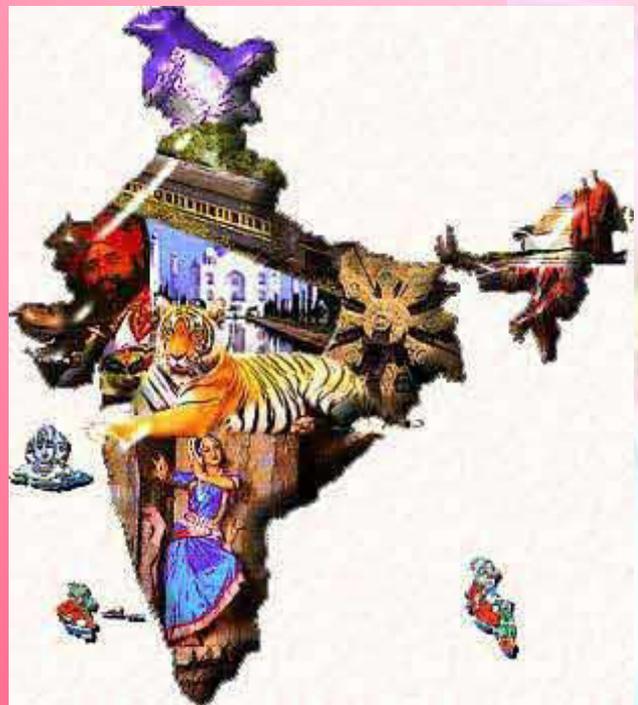
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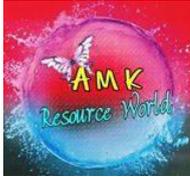
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## Fundamental Concepts of Geometry

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## Fundamental Concepts of Geometry

- **Axiom** : The basic facts which are taken for granted without proof
  - ... Things which are equal to the same thing are equal to one another
  - ... If equals are added to equals, the wholes are equal
  - ... If equals are taken from equals, the remainders are equal
  - ... The whole is greater than its parts
  - ... Halves of equals are equal
  - ... If equals are added to unequals the wholes are unequals
  - ... A line contains infinitely many points
- **Postulates** : The statements which specify relationships among basic geometric facts and concepts which are assumed to be true without logical proof
  - ... One and only one straight line can be drawn through two points
  - ... Any number of lines can be drawn through a point
  - ... A straight line may be produced to any length on either side
  - ... Angle at a common end point of two opposite rays is  $180^\circ$
  - ... Any two straight lines can have either only one point in common or no point in common
  - ... Two parallel lines in a plane do not meet even if produced infinitely on either side
- **Statement** : A sentence which can be judged to be true or false
  - ... If a ray stands on a straight line, then the sum of the adjacent angles formed is  $180^\circ$
  - ... If two straight lines intersect, then the Vertically Opposite Angles are equal
  - ... If a transversal cuts two parallel lines, then corresponding angles are equal
- **Theorem** : A statement that requires a proof
- **Corollary** : A statement whose truth can easily be deduced from a theorem
- **Point** : A point is a mark of position

- **Line** : A line segment when extended indefinitely in both the directions
- **Line Segment** : The straight path between two points
- **Ray** : A Line Segment which is extended indefinitely in one direction
- **Intersecting Lines** : Two lines having a common point
- **Point of intersection** : The point common to two intersecting lines
- **Concurrent Lines**: Three or more lines in a plane are said to be concurrent if all of them intersect at the same point.
- **Area** : The amount of surface enclosed by the sides of a closed bounded figure
- **Plane** : A flat surface extended endlessly in all the four directions
- **Collinear points** : Three or more points are said to be collinear, if there is a line which contains them all
- **Parallel Lines** : Two lines in a plane are said to be parallel, if they do not intersect no matter how long they are produced on either side
- **Angle** : Two rays having a common end point
- **Right Angle** : An angle whose measure is  $90^\circ$
- **Acute Angle** : An angle whose measure is more than  $0^\circ$  but less than  $90^\circ$
- **Obtuse Angle** : An angle whose measure is more than  $90^\circ$  but less than  $180^\circ$
- **Straight Angle** : An angle whose measure is  $180^\circ$
- **Reflex Angle** : An angle whose measure is more than  $180^\circ$  but less than  $360^\circ$
- **Complete Angle** : An angle whose measure is  $360^\circ$
- **Equal Angles** : Two angles are said to be equal, if they have the same measure
- **Complementary Angles** : Two angles are said to be complementary, if sum of their measures is  $90^\circ$
- **Supplementary Angles** : Two angles are said to be Supplementary, if sum of their measures is  $180^\circ$
- **Adjacent Angles** : Two angles are said to be Adjacent angles, if they have a common vertex and a common arm such that the other arms of the two angles are on either side of their common arm
- **Linear Pair** : If the sum of two adjacent angles is  $180^\circ$ , they are said to form a Linear pair
- **Linear Pair Axiom** : If a ray stands on a straight line then the sum of the adjacent angles so formed is  $180^\circ$
- **Angles at a point** : The sum of all angles at a point is  $360^\circ$
- **Vertically Opposite Angles** : The angles formed on the opposite side of a point intersecting the two lines
- **Perpendicular Lines** : When two lines intersect and make a right angle, they are said to be perpendicular to each other

- **Perpendicular Bisector** : The Perpendicular Bisector of a line segment is a line, perpendicular to the given line segment and passing through its mid point
- **Traversal** : A straight line that intersect two or more lines
- **Condition of Parallelism** : If two parallel lines are intersected by a traversal, then
  - ... Corresponding Angles are Equal
  - ... Alternate Interior Angles are Equal
  - ... Alternate Exterior angles are Equal
  - ... Co – Interior Angles are Supplementary
- **Polygon** : A closed plane figure bounded by three or more line segments
- **Sides of a Polygon** : The line segments forming a polygon
- **Vertex of a Polygon** : The point of intersection of two consecutive sides of a polygon
- **Diagonal of a polygon** : A line segment joining any two non consecutive vertices of a polygon
- **Interior Angle of a Polygon** : An angle formed by two consecutive sides of a polygon
- **Convex Polygon** : If each angle of a polygon is less than 180°, then it is Convex Polygon
- **Concave Polygon** : If at least one angle of a polygon is a reflex angle (more than 180°) then it is Concave Polygon
- **Exterior Angle of a Polygon** : An angle formed on producing a side of a polygon
- **Regular Polygon** : A Polygon having all sides equal and all angles equal
- **Sum of Interior angles of a polygon of n sides** :  $(2n - 4)$  right angles
- **Sum of Exterior angles of a polygon of n sides** :  $[2n - (2n - 4)]$  right angles i.e 360°
- **Formula to find each Interior Angle of n sides** :  $\frac{(2n - 4) \times 90}{n}$
- **Formula to find each Exterior Angle of n sides** :  $\frac{360}{n}$
- **Triangle** : A plane closed figure bounded by three line segments, A triangle has three sides, three angles and three vertices
- **Scalene Triangle** : A triangle in which all the sides are of different lengths
- **Isosceles Triangle** : A triangle having two sides equal
- **Equilateral Triangle** : A triangle having all sides equal
- **Acute Angled Triangle** : A triangle in which every angle measure is more than 0° but less than 90°
- **Right Angled Triangle** : A triangle in which one of the angle measures is 90°
- **Obtuse Angled Triangle** : A triangle in which one of the angles measures more than 90° but less than 180°

- **Median** : A line segment joining a vertex to the mid point of the opposite side of a triangle, A triangle has 3 Medians
- **Centroid** : The point of intersection or concurrence of the three medians of a triangle
- **Altitude** : The length of perpendicular from a vertex to the opposite side of a triangle
- **Orthocenter** : The point of intersection of the three altitudes of a triangle
- **Incenter** : The point of intersection of the internal bisectors of the angles of a triangle
- **Circumcenter** : The point of intersection of the perpendicular bisectors of the sides of a triangle
- **Angle Sum Property** : The sum of Angles of a triangle is 180°
- **Exterior Angle Property** : If one side of a triangle is produced, then the exterior angle so formed is equal to the sum of the interior opposite angles
- **Similar Figures** : Two figures having exactly the same shape but can be of different size
- **Similar Triangles** : Two triangles are similar, if their corresponding angles are equal
- **Congruent Figures** : Geometrical figures which have exactly the same shape and the same size
- **Congruent Line Segments** : Two lines are said to be congruent if they have same measure
- **Congruent Angles** : Two angles are said to be congruent if they have equal measures
- **Congruent Triangles** : Two triangles are congruent if pairs of corresponding sides and corresponding angles are equal
- **Side Angle Side Axiom of Congruency** : If two sides and included angle of one triangle are equal respectively to the corresponding two sides and included angle of the other, then the two triangles are congruent
- **Angle Side Angle Axiom of Congruency** : If two angles and the included side of one triangle are equal respectively to the corresponding two angles and the included side of the other, then two triangles are congruent
- **Side Side Side Axiom of Congruency** : If three sides of one triangle are equal respectively to the corresponding three sides of the other, then the two triangles are congruent
- **Angle Angle Side Axiom of Congruency** : If two angles and any one side of one triangle are equal respectively to the corresponding two angles and the corresponding side of the other, then the two triangles are congruent
- **Right Angle Hypotenuse Side Axiom of congruency** : If the hypotenuse and one side of one right angled triangle are equal respectively to the corresponding hypotenuse and one side of the other right triangle, then the two triangles are congruent

- **Pythagoras Theorem** : In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the remaining two sides ( $AC^2 = AB^2 + BC^2$ )
- **Converse of Pythagoras Theorem** : If , in any triangle the square of the largest side is equal to the sum of the squares of the remaining two sides, then the triangle is right angled triangle
- **Quadrilateral** : A closed figure bounded by four line segments, it has 4 vertices, 4 sides, 4 angles and 2 diagonals
- **Adjacent Sides of Quadrilateral** : Two sides of a quadrilateral having a common end point
- **Opposite Angles of a Quadrilateral** : Two sides of a quadrilateral having no common end point
- **Adjacent Angles of a Quadrilateral** : Two angles of a quadrilateral having a common arm
- **Opposite Angles of a Quadrilateral** : Two angles of a quadrilateral not having a common arm
- **Important Results of Quadrilaterals**
  - ... The sum of all the angles of a quadrilateral is 360°
  - ... In a Parallelogram opposite sides and opposite angles are equal
  - ... The diagonals of a Parallelogram bisect each other
  - ... If a pair of opposite sides of a quadrilateral are equal and parallel, it is a Parallelogram
  - ... The diagonals of a rectangle are equal
  - ... The diagonals of a Square are equal and perpendicular to each other
  - ... The diagonals of a Rhombus bisect each other at right angles
- **Parallelogram** : A quadrilateral in which both pairs of opposite sides are parallel
- **Rectangle** : A Parallelogram each of whose angles measure 90°
- **Square** : A rectangle having all sides equal,
- **Rhombus** : A Parallelogram having all sides equal
- **Trapezium** : A quadrilateral in which two opposite sides are parallel
- **Isosceles Trapezium** : If the non parallel sides of a trapezium are equal, then it is Isosceles Trapezium
- **Kite** : A quadrilateral in which two pairs of adjacent sides are equal
- **Base of a Parallelogram** : Any side of a Parallelogram is called its Base
- **Altitude of a Parallelogram** : The length of the line segment which is perpendicular to the base from the opposite side
- **Results on Area**

... Parallelograms on the same base and between the same parallels are equal in area

1. Area of Parallelogram = base x height

... If a triangle and a parallelogram are on the same base and between the same parallels then

1. Area of Triangle =  $\frac{1}{2}$  x base x height

... Two triangles on the same base and between the same parallels are equal in area

... Two triangles having equal bases and lying between the same parallels are equal in area

- **Circle** : A simple closed curve consisting of all points in a plane which are at a fixed distance from a fixed point inside it
- **Interior of circle** : The part of the plane consisting of all those points whose distance from the center of the circle is less than the radius of the circle
- **Exterior of the circle** : The part of the plane consisting of all those points whose distance from the center of the circle is greater than the radius of the circle
- **Center of the circle** : The fixed point inside the circle
- **Circular Region** : The set of all points of the plane which lie either on the circle or inside the circle forms Circular Region
- **Radius** : A line segment joining any two point on the circle from its center
- **Chord** : A line segment whose end points lie on a circle
- **Diameter** : A chord of a circle passing through its center
  1. **Diameter** :  $2 \times$  Radius
- **Tangent** : A line which touches a circle at one point only
- **Point of Contact (Tangent)** : The point at which the tangent touches the circle
- **Arc** : Any part of a circle
- **Minor Arc** : An arc less than one half of the whole are of a circle
- **Major Arc** : An arc greater than one half of the whole of a circle
- **Semicircle** : One half of he whole arc of a circle
- **Semicircular Region** : The region enclosed by the diameter together with semicircle
- **Circumference** : The whole arc of the Circle
  1.  $C = 2\pi r$ ,  $r =$  Radius
- **Central Angle** : The angle formed by two radii at the ends of an arc of the circle, at the center of the circle is called the Angel Subtended by the Arc
- **Segment of a Circle** : The part of the circular region enclosed by an arc and Chord joining the end points of the arc together with the arc and the chord
- **Major Segment** : The segment containing the Major Arc

- **Minor Segment** : The segment containing the Minor Arc
- **Sector of a Circle** : The part of the circular region bounded by the arc and the two radii at the ends of the arc together with the arc and the radii
- **Minor Sector** : The sector containing the Minor Arc
- **Major Sector** : The sector Containing the Major Arc
- **Quadrant** : If two radii of a circle are at right angles to each other, then the sector is called Quadrant of the circle
- **Results on Similar Triangles**
  - ... If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio
  - ... If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side
  - ... If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio or proportion and hence the two triangles are similar
  - ... If in two triangles sides of one triangle are proportional to the sides of the other triangle then their corresponding angles are equal and hence the two triangles are Similar
  - ... If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar
  - ... The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides
- **Number of Tangents from a point on a circle**
  - ... There is no tangent to a circle passing through a point lying inside the circle
  - ... There is 1 and only 1 tangent to a circle passing through a point lying on the circle
  - ... There are exactly 2 tangents to a circle through a point lying outside the circle
- **Results on Tangents**
  - ... The tangent at any point of a circle is perpendicular to the radius through the point of contact
  - ... The length of tangents drawn from an external point to a circle are equal
  - ... If two circles touch each other, the point of contact lies on the straight line through their centers
  - ... If two tangents are drawn to a circle from an exterior point, then
    - i. The tangents are equal in length

- ii. The tangents subtend equal angles at the center
- iii. The tangents are equally inclined to the line joining the point and the center of the circle

- **Line Symmetry** : A figure is said to have Line Symmetry if its image coincides with the pre image under reflection in the particular line
- **Square** : A Square has 4 line symmetry lines, the two diagonals and perpendicular bisectors of each pair of opposite sides
- **Lines** : A line has an infinite number of symmetry lines, all perpendicular to it
- **Segments** : A segment has two symmetry lines, its perpendicular bisector and the segment itself
- **Angles** : An angle has one symmetry line, its angular bisector
- **Isosceles triangle** : An Isosceles triangle has one symmetry line, the angular bisector of the vertex, which is also the perpendicular bisector of the base
- **Equilateral triangle** : An Equilateral triangle has 3 symmetry lines, the angular bisector of each of the vertices
- **Isosceles Trapezium** : An Isosceles Trapezium has 1 symmetry line, the Perpendicular bisector of the parallel side
- **Rhombus** : A Rhombus has 2 symmetry lines, its diagonals
- **Rectangle** : A rectangle has 2 symmetry lines, the perpendicular bisectors of each pair of opposite sides
- **Circle** : A circle has infinite set of symmetry lines through the center
- **Semi circle** : 1 line of symmetry through the center
- **Regular Pentagon** : 5 lines of symmetry, the perpendiculars drawn from the opposite sides
- **Regular Hexagon** : 6 lines of Symmetry, namely 3 diagonals and 3 lines each joining the mid points of opposite sides of the hexagon
- **Results on Areas of Similar Triangles**
  - ... The area of two similar triangles are proportional to the squares on their corresponding sides
  - ... The areas of two similar triangles are proportional to the squares on their corresponding altitudes
  - ... The areas of two similar triangles are proportional to the squares on their corresponding Medians
  - ... The areas of two similar triangles are proportional to the squares on their corresponding angle bisector segments
- **Touching Circles** : Two circles are said to touch each other if and only if they have one and only one point in common

- **Direct Common Tangent** : A common tangent to two circles is called a Direct Common Tangent if both the circles lie on the same side of it
- **Transverse Common Tangent** : A common tangent to two circles is called Transverse Common Tangent if the circles lie on its opposite sides
- **Congruent Arcs** : Two arcs are said to be congruent if they have same degree measure
- **Congruent Circles** : Two circles of equal radii are said to be congruent
- **Concentric Circles** : Circles having same center but different radii
- **Concyclic Points** : The points which lie on the circumference of the same circle
- **Alternate Segments of a Circle** : The minor and major segments of a circle are called Alternate segments of each other
- **Chord Properties of a Circle**
  - ... The straight line drawn from the center of a circle to bisect a chord, which is not a diameter, is perpendicular to the chord
  - ... The perpendicular to a chord from the center of a circle bisects the chord
  - ... One and only one circle can be drawn passing through three non collinear points
  - ... Equal chords of a circle are equidistant from the center
  - ... Chords of a circle that are equidistant from the center of the circle, are equal
- **Angle properties of a Circle**
  - ... The angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circle
  - ... Angles in the same segment of a circle are equal
  - ... The angle in a Semicircle is a Right Angle
  - ... The opposite angles of a quadrilateral inscribed in a circle are Supplementary
  - ... The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle
- **Arc Properties of Circle**
  - ... In equal circles or in the same circle, if two arcs subtend equal angles at the center they are equal
  - ... In equal circles if two chords are equal, they cut off equal areas
- **Cyclic Quadrilateral** : If all the four vertices of a quadrilateral lie on a circle, then such a quadrilateral is called Cyclic Quadrilateral
- **Intersecting Chords and Tangents**
  - ... If two chords of a circle intersect internally or externally, then the products of the lengths of their segments are equal

- ... If a chord and a tangent intersect externally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection
- ... The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment
- **Circumcircle of a Triangle** : The circle passing through the vertices of a triangle
- **Results of Isosceles Triangle**
  - ... If two sides of a triangle are equal, then the angles opposite to them are also equal
  - ... If two angles of a triangle are equal, then the sides opposite to them are also equal
- **Inequalities**
  - ... If two sides of a triangle are unequal, then the greater side has greater angle opposite to it
  - ... If two angles of a triangle are unequal, then the greater angle has greater side opposite to it
  - ... The sum of any two sides of a triangle is greater than its third side
  - ... Of all the line segments that can be drawn to a given straight line from a given point outside it, the perpendicular is the shortest
- **Mid Point Theorem** : The line segment joining the mid points of any two sides of a triangle is parallel to the third side and equal to half of it
- **Intercept Theorem** : If a transversal makes equal intercepts on three or more parallel lines then any other line cutting them will also make equal intercepts
- **Angle Angle Axiom of Similarity** : If two triangles have two pairs of corresponding angles equal, then the triangles are similar
- **Basic Proportionality Theorem** : A line drawn parallel to one side of a triangle divides the other two sides proportionately
- **Converse of Basic Proportionality Theorem** : If a line divides any two sides of a triangle proportionally, the line is parallel to the third side
- **Results on Rectangle, Rhombus and Square**
  - ... If the diagonals of a parallelogram are equal then it is a rectangle
  - ... The diagonals of a rhombus bisect each other at right angles
  - ... If the diagonals of a parallelogram cut at right angles then it is a Rhombus
  - ... The diagonals of a square are equal and perpendicular to each other
  - ... If the diagonals of a parallelogram are equal and intersect at right angles, then the parallelogram is a square
- **Polygon Region** : A polygon region is the union of a polygon and its interior

- **Equal Figures** : Two plane figures having equal area are said to be equal
- **Results of a Polygon Region**
  - ... Parallelograms on the same base and between the same parallels are equal in area
  - ... The area of a parallelogram is equal to the area of the rectangle on the same base and of the same altitude i.e between the same parallels
  - ... A diagonal of a parallelogram divides it into two triangles
  - ... Triangles on the same base and between the same parallels are equal in area
- **Results of Traversal Line** : If a traversal intersects two parallel lines then
  - ... Each pair of corresponding angles are equal
  - ... Each pair of Alternate angles are equal
  - ... The sum of the interior angles on the same side of the traversal are 180°
- **Rectangular property of a circle**
  - ... If two chords of a circle intersect internally, then the product of the length of the segments are equal
  - ... If two chords of a circle intersect externally, then the product of the lengths of the two segments are equal
- **Alternate Segment Theorem** : If a straight line touches a circle and from the point of contact, a chord is drawn, the angle between the tangent and the chord is equal to an angle in the alternate segment
- **Right Circular Cone** : If a right angled triangle is revolved about one of the sides containing a right angle, the solid thus formed is Right Circular Cone
- **Sphere** : When a circle is revolved about a diameter the solid formed is Sphere
- **Hemisphere** : When a solid sphere is cut through its center into two equal halves, each half is called Hemisphere
- **Spherical Shell** : The solid enclosed between the two concentric spheres
- **Hemispherical Shell** : When a spherical shell is cut into two equal halves, a hemispherical shell is formed
- **Corresponding Angles** : Two angles on the same side of a traversal, if both lie either above the two lines or below the two lines
- **Apollonius Theorem** : In any triangle the sum of the squares on any two sides is equal to twice the square on half of the third side together with twice the square on the median which bisects the third side
- **Perimeter** : The Perimeter of a plane figure is the length of its boundary
- **Solids** : The bodies occupying space are called Solids
- **Volume of Solid** : The space occupied by the solid body
- **Cuboid** : A solid bounded by six rectangular plane surfaces

- **Cube** : A cuboid whose length, breadth and height are all equal
- **Curves** : The figure traced out with the help of the sharp edge of a pencil without lifting the pencil
- **Closed Figures** : The curves that begin and end at the same point
- **Open Figures** : The curves do not end at the starting point
- **Plane Figure** : A closed figure which does not intersect itself
- **Diagonal of a cuboid** : The line joining opposite corners of a cuboid
- **Lateral Surface area of Cuboid** : The sum of areas of the four walls of a cuboid
- **Total Surface Area of a Cuboid** : The sum of areas of all the six faces of a cuboid
- **Cross Section** : A cut which is made through a solid perpendicular to its length
- **Right Circular Cylinder** : A solid generated by the revolution of a rectangle about its side
- **Axis of cylinder** : The line joining the centers of circular cross sections at the ends of the cylinder
- **Height of cylinder** : The length of the axis of the cylinder
- **Radius of Cylinder** : The radius of any circular cross section
- **Solid Shapes Important Points.**
  - ... A cuboid has three pairs of identical (Congruent) faces
  - ... A cylinder has two identical (Congruent) faces
  - ... A cube has all faces congruent
- **Angles in circle**
  - ... Minor Arc subtends Acute Angle
  - ... Semi Circle Subtends Right Angle
  - ... Major Arc Subtends Obtuse Angle
- **Distance between the centers of Touching Circles**
  - ... If two circles touch each other externally, the distance between their centers is equal to the sum of their radii [  $d = R + r$  ]
  - ... If two circles touch each other internally, the distance between their centers is equal to the difference of their radii [  $d = R - r$  ]
- **Hollow Cylinder** : A cylinder formed by the lateral surface only
- **Solid Cylinder** : The region bounded by two circular plane surfaces and also lateral surface
- **Features of Right Circular Cylinder**
  - ... A right circular cylinder has two plane surfaces, circular in shape
  - ... The two circular planes are parallel to each other and also congruent
  - ... Radius of the circular plane is the radius of the cylinder
  - ... The curved surface joining the plane surface is the lateral surface of cylinder

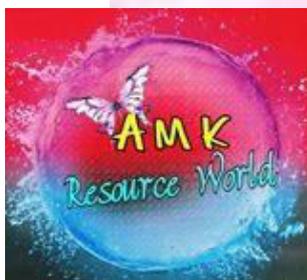
- **Features of Right Circular Cone**

- ... A cone has a circular plane at its base
- ... The point of intersection of the axis of the cone and slant height is the vertex of the cone
- ... The line joining the vertex and the center of the circular base is the height of the cone
- ... The curved surface which connects the vertex and circular edge of the base is the lateral surface of the cone
- ... The distance between the vertex and any point on the circumference of the base is the slant height

- **Classification of Polygons**

3	4	5	6	7	8	9	10	n
Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	Nonagon	Decagon	n - gon

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