HOW TO CALCULATE AND SIMULATE THE PERFORMANCE OF A HEAT PIPE

A heat pipe is a very efficient two-phase heat conductor consisting of a vessel in which its inner walls are lined with a wick structure. In manufacture, the heat pipe vessel is first vacuumed, then charged with a working fluid, and hermetically sealed. It can then be bent and flattened in order to meet tight design constraints [1] that may be encountered in small form factor or low space claim systems.

When the heat pipe is heated at one end, the working fluid evaporates from liquid to a vapour phase. Increased vapour pressure in the evaporator section causes the vapour to travel to the condenser end of the heat pipe. Here, the vapour condenses back to liquid and releases the heat absorbed in the evaporator section. The liquid then travels back from the condenser to the evaporator section of the heat pipe via the wick by capillary action [1].

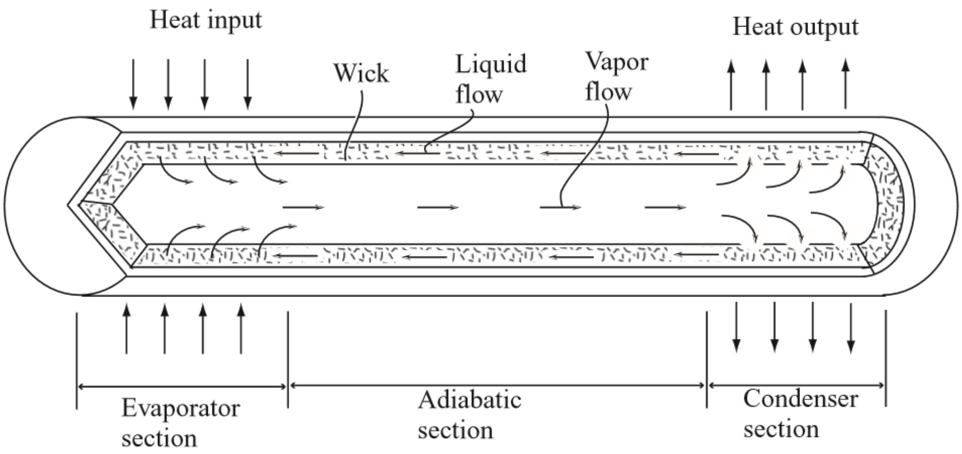

FIGURE 1 - CROSS SECTION OF A HEAT PIPE [2]

THE ELECTROTHERMAL NETWORK OF A HEATPIPE

For design engineers, understanding the temperature loss across a heat pipe of a given size is very useful for system design. A common metric for evaluating the performance of a heatpipe is the thermal resistance, which can be readily calculated by utilising an analogous electrothermal network. The overall thermal resistance of the heat pipe is composed of nine different resistances arranged in a series-parallel combination [3]. The external resistances identified $R_{ext,e}$ and $R_{ext,c}$ refer to any connecting resistances to the pipe, such as a heatsink or TIM in contact with the pipe, and are not evaluated further in this article.

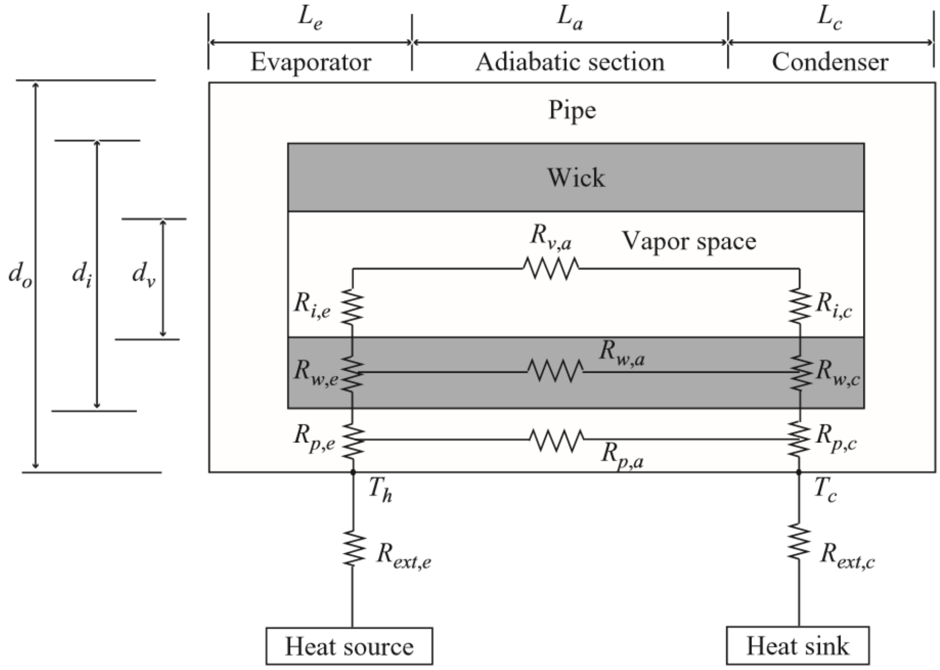

FIGURE 2 – HEAT PIPE THERMAL RESISTANCE NETWORK [2]

Where,
$R_{p,e}$ = Radial conduction resistance of the heat pipe wall at the evaporator
$R_{w,e}$ = Radial conduction resistance of the liquid-wick combination at the evaporator
$R_{i,e}$ = Radial convective resistance of the liquid-vapour interface at the evaporator
$R_{v,a}$ = Axial vapour thermal resistance of the adiabatic section
$R_{i,c}$ = Radial convective resistance of the liquid-vapour interface at the condenser
$R_{w,c}$ =Radial conduction resistance of the liquid-wick combination at the condenser
$R_{p,c}$ = Radial conduction resistance of the heat pipe wall at the condenser
$R_{w,a}$ = Axial conduction resistance of the liquid-wick combination at the adiabatic section
$R_{p,a}$ = Axial conduction resistance of the heat pipe wall at the adiabatic section

The overall thermal resistance, R_t, of the series-parallel combination is presented below:

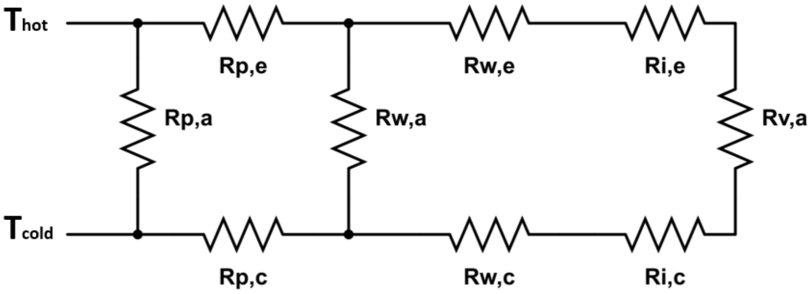

FIGURE 3 – ELECTROTHERMAL NETWORK OF A HEAT PIPE

Which can be calculated as,

$$R_t = \frac{1}{\frac{1}{R_{p,a}} + \frac{1}{\frac{1}{\frac{1}{R_{w,a}} + \frac{1}{R_{w,e} + R_{i,e} + R_{v,a} + R_{i_c} + R_{w,C}}} + R_{p,e} + R_{p,c}}} \quad (1)$$

In an electrical circuit in parallel, the resistances with large values may be treated as open circuit (infinitely high resistance), while in a circuit in series the resistances with small values may be treated as short circuit (zero resistance). Typical values for these resistances described above can be found in Table 1 below:

TABLE 1 - COMPARATIVE VALUES FOR HEAT PIPE RESISTANCES [2]

Resistance	°C/W
$R_{p,e}$ and $R_{p,c}$	10^{-1}
$R_{w,e}$ and $R_{w,c}$	10^{+1}
$R_{i,e}$ and $R_{i,c}$	10^{-5}
$R_{v,a}$	10^{-8}
$R_{p,a}$	10^{+2}
$R_{w,a}$	10^{+4}

Considering the values in Table 1, the circuit in parallel can be simplified to a circuit in series by omitting the two axial thermal resistances $R_{p,a}$ and $R_{w,a}$. The new circuit in series can be simplified further by making a short circuit of $R_{i,e}$, $R_{v,a}$ and $R_{i,c}$ due to the comparatively small values [2].

Hence the total thermal resistance of the heat pipe, R_t, becomes the sum of the series resistances below:

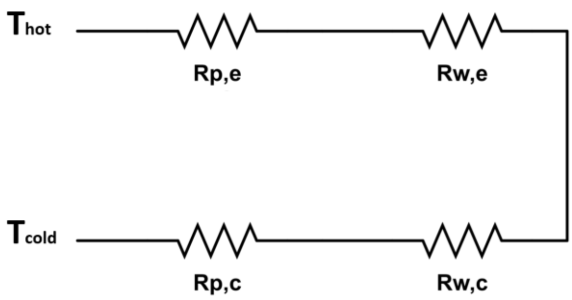

FIGURE 4 – SIMPLIFIED ELECTROTHERMAL NETWORK OF A HEAT PIPE

Represented as,

$$R_t = R_{p,e} + R_{w,e} + R_{w,c} + R_{p,c} \qquad (2)$$

CALCULATING RESISTANCE VALUES IN THE HEATPIPE NETWORK

Calculation of the resistance of each of these values is dependent on many factors, including: the shape of the pipe, the materials involved and the heat transfer method. The following section details the fundamental equations that drive each section of heat transfer as described by Equation (2).

For a plate (flattened) heat pipe, the conduction resistance at the evaporator is linear [2] and so:

$$R_{p,e} = \frac{\Delta x_e}{k_p A} \qquad (3)$$

Where,
$R_{p,e}$ = conduction resistance of the plate heat pipe at the evaporator (°C/W)

k_p= thermal conductivity of the heat pipe material (W/mK)
Δx_e = thickness of the heat pipe wall (m)
A = cross sectional area (m^2)

For a cylindrical heat pipe, the radial conduction resistance at the evaporator through the heat pipe wall is [2]:

$$R_{p,e} = \frac{\ln \frac{d_o}{d_i}}{2\pi l_e k_p} \qquad (4)$$

Where,
$R_{p,e}$ = radial conduction resistance of the heat pipe wall at the evaporator (°C/W)
k_p= thermal conductivity of the heat pipe material (W/mK)
d_o = outer diameter of the pipe (m)
d_i = inner diameter of the pipe (m)
l_e = length of evaporator (m)

For a cylindrical heat pipe, the radial conduction resistance at the evaporator through the wick is [2]:

$$R_{w,e} = \frac{\ln \frac{d_i}{d_v}}{2\pi l_e k_{wick}} \qquad (5)$$

Where,
$R_{w,e}$= Resistance of the liquid-wick combination at the evaporator (°C/W)
k_{wick}= thermal conductivity of the wick material (W/mK)
d_i= inner diameter of the pipe (m)
d_v= inner diameter of the wick (m)
l_e = length of evaporator (m)

The exact geometric configuration of a sintered wick is unknown because of the random dispersion of the particles and the varying degree of deformation and fusion which occurs during the sintering process. For this reason, it is suggested that the sintered wick be represented by a continuous solid phase containing a random dispersion of randomly sized spheres of liquid. Maxwell has derived an expression that gives the thermal conductivity of such a heterogeneous material [5].

$$k_{wick} = k_s \left[\frac{2 + \frac{k_l}{k_s} - 2\varepsilon(1 - \frac{k_l}{k_s})}{2 + \frac{k_l}{k_s} + \varepsilon(1 - \frac{k_l}{k_s})} \right] \qquad (6)$$

Where,
k_s= thermal conductivity of the solid wick material (W/mK)
k_l= thermal conductivity of liquid phase in the wick (W/mK)
ε= volume fraction of liquid phase

Equations for the condenser end would be similar to these described, with any variation of diameter or length considered. If the evaporator and condenser are the same length, Equation (2) simplifies to:

$$R_t = 2(R_{p,e} + R_{w,e}) \qquad (7)$$

Although typically discounted from heatpipe calculations due to the low circuit resistance the axial vapour thermal resistance, $R_{v,a}$, can be calculated in practise, using the Clausius-Clapeyron criterion [4] shown below for reference:

$$R_{v,a} = \frac{\pi r_v^2 T_v F_v \left(\frac{1}{6} L_e + L_a + \frac{1}{6} L_c\right)}{\rho_v \lambda} \qquad (8)$$

Given,

$$F_v = \left(\frac{(f_v Re)\mu_v}{2 r_v^2 A_v \rho_v \lambda}\right) \qquad (9)$$

Where,
$f_v Re$= is the friction coefficient (dimensionless)
r_v= hydraulic radius of the vapour section (m)
T_v= vapour temperature (K)
ρ_v= vapour density (kg/m^3)
μ_v= vapour viscosity (Pa.s)
λ= latent heat of vaporisation (J/kg)
A_v= cross sectional area of the vapour section (m^2)
l_e= length of evaporator region (m)
l_a= length of adiabatic region (m)
l_c= length of condenser region (m)

THE EFECTIVE THERMAL CONDUCVITIVY OF A HEATPIPE

The total effective thermal conductivity, k_{eff}, of a heat pipe as a single piece is very useful in both estimating and simulating its performance. Once the overall thermal resistance, R_t, is known it is possible to determine the effective thermal conductivity, k_{eff}, and the temperature drop across the pipe, ΔT ,required to drive Q [1].

$$\Delta T = R_t Q \qquad (10)$$

Where,
R_t = total thermal resistance in (°C/W)
ΔT= total temperature drop from source to sink (°C)
Q = heat transfer rate through the heat pipe (W)

Q_{max} is dependent on the capillary pumping pressure of the system, and although its determination is out of scope for this technical paper please refer to Entropy's document on how to calculate the maximum power of a heat pipe for details. Through manipulation of Fourier's law of conduction and using Equation (10) above, a value for k_{eff} is calculated which can be used to estimate the overall conductivity of the heat pipe:

$$Q = kA\frac{\Delta T}{\Delta x} \qquad (11)$$

Hence,

$$k_{eff} = \frac{\Delta x}{R_t A} \qquad (12)$$

Where,
R_t = total thermal resistance in (°C/W)
Δx = effective length, l_{eff}, of the heat-pipe (m)
$A = \pi r^2$, cross sectional area of heat pipe (m^2)

Since mass flow will vary in both the evaporator and the condenser region, an effective length rather than the geometrical length must be used for these regions [5]. This value assumes that the vapour flow reduces linearly towards the end of the pipe across the length of the evaporator or condenser, so it is assumed the functional length is half.

$$l_{eff} = l_a + \frac{l_e + l_c}{2} \qquad (13)$$

Where,
l_a = length of adiabatic region (m)
l_e = length of evaporator region (m)
l_c = length of condenser region (m)

Using Equation (12), the heat pipe geometry can be simplified as a steady state solid single body model for calculation and thermal simulation. The effective length l_{eff} is used to calculate k_{eff}, but the actual geometrical Δx is used to represent the single body model, see Figure 5 below:

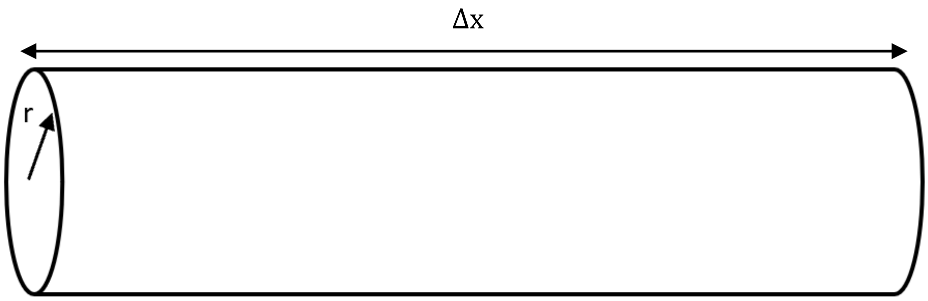

FIGURE 5 – SIMPLIFIED 1D STEADY STATE CONDUCTION SIMULATION MODEL OF A HEAT PIPE

It should be noted this method works only for heat pipes with a single evaporator and condenser, however it is extremely useful for evaluating the comparative impact of making geometrical and structural changes to a pipe for the benefit of performance. These calculations provide a good estimate of the performance of a common linear tube heat pipe, and the impact of heat pipe bending and flattening is not discussed within this report. For further evaluation and discuss of heat pipe use and technology, please refer to other relevant Entropy media.

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