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SECTION I
MEASUREMENT
**Chapter 1: Measurement**  
- SI Units  
- Errors and Uncertainties  
- Scalars and Vectors

### a. Recall the following base quantities and their units; mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).

<table>
<thead>
<tr>
<th>Base Quantities</th>
<th>SI Units</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>metre</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Amount of substance</td>
<td>mole</td>
<td>mol</td>
</tr>
<tr>
<td>Temperature</td>
<td>Kelvin</td>
<td>K</td>
</tr>
<tr>
<td>Current</td>
<td>ampere</td>
<td>A</td>
</tr>
<tr>
<td>Luminous intensity</td>
<td>candela</td>
<td>cd</td>
</tr>
</tbody>
</table>

### b. Express derived units as products or quotients of the base units and use the named units listed in ‘Summary of Key Quantities, Symbols and Units’ as appropriate.

A derived unit can be expressed in terms of products or quotients of base units.

<table>
<thead>
<tr>
<th>Derived Quantities</th>
<th>Equation</th>
<th>Derived Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (A)</td>
<td>$A = L^2$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Volume (V)</td>
<td>$V = L^3$</td>
<td>m$^3$</td>
</tr>
<tr>
<td>Density ($\rho$)</td>
<td>$\rho = \frac{m}{V}$</td>
<td>kg m$^{-3}$ = kg m$^3$</td>
</tr>
<tr>
<td>Velocity (v)</td>
<td>$v = \frac{L}{t}$</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>Acceleration (a)</td>
<td>$a = \frac{\Delta v}{t}$</td>
<td>m s$^{-2}$</td>
</tr>
<tr>
<td>Momentum (p)</td>
<td>$p = m \times v$</td>
<td>(kg)(m s$^{-1}$) = kg m s$^{-1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Derived Quantities</th>
<th>Equation</th>
<th>Derived Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force (F)</td>
<td>$F = \frac{\Delta p}{t}$</td>
<td>Newton N</td>
</tr>
<tr>
<td>Pressure (p)</td>
<td>$p = \frac{F}{A}$</td>
<td>Pascal Pa</td>
</tr>
<tr>
<td>Energy (E)</td>
<td>$E = F \times d$</td>
<td>joule J</td>
</tr>
<tr>
<td>Power (P)</td>
<td>$P = \frac{E}{t}$</td>
<td>watt W</td>
</tr>
<tr>
<td>Frequency (f)</td>
<td>$f = \frac{1}{T}$</td>
<td>hertz Hz</td>
</tr>
<tr>
<td>Charge (Q)</td>
<td>$Q = I \times t$</td>
<td>coulomb C</td>
</tr>
<tr>
<td>Potential Difference (V)</td>
<td>$V = \frac{E}{Q}$</td>
<td>volt V</td>
</tr>
<tr>
<td>Resistance (R)</td>
<td>$R = \frac{V}{I}$</td>
<td>ohm $\Omega$</td>
</tr>
</tbody>
</table>

**www.megalecture.com**

mob: +92 323 509 4443, email: megalecture@gmail.com

**www.youtube.com/megalecture**
c. Show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication SI Units, Signs, Symbols and Systematics (The ASE Companion to 5-16 Science, 1995).

Self-explanatory

d. Use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (K), mega (M), giga (G), tera (T).

<table>
<thead>
<tr>
<th>Multiplying Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-12}$</td>
<td>pico</td>
<td>p</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro</td>
<td>μ</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>deci</td>
<td>d</td>
</tr>
<tr>
<td>$10^{1}$</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>$10^{3}$</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>$10^{6}$</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>tera</td>
<td>T</td>
</tr>
</tbody>
</table>

e. Make reasonable estimates of physical quantities included within the syllabus.

When making an estimate, it is only reasonable to give the figure to 1 or at most 2 significant figures since an estimate is not very precise.

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Reasonable Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of 3 cans (330 ml) of Coke</td>
<td>1 kg</td>
</tr>
<tr>
<td>Mass of a medium-sized car</td>
<td>1000 kg</td>
</tr>
<tr>
<td>Length of a football field</td>
<td>100 m</td>
</tr>
<tr>
<td>Reaction time of a young man</td>
<td>0.2 s</td>
</tr>
</tbody>
</table>

- Occasionally, students are asked to estimate the area under a graph. The usual method of counting squares within the enclosed area is used. (eg. Topic 3 (Dynamics), N94P2Q1c)
- Often, when making an estimate, a formula and a simple calculation may be involved.

**EXAMPLE 1E1**

Estimate the average running speed of a typical 17-year-old’s 2.4-km run.

\[
\text{velocity} = \frac{\text{distance}}{\text{time}} = \frac{2400}{12.5 \times 60} = 3.2
\]

\[
\approx 3.2 \text{ m s}^{-1}
\]

**EXAMPLE 1E2 (N08/1/2)**

Which estimate is realistic?

<table>
<thead>
<tr>
<th>Option</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>The kinetic energy of a bus travelling on an expressway is 30 000 J. A bus of mass $m$ travelling on an expressway will travel between 50 to 80 km h$^{-1}$, which is 13.8 to 22.2 m s$^{-1}$. Thus, its KE will be approximately $\frac{1}{2} m (18^2) = 162m$. Thus, for its KE to be 30 000J: $162m = 30 000$. Thus, $m = 185kg$, which is an absurd weight for a bus; ie. This is not a realistic estimate.</td>
</tr>
<tr>
<td>B</td>
<td>The power of a domestic light is 300W. A single light bulb in the house usually runs at about 20 W to 60 W. Thus, a domestic light is unlikely to run at more than 200W; this estimate is rather high.</td>
</tr>
<tr>
<td>C</td>
<td>The temperature of a hot oven is 300 K. 300K = 27°C. Not very hot.</td>
</tr>
</tbody>
</table>
The volume of air in a car tyre is 0.03 m$^3$.

Estimating the width of a tyre, $t$, is 15 cm or 0.15 m, and estimating $R$ to be 40 cm and $r$ to be 30 cm, volume of air in a car tyre is

\[ = \pi(R^2 - r^2)t \]

\[ = \pi(0.4^2 - 0.3^2)(0.15) \]

\[ = 0.033 \text{ m}^3 \approx 0.03 \text{ m}^3 \text{ (to one sig. fig.)} \]

- **f. g.** Show an understanding of the distinction between systematic errors (including zero errors) and random errors.

- **f. g.** Show an understanding of the distinction between precision and accuracy.

**Random error** is the type of error which causes readings to scatter about the true value.

**Systematic error** is the type of error which causes readings to deviate in one direction from the true value.

**Precision**: refers to the degree of agreement (scatter, spread) of repeated measurements of the same quantity. (NB: regardless of whether or not they are correct.)

**Accuracy** refers to the degree of agreement between the result of a measurement and the true value of the quantity.

- → R Error Higher → → → Less Accurate → → →
- → → → S Error Higher → → → Less Precise → → →

- **h.** Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties (a rigorous statistical treatment is not required).

For a quantity $x = (2.0 \pm 0.1)$ mm,

**Actual/ Absolute uncertainty**, \( \Delta x \) = ± 0.1 mm

**Fractional uncertainty**, \( \frac{\Delta x}{x} \) = 0.05

**Percentage uncertainty**, \( \frac{\Delta x}{x} \times 100\% \) = 5%

If $p = \frac{2x + y}{3}$ or $p = \frac{2x - y}{3}$, \( \Delta p = \frac{2\Delta x + \Delta y}{3} \)

If $r = 2xy^3$ or $r = \frac{2x}{y}$, \( \frac{\Delta r}{r} = \frac{\Delta x}{x} + \frac{3\Delta y}{y} \)

Actual error must be recorded to only 1 significant figure, & The number of decimal places a calculated quantity should have is determined by its actual error.
For eg, suppose \( g \) has been initially calculated to be 9.80645 m s\(^{-2}\) \& \( \Delta g \) has been initially calculated to be 0.04848 m s\(^{-2}\). The final value of \( \Delta g \) must be recorded as 0.05 m s\(^{-2}\) (1 sf), and the appropriate recording of \( g \) is (9.81 ± 0.05) m s\(^{-2}\).

<table>
<thead>
<tr>
<th>Type</th>
<th>Scalar</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
<td>A <strong>scalar</strong> quantity has a <strong>magnitude only</strong>. It is completely described by a certain number and a unit.</td>
<td>A <strong>vector</strong> quantity has both <strong>magnitude and direction</strong>. It can be described by an arrow whose length represents the magnitude of the vector and the arrow-head represents the direction of the vector.</td>
</tr>
<tr>
<td>Examples</td>
<td>Distance, speed, mass, time, temperature, <strong>work done</strong>, kinetic energy, pressure, <strong>power</strong>, electric charge etc.</td>
<td>Displacement, velocity, moments (or torque), momentum, force, electric field etc.</td>
</tr>
</tbody>
</table>

**Common Error:**
Students tend to associate kinetic energy and pressure with vectors because of the vector components involved. However, such considerations have no bearings on whether the quantity is a vector or scalar.

j. k. **Add and subtract coplanar vectors.**
Represent a vector as two perpendicular components.

In the diagram below, XY represents a flat kite of weight 4.0 N. At a certain instant, XY is inclined at 30° to the horizontal and the wind exerts a steady force of 6.0 N at right angles to XY so that the kite flies freely.

![Diagram of kite](image)

**By accurate scale drawing**
Draw a scale diagram to find the magnitude and direction of the resultant force acting on the kite.

Scale: 1 cm = 1.0 N
resultant, \( R \)

**By calculations using sine and cosine rules, or Pythagoras’ theorem**

Using cosine rule,
\[
\begin{align*}
R^2 &= a^2 + b^2 - 2ab \cos \alpha \\
R &= \sqrt{a^2 + b^2 - 2ab \cos \alpha} \\
R &= \sqrt{4.0^2 + 6.0^2 - 2(4.0)(6.0)\cos 30°} \\
R &\approx 3.23 \text{ N}
\end{align*}
\]
\[ R = 3.2 \text{ N (3.2 cm)} \]
\[ \text{at } \theta = 112^\circ \text{ to the 4 N vector.} \]

Using sine rule,

\[ \frac{a}{\sin A} = \frac{b}{\sin B} \]
\[ \frac{6}{\sin \alpha} = \frac{3.23}{\sin 30^\circ} \]
\[ \alpha = 68^\circ \text{ or } 112^\circ \]
\[ = 112^\circ \text{ to the 4 N vector} \]

**Summing Vector Components**

\[ F_x = -6 \sin 30^\circ \]
\[ = -3 \text{ N} \]
\[ F_y = 6 \cos 30^\circ - 4 \]
\[ = 1.2 \text{ N} \]

\[ R = \sqrt{(-3)^2 + (1.2)^2} \]
\[ = 3.23 \text{ N} \]

\[ \tan \theta = \frac{1.2}{3} \]
\[ \theta = 22^\circ \]

\[ R \text{ is at an angle } 112^\circ \text{ to the 4 N vector. (90^\circ + 22^\circ)} \]
SECTION II
NEWTONIAN MECHANICS
Chapter 2: Kinematics
- Rectilinear Motion
- Non-linear Motion

a. Define displacement, speed, velocity and acceleration.

Distance: Total length covered irrespective of the direction of motion.

Displacement: Distance moved in a certain direction

Speed: Distance travelled per unit time.

Velocity: is defined as the rate of change of displacement, or displacement per unit time

(Note: displacement over time, nor, displacement per second, nor, rate of change of displacement per unit time)

Acceleration: is defined as the rate of change of velocity.

b. Use graphical methods to represent distance travelled, displacement, speed, velocity and acceleration.

Self-explanatory

c. Find displacement from the area under a velocity-time graph.

The area under a velocity-time graph is the change in displacement.

d. Use the slope of a displacement-time graph to find velocity.

The gradient of a displacement-time graph is the (instantaneous) velocity.

e. Use the slope of a velocity-time graph to find acceleration.

The gradient of a velocity-time graph is the acceleration.

f. Derive, from the definitions of velocity and acceleration, equations that represent uniformly accelerated motion in a straight line.

Solve problems using equations which represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without acceleration.

1. \( v = u + at \): derived from definition of acceleration: \( a = \frac{(v - u)}{t} \)
2. \( s = \frac{1}{2} (u + v) t \): derived from the area under the \( v-t \) graph
3. \( v^2 = u^2 + 2as \): derived from equations (1) and (2)
4. \( s = ut + \frac{1}{2} at^2 \): derived from equations (1) and (2)

These equations apply only if the motion takes place along a straight line and the acceleration is constant (hence, for eg., air resistance must be negligible.)

h. Describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance.

Consider a body moving in a uniform gravitational field under 2 different conditions:

A WITHOUT AIR RESISTANCE

Assuming negligible air resistance, whether the body is moving up, or at the highest point or moving down, the weight of the body, \( W \), is the only force acting on it, causing it to experience a constant acceleration. Thus, the gradient of the \( v-t \) graph is constant throughout its rise and fall. The body is said to undergo free
B WITH AIR RESISTANCE

If air resistance is NOT negligible and if it is projected upwards with the same initial velocity, as the body moves upwards, both air resistance and weight act downwards. Thus its speed will decrease at a rate greater than 9.81 m s$^{-2}$. This causes the time taken to reach its maximum height reached to be lower than in the case with no air resistance. The max height reached is also reduced.

At the highest point, the body is momentarily at rest; air resistance becomes zero and hence the only force acting on it is the weight. The acceleration is thus 9.81 m s$^{-2}$ at this point.

As a body falls, air resistance opposes its weight. The downward acceleration is thus less than 9.81 m s$^{-2}$. As air resistance increases with speed (Topic 5), it eventually equals its weight (but in opposite direction). From then there will be no resultant force acting on the body and it will fall with a constant speed, called the terminal velocity.

i. Describe and explain motion due to a uniform velocity in one direction and uniform acceleration in a perpendicular direction.

Equations that are used to describe the horizontal and vertical motion

<table>
<thead>
<tr>
<th>s (displacement)</th>
<th>$x$ direction (horizontal – axis)</th>
<th>$y$ direction (vertical – axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_x$</td>
<td>$s_x = u_x t$</td>
<td>$s_y = u_y t + \frac{1}{2} a_y t^2$</td>
</tr>
<tr>
<td></td>
<td>$s_x = u_x t + \frac{1}{2} a_x t^2$</td>
<td>(Note: If projectile ends at same level as the start, then $s_y = 0$)</td>
</tr>
<tr>
<td>u (initial velocity)</td>
<td>$u_x$</td>
<td>$u_y$</td>
</tr>
<tr>
<td>v (final velocity)</td>
<td>$v_x = u_x + a_x t$</td>
<td>$v_y = u_y + at$</td>
</tr>
<tr>
<td></td>
<td>(Note: At max height, $v_x = 0$)</td>
<td>$v_y^2 = u_y^2 + 2 a_y s_y$</td>
</tr>
<tr>
<td>a (acceleration)</td>
<td>$a_x$</td>
<td>$a_y$</td>
</tr>
<tr>
<td></td>
<td>(Note: Exists when a force in x direction present)</td>
<td>(Note: If object is falling, then $a_y = -g$)</td>
</tr>
<tr>
<td>t (time)</td>
<td>$t$</td>
<td>$t$</td>
</tr>
</tbody>
</table>

Parabolic Motion: $\tan \theta = \frac{v_y}{v_x}$

$\theta$: direction of tangential velocity  
{NOT: $\tan \theta = \frac{s_y}{s_x}$}
Chapter 3: Dynamics
- Newton’s laws of motion
- Linear momentum and its conservation

a. State each of Newton’s laws of motion.

**Newton’s First Law**
Every body continues in a state of rest or uniform motion in a straight line unless a net (external) force acts on it.

**Newton’s Second Law**
The rate of change of momentum of a body is directly proportional to the net force acting on the body, and the momentum change takes place in the direction of the net force.

**Newton’s Third Law**
When object X exerts a force on object Y, object Y exerts a force of the same type that is equal in magnitude and opposite in direction on object X.

The two forces ALWAYS act on different objects and they form an action-reaction pair.

b. Show an understanding that mass is the property of a body which resists change in motion.

**Mass:** is a measure of the amount of matter in a body, & is the property of a body which resists change in motion.

c. Describe and use the concept of weight as the effect of a gravitational field on a mass.

**Weight:** is the force of gravitational attraction (exerted by the Earth) on a body.

d. Define linear momentum and impulse.

**Linear momentum** of a body is defined as the product of its mass and velocity ie \( p = m \cdot v \)

**Impulse of a force** \( I \) is defined as the product of the force and the time \( \Delta t \) during which it acts

\[ I = F \times \Delta t \]

{for force which is \( \text{const} \) over the duration \( \Delta t \)}

For a variable force, the impulse = Area under the \( F-t \) graph (\( \int F \cdot dt \); may need to “count squares”)

Impulse is equal in magnitude to the change in momentum of the body acted on by the force. Hence the change in momentum of the body is equal in magnitude to the area under a (net) force-time graph. {Incorrect to define impulse as change in momentum}

e. Define force as rate of change of momentum.

**Force** is defined as the rate of change of momentum, ie \( F = \frac{m(v - u)}{t} = ma \) or \( F = \frac{dv}{dt} \)

The (one) Newton is defined as the force needed to accelerate a mass of 1 kg by 1 m s\(^{-2}\).

f. Recall and solve problems using the relationship \( F = ma \) appreciating that force and acceleration are always in the same direction.

Self-explanatory

g. State the principle of conservation of momentum.

**Principle of Conservation of Linear Momentum:** When objects of a system interact, their total momentum before and after interaction are equal if no net (external) force acts on the system.

or, The total momentum of an **isolated** system is constant

\[ m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \]

if net \( F = 0 \) {for all collisions}

NB: Total momentum **DURING** the interaction/collision is also conserved.

h. Apply the principle of conservation of momentum to solve problems including elastic and inelastic
Interactions between two bodies in one dimension. (Knowledge of coefficient of restitution is not required.)

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(Perfectly) elastic collision:</strong></td>
<td>Both momentum &amp; kinetic energy of the system are conserved.</td>
</tr>
<tr>
<td>Inelastic collision:</td>
<td>Only momentum is conserved, total kinetic energy is not conserved.</td>
</tr>
<tr>
<td>Perfectly inelastic collision:</td>
<td>Only momentum is conserved, and the particles stick together after collision. (i.e. move with the same velocity.)</td>
</tr>
</tbody>
</table>

i. Recognise that, for a perfectly elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

For all elastic collisions, \( u_1 - u_2 = v_2 - v_1 \)

ie. **relative speed of approach = relative speed of separation**

or, \( \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \)

j. Show an understanding that, whilst the momentum of a system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

In inelastic collisions, total energy is conserved but Kinetic Energy may be converted into other forms of energy such as sound and heat energy.
Chapter 4: Forces
- Types of force
- Equilibrium of force
- Centre of gravity
- Turning effects of forces

a. Recall and apply Hooke’s Law to new situations or to solve related problems.

Within the limit of proportionality, the extension produced in a material is directly proportional to the force/load applied

\[ F = kx \]

Force constant \( k \) = force per unit extension \((F/x)\) \{N08P3Q6b(ii)\}

b. Deduce the elastic potential energy in a deformed material from the area under a force-extension graph.

Elastic potential energy/strain energy = Area under the F-x graph \( \text{(May need to “count the squares”)\} \)

For a material that obeys Hooke’s law,

Elastic Potential Energy, \( E = \frac{1}{2} F x = \frac{1}{2} k x^2 \)

c. Describe the forces on mass, charge and current in gravitational, electric and magnetic fields, as appropriate.

Forces on Masses in Gravitational Fields - A region of space in which a mass experiences an (attractive) force due to the presence of another mass.

Forces on Charge in Electric Fields - A region of space where a charge experiences an (attractive or repulsive) force due to the presence of another charge.

Forces on Current in Magnetic Fields - Refer to Chapter 15

d. Solve problems using \( p = \rho g h \).

Hydrostatic Pressure \( p = \rho g h \)

\{or, pressure difference between 2 points separated by a vertical distance of \( h \)\}

e. f. Show an understanding of the origin of the upthrust acting on a body in a fluid.

State that an upthrust is provided by the fluid displaced by a submerged or floating object.

Upthrust: An upward force exerted by a fluid on a submerged or floating object; arises because of the difference in pressure between the upper and lower surfaces of the object.

g. h. Calculate the upthrust in terms of the weight of the displaced fluid.

Recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal to the weight of the new object to new situations or to solve related problems.

Archimedes’ Principle: Upthrust = weight of the fluid displaced by submerged object.

\[ \text{ie} \quad \text{Upthrust} = \text{Vol}_{\text{submerged}} \times \rho_{\text{fluid}} \times g \]

i. Show a qualitative understanding of frictional forces and viscous forces including air resistance. \( \text{(No treatment of the coefficients of friction and viscosity is required.)}\)

Frictional Forces:
- The contact force between two surfaces = \((\text{friction}^2 + \text{normal reaction}^2)^{1/2}\)
- The component along the surface of the contact force is called friction.
- Friction between 2 surfaces always opposes relative motion (or attempted motion), and
- Its value varies up to a maximum value (called the static friction)

Viscous Forces:
- A force that opposes the motion of an object in a fluid;
- Only exists when there is (relative) motion;
- Magnitude of viscous force increases with the speed of the object

j. **Use a vector triangle to represent forces in equilibrium.**

See Chapter 1j, 1k

k. **Show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity.**

Centre of Gravity of an object is defined as that pt through which the entire weight of the object may be considered to act.

l. **Show an understanding that a couple is a pair of forces which tends to produce rotation only.**

A couple is a pair of forces which tends to produce rotation only.

m. **Define and apply the moment of a force and the torque of a couple.**

- **Moment of a Force:** The product of the force and the perpendicular distance of its line of action to the pivot
- **Torque of a Couple:** The produce of one of the forces of the couple and the perpendicular distance between the lines of action of the forces. (WARNING: NOT an action-reaction pair as they act on the same body.)

n. **Show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.**

**Conditions for Equilibrium** (of an extended object):
1. The resultant force acting on it in any direction equals zero
2. The resultant moment about any point is zero.

If a mass is acted upon by 3 forces only and remains in equilibrium, then
1. The lines of action of the 3 forces must pass through a common point.
2. When a vector diagram of the three forces is drawn, the forces will form a closed triangle (vector triangle), with the 3 vectors pointing in the same orientation around the triangle.

o. **Apply the principle of moments to new situations or to solve related problems.**

- **Principle of Moments:** For a body to be in equilibrium, the sum of all the anticlockwise moments about any point must be equal to the sum of all the clockwise moments about that same point.
## Chapter 5: Work, Energy and Power

- Work
- Energy conversion and conservation
- Potential energy and kinetic energy
- Power

### a. Show an understanding of the concept of work in terms of the product of a force and displacement in the direction of the force.

Work Done by a force is defined as the product of the force and displacement (of its point of application) in the direction of the force.

\[ W = F \cdot s \cdot \cos \theta \]

Negative work is said to be done by a force if its line of action is anti-parallel to the displacement.

If a variable force \( F \) produces a displacement in the direction of \( F \), the work done is determined from the area under \( F \times x \) graph. (May need to find area by “counting the squares”.)

### b. Calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: \( W = p \Delta V \).

**Work Done by a force** is defined as the product of the force and displacement (of its point of application) in the direction of the force.

\[ W = F \cdot s \cdot \cos \theta \]

### c. Give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation to simple examples.

By Principle of Conservation of Energy,

**Work Done on a system** = KE gain + GPE gain + Thermal Energy generated (ie Work done against friction)

### d. Derive, from the equations of motion, the formula \( E_k = \frac{1}{2}mv^2 \).

Consider a rigid object of mass \( m \) that is initially at rest. To accelerate it uniformly to a speed \( v \), a constant net force \( F \) is exerted on it, parallel to its motion over a displacement \( s \).

Since \( F \) is constant, acceleration is constant.

Therefore, using the equation:

\[ \begin{align*}
    v^2 &= u^2 + 2a \cdot s \\
    a \cdot s &= \frac{1}{2} (v^2 - u^2)
\end{align*} \]

Since kinetic energy is equal to the work done on the mass to bring it from rest to a speed \( v \),

The kinetic energy, \( E_k = \) Work done by the force \( F \)

\[ E_k = F \cdot s = m \cdot a \cdot s = \frac{1}{2} m(v^2 - u^2) \]

### e. Recall and apply the formula \( E_k = \frac{1}{2}mv^2 \).

Self-explanatory

### f. Distinguish between gravitational potential energy, electric potential energy and elastic potential energy.

**Gravitational potential energy**: this arises in a system of masses where there are attractive gravitational forces between them. The gravitational potential energy of an object is the energy it possesses by virtue of its position in a gravitational field.

**Elastic potential energy**: this arises in a system of atoms where there are either attractive or repulsive short-range inter-atomic forces between them. (From Topic 4, E. P. E. = \( \frac{1}{2} k x^2 \).

**Electric potential energy**: this arises in a system of charges where there are either attractive or repulsive...
g. **Show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems.**

The potential energy, $U$, of a body in a force field (whether gravitational or electric field) is related to the force $F$ it experiences by: 

$$ F = \frac{dU}{dx} \cdot $$

h. **Derive, from the defining equation $W = Fs$ the formula $E_p = mgh$ for potential energy changes near the Earth’s surface.**

Consider an object of mass $m$ being lifted vertically by a force $F$, without acceleration, from a certain height $h_1$ to a height $h_2$. Since the object moves up at a constant speed, $F$ is equal to $mg$.

The change in potential energy of the mass = Work done by the force $F$

$$ = F \cdot s $$

$$ = F \cdot h $$

$$ = m \cdot g \cdot h $$

i. **Recall and use the formula $E_p = mgh$ for potential energy changes near the Earth’s surface.**

Self-explanatory

j. **Show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems.**

**Efficiency:** The ratio of (useful) output energy of a machine to the input energy.

$$ \text{Efficiency} = \frac{\text{Useful Output Energy}}{\text{Input Energy}} \times 100\% = \frac{\text{Useful Output Power}}{\text{Input Power}} \times 100\% $$

k. **Define power as work done per unit time and derive power as the product of force and velocity.**

**Power** (instantaneous) is defined as the work done per unit time.

$$ P = \frac{\text{Total Work Done}}{\text{Total Time}} $$

$$ = \frac{W}{t} $$

Since work done $W = F \times s$,

$$ P = \frac{F \times s}{t} $$

$$ = F \cdot \frac{s}{t} $$

- for object moving at **const speed:** $F = \text{Total resistive force (equilibrium condition)}$
- for object beginning to **accelerate:** $F = \text{Total resistive force} + ma \text{ [N07p1Q10,N88p1Q5]}$
Chapter 6: Motion in a Circle
- Kinematics of uniform circular motion
- Centripetal acceleration
- Centripetal force

a. Express angular displacement in radians.

Radian (rad) is the S.I. unit for angle, θ and it can be related to degrees in the following way. In one complete revolution, an object rotates through 360°, or 2π rad.

As the object moves through an angle θ, with respect to the centre of rotation, this angle θ is known as the angular displacement.

b. Understand and use the concept of angular velocity.

Angular velocity (ω) of the object is the rate of change of angular displacement with respect to time.

\[ \omega = \frac{\theta}{t} = \frac{2\pi}{T} \quad \text{(for one complete revolution)} \]

Note: (i) The direction of the linear velocity is at a tangent to the circle described at that point. Hence it is sometimes referred to as the tangential velocity.

(ii) ω is the same for every point in the rotating object, but the linear velocity v is greater for points further from the axis.

c. Recall and use \( v = r\omega \).

Linear velocity, v, of an object is its instantaneous velocity at any point in its circular path.

\[ v = \frac{\text{arc length}}{\text{time taken}} = \frac{r\theta}{T} = r\omega \]

Note: (i) The direction of the linear velocity is at a tangent to the circle described at that point. Hence it is sometimes referred to as the tangential velocity.

(ii) ω is the same for every point in the rotating object, but the linear velocity v is greater for points further from the axis.

d. Describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of a uniform motion in a circle.

A body moving in a circle at a constant speed changes velocity (since its direction changes). Thus, it always experiences an acceleration, a force and a change in momentum.

e. Recall and use centripetal acceleration \( a = r\omega^2 \), \( a = \frac{v^2}{r} \).

Centripetal acceleration \( a = r\omega^2 = \frac{v^2}{r} \) (in magnitude)

f. Recall and use centripetal force \( F = mr\omega^2 \), \( F = \frac{mv^2}{r} \).

Centripetal force is the resultant of all the forces that act on a system in circular motion.

{It is not a particular force; “centripetal” means “centre-seeking”. Also, when asked to draw a diagram showing all the forces that act on a system in circular motion, it is wrong to include a force that is labelled as “centripetal force”.}

\[ \text{Centripetal force, } F = mr\omega^2 = \frac{mv^2}{r} \quad \text{(in magnitude)} \]

A person in a satellite orbiting the Earth experiences “weightlessness” although the gravity field strength at that height is not zero because the person and the satellite would both have the same acceleration; hence the contact force between man & satellite/normal reaction on the person is zero. (Not because the field strength is negligible.)
Chapter 7: Gravitation

- Gravitational Field
- Force between point masses
- Field of a point mass
- Field near to the surface of the Earth
- Gravitational Potential

a. Show an understanding of the concept of a gravitational field as an example of field of force and define gravitational field strength as force per unit mass.

Gravitational field strength at a point is defined as the gravitational force per unit mass at that point.

b. Recall and use Newton’s law of gravitation in the form \( F = \frac{GMm}{r^2} \)

Newton’s law of gravitation:
The (mutual) gravitational force \( F \) between two point masses \( M \) and \( m \) separated by a distance \( r \) is given by

\[
F = \frac{GMm}{r^2}
\]

or, the gravitational force of between two point masses is proportional to the product of their masses & inversely proportional to the square of their separation.

c. Derive, from Newton’s law of gravitation and the definition of gravitational field strength, the equation \( g = \frac{GM}{r^2} \) for the gravitational field strength of a point mass.

Gravitational field strength at a point is the gravitational force per unit mass at that point. It is a vector and its S.I. unit is \( \text{N kg}^{-1} \).

By definition, \( g = \frac{F}{m} \)

By Newton Law of Gravitation, \( F = \frac{GMm}{r^2} \)

Combining, magnitude of \( g = \frac{GM}{r^2} \)

Therefore \( g = \frac{GM}{r^2} \), \( M \) = Mass of object “creating” the field

d. Recall and apply the equation \( g = \frac{GM}{r^2} \) for the gravitational field strength of a point mass to new situations or to solve related problems.

Example 7D1
Assuming that the Earth is a uniform sphere of radius 6.4 x 10^6 m and mass 6.0 x 10^{24} kg, find the gravitational field strength \( g \) at a point

(a) on the surface,

\[
g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6.4 \times 10^6)^2} = 9.77 \text{ m s}^{-2}
\]

(b) at height 0.50 times the radius of above the Earth’s surface,

\[
g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(1.5 \times 6.4 \times 10^6)^2} = 4.34 \text{ m s}^{-2}
\]

Example 7D2
The acceleration due to gravity at the Earth’s surface is 9.80 m s^{-2}. Calculate the acceleration due to gravity on a planet which has the same density but twice the radius of Earth.
\[ g = \frac{GM}{r^2} \]
\[ g_P = \frac{M_{\text{Earth}}}{M_{\text{Earth}}^2} \]
\[ g_E = \frac{4}{3}\pi\rho \frac{r_P^3}{r_E^2} \]
\[ = \frac{4}{3}\pi\rho \frac{r_P^3}{r_E^2} \]
\[ = \frac{r_P}{r_E} \]
\[ = 2 \]

Hence \( g_P = 2 \times 9.81 = 19.6 \text{ m s}^{-2} \).

e. Show an appreciation that on the surface of the Earth \( g \) is approximately constant and is called the acceleration of free fall.

Assuming that Earth is a uniform sphere of mass \( M \). The magnitude of the gravitational force from Earth on a particle of mass \( m \), located outside Earth a distance \( r \) from the centre of the Earth is
\[ F = \frac{GMm}{r^2} \]. When a particle is released, it will fall towards the centre of the Earth, as a result of the gravitational force with an acceleration \( a_g \).

\[ \text{i.e.} \quad F = ma_g \]
\[ a_g = \frac{GM}{r^2} \]

Hence \( a_g = g \)

Thus gravitational field strength \( g \) is also numerically equal to the acceleration of free fall.

Example 7E1
A ship is at rest on the Earth’s equator. Assuming the earth to be a perfect sphere of radius \( R \) and the acceleration due to gravity at the poles is \( g_0 \), express its apparent weight, \( N \), of a body of mass \( m \) in terms of \( m, g_0, R \) and \( T \) (the period of the earth’s rotation about its axis, which is one day).

Ans:
At the North Pole, the gravitational attraction is
\[ F = \frac{GMm}{R^2} = mg_0 \]

At the equator,
Normal Reaction
Force on ship by Earth = Gravitational attraction – centripetal force
\[ N = mg_0 - mR\omega^2 \]
\[ = mg_0 - mR \left( \frac{2\pi}{T} \right)^2 \]

f. Define potential at a point as the work done in bringing unit mass from infinity to the point.

Gravitational potential at a point is defined as the work done (by an external agent) in bringing a unit mass from infinity to that point (without changing its kinetic energy).

\[ \Phi = \frac{W}{m} = -\frac{GM}{r} \]

Why gravitational potential values are always negative?
- As the gravitational force on the mass is attractive, the work done by an ext agent in bringing unit mass from infinity to any point in the field will be negative work (as the force exerted by the ext agent is opposite in direction to the displacement to ensure that \( \Delta KE = 0 \))

- Hence by the definition of negative work, all values of \( \Phi \) are negative.
Relation between g and \( \phi \):
\[
    g = -\frac{d\phi}{dr} = \text{gradient of } \phi - r \text{ graph} \quad \text{(Analogy: } E = -dV/dx)\]

Gravitational potential energy \( U \) of a mass \( m \) at a point in the gravitational field of another mass \( M \), is the work done in bringing that mass \( m \) (NOT: unit mass, or a mass) from infinity to that point.

\[
    \rightarrow U = m \phi = -\frac{GMm}{r}
\]

Change in GPE, \( \Delta U = mg \Delta h \) only if \( g \) is constant over the distance \( h \); \( \Rightarrow h \ll \text{ radius of planet} \)
otherwise, must use: \( \Delta U = m \phi_f - m \phi_i \)

**h. Recognise the analogy between certain qualitative and quantitative aspects of gravitational and electric fields.**

<table>
<thead>
<tr>
<th>Aspects</th>
<th>Electric Field</th>
<th>Gravitational Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>Quantity interacting with or producing the field</strong></td>
<td>Charge ( Q )</td>
<td>Mass ( M )</td>
</tr>
<tr>
<td>2. <strong>Definition of Field Strength</strong></td>
<td>Force per unit positive charge ( E = \frac{F}{q} )</td>
<td>Force per unit mass ( g = \frac{F}{M} )</td>
</tr>
<tr>
<td>3. <strong>Force between two Point Charges or Masses</strong></td>
<td>Coulomb’s Law: ( F = \frac{Q_1Q_2}{4\pi\varepsilon_0r^2} )</td>
<td>Newton’s Law of Gravitation: ( F_g = \frac{GMm}{r^2} )</td>
</tr>
<tr>
<td>4. <strong>Field Strength of isolated Point Charge or Mass</strong></td>
<td>( E = \frac{Q}{4\pi\varepsilon_0r^2} )</td>
<td>( g = \frac{GM}{r^2} )</td>
</tr>
<tr>
<td>5. <strong>Definition of Potential</strong></td>
<td>Work done in bringing a unit positive charge from infinity to the point. ( V = \frac{W}{Q} )</td>
<td>Work done in bringing a unit mass from infinity to the point. ( \phi = \frac{W}{M} )</td>
</tr>
<tr>
<td>6. <strong>Potential of isolated Point Charge or Mass</strong></td>
<td>( V = \frac{Q}{4\pi\varepsilon_0r} )</td>
<td>( \phi = -\frac{G}{r} )</td>
</tr>
<tr>
<td>7. <strong>Change in Potential Energy</strong></td>
<td>( \Delta U = q \Delta V )</td>
<td>( \Delta U = m \Delta \phi )</td>
</tr>
</tbody>
</table>

**i. Analyse circular orbits in inverse square law fields by relating the gravitational force to the centripetal acceleration it causes.**

**Total Energy** of a Satellite = GPE + KE = \( \left( \frac{GMm}{r} \right) + \left( \frac{1}{2}GMm \right) \)

**Escape Speed of a Satellite**

By Conservation of Energy,

\[
    \frac{1}{2}mv_E^2 + \left( \frac{GMm}{r} \right) = 0 + 0
\]

Thus escape speed, \( v_E = \sqrt{\frac{2GM}{r}} \)

Note: Escape speed of an object is independent of its mass

For a satellite in circular orbit, "the centripetal force is provided by the gravitational force."

(Must always state what force is providing the centripetal force before following eqn is used!)

Hence \( \frac{GMm}{r^2} = \frac{mv_E^2}{r} = mr^2\omega^2 = m \left( \frac{2\pi}{T} \right)^2 \)

A satellite does not move in the direction of the gravitational force (ie it stays in its circular orbit) because: the gravitational force exerted by the Earth on the satellite is just sufficient to cause the centripetal acceleration but not enough to also pull it down towards the Earth.
j. **Show an understanding of geostationary orbits and their application.**

**Geostationary satellite** is one which is **always above a certain point on the Earth** (as the Earth rotates about its axis.)

For a **geostationary** orbit: \( T = 24 \) hrs, orbital radius (& height) are fixed values from the centre of the Earth, ang velocity \( \omega \) is also a fixed value; rotates fr west to east. However, the **mass** of the satellite is **NOT a particular value** & hence the ke, gpe, & the centripetal force are also not fixed values (ie their values depend on the mass of the geostationary satellite.)

A geostationary orbit must lie in the **equatorial plane** of the earth because it must accelerate in a plane where the **centre** of Earth lies since the **net force** exerted on the satellite is the **Earth’s gravitational force**, which is directed towards the **centre** of Earth.

(Alternatively, may explain by showing why it’s impossible for a satellite in a non-equatorial plane to be geostationary.)
Chapter 8: Oscillations
- Simple harmonic motion
- Energy in simple harmonic motion
- Damped and forced oscillations: resonance

a. Describe simple examples of free oscillations.

Self-explanatory

b. Investigate the motion of an oscillator using experimental and graphical methods.

Self-explanatory

c. Understand and use the terms amplitude, period, frequency, angular frequency and phase difference and express the period in terms of both frequency and angular frequency.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>is defined as the time taken for one complete oscillation.</td>
</tr>
<tr>
<td>Frequency</td>
<td>is defined as the number of oscillations per unit time,</td>
</tr>
<tr>
<td></td>
<td>$f = \frac{1}{T}$</td>
</tr>
<tr>
<td>Angular frequency</td>
<td>$\omega$: is defined by the eqn. $\omega = 2\pi f$. It is thus the rate of change of angular displacement (measured in radians per sec)</td>
</tr>
<tr>
<td>Amplitude</td>
<td>The maximum displacement from the equilibrium position.</td>
</tr>
<tr>
<td>Phase difference</td>
<td>$\phi = \frac{2\pi x}{\lambda} = \frac{t}{T} \times 2\pi$ (x = separation in the direction of wave motion between the 2 particles)</td>
</tr>
</tbody>
</table>

d. Recognise and use the equation $a = -\omega^2 x$ as the defining equation of simple harmonic motion.

Simple harmonic motion: An oscillatory motion in which the acceleration (or restoring force) is
- always proportional to, and
- opposite in direction to the displacement from a certain fixed point/ equilibrium position

ie $a = -\omega^2 x$ (Defining equation of S.H.M)

e. Recall and use $x = x_0 \sin(\omega t)$ as a solution to the equation $a = -\omega^2 x$.

f. Recognise and use $v = v_0 \cos(\omega t)$ and $v = \pm \omega \sqrt{x_0^2 - x^2}$

<table>
<thead>
<tr>
<th>“Time Equations”</th>
<th>“Displacement Equations”</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = x_0 \sin(\omega t)$ or $x = x_0 \cos(\omega t)$ (depending on the initial condition)</td>
<td>$v = \pm \omega \sqrt{x_0^2 - x^2}$ (In Formula List)</td>
</tr>
<tr>
<td>$\frac{dx}{dt} = \omega x_0 \cos(\omega t)$ (assuming $x = x_0 \sin(\omega t)$)</td>
<td>(v - x graph is an ellipse)</td>
</tr>
<tr>
<td>$a = -\omega^2 x$</td>
<td>$a = -\omega^2 x$</td>
</tr>
<tr>
<td>KE = $\frac{1}{2} mv^2$ = $\frac{1}{2} m(\omega x_0 \cos(\omega t))^2$</td>
<td>KE = $\frac{1}{2} mv^2$ = $\frac{1}{2} m \omega^2 (x_0^2 - x^2)$</td>
</tr>
</tbody>
</table>

 KE = $\frac{1}{2} mv^2$ = $\frac{1}{2} m \omega^2 (x_0^2 - x^2)$ (KE - x graph is a parabola)

g. Describe with graphical illustrations, the changes in displacement, velocity and acceleration during simple harmonic motion.
h. Describe the interchange between kinetic and potential energy during simple harmonic motion.

The energy of the oscillator changes from potential to kinetic and back to potential in every half-cycle interval. At any point of its motion, the sum of the PE and KE is equal to the total energy. At the equilibrium position, the mass has a maximum KE because its speed is greatest and zero PE as the spring neither compressed nor stretched. At either A or B where the mass stops, its KE is zero while its PE is maximum.

The constant interchange of energies during SHM can be represented graphically as follows.

It can be shown that

\[ \text{total energy} = \text{maximum KE} = \text{maximum PE} = \frac{1}{2}m\omega^2x_0^2 \]
Damping refers to the loss of energy from an oscillating system to the environment due to dissipative forces (e.g., friction, viscous forces, eddy currents).

**Light Damping:** The system oscillates about the equilibrium position with decreasing amplitude over a period of time.

**Critical Damping:** The system does not oscillate & damping is just adequate such that the system returns to its equilibrium position in the shortest possible time.

**Heavy Damping:** The damping is so great that the displaced object never oscillates but returns to its equilibrium position very very slowly.

### j. Describe practical examples of forced oscillations and resonance.

**Free Oscillation:** An oscillating system is said to be undergoing *free oscillations* if its oscillatory motion is not subjected to an external periodic driving force. The system oscillates at its natural freq.

**Forced Oscillation:** In contrast to free oscillations, an oscillating system is said to undergo *forced oscillations* if it is subjected to an input of energy from an external periodic driving force. The freq of the forced (or driven) oscillations will be at the freq of the *driving force* (called the driving frequency) i.e., no longer at its own natural frequency.

**Resonance:** A phenomenon whereby the amplitude of a system undergoing forced oscillations increases to a maximum. It occurs when the frequency of the periodic driving force is equal to the natural frequency of the system.

**Effects of Damping on Freq Response of a system undergoing forced oscillations**

1. Resonant frequency decreases
2. Sharpness of resonant peak decreases
3. Amplitude of forced oscillation decreases

### k. Describe graphically how the amplitude of a forced oscillation changes with frequency near to the natural frequency of the system, and understand qualitatively the factors which determine the frequency response and sharpness of the resonance.

![Graph showing frequency response and resonance](image)

**Amplitude of forced oscillation / m**

**f₀**  Driver's frequency / Hz

- No damping
- Light damping
- Heavy damping

### l. Show an appreciation that there are some circumstances in which resonance is useful and other circumstances in which resonance should be avoided.
### Examples of Useful Purposes of Resonance

(a) Oscillation of a child's swing.

(b) Tuning of musical instruments.

(c) Tuning of radio receiver - Natural frequency of the radio is adjusted so that it responds resonantly to a specific broadcast frequency.

(d) Using microwave to cook food - Microwave ovens produce microwaves of a frequency which is equal to the natural frequency of water molecules, thus causing the water molecules in the food to vibrate more violently. This generates heat to cook the food but the glass and paper containers do not heat up as much.

(e) Magnetic Resonance Imaging (MRI) is used in hospitals to create images of the human organs.

(f) Seismography - the science of detecting small movements in the Earth's crust in order to locate centres of earthquakes.

### Examples of Destructive Nature of Resonance

(a) An example of a disaster that was caused by resonance occurred in the United States in 1940. The Tarcoma Narrows Bridge in Washington was suspended by huge cables across a valley. Shortly after its completion, it was observed to be unstable. On a windy day four months after its official opening, the bridge began vibrating at its resonant frequency. The vibrations were so great that the bridge collapsed.

(b) High-pitched sound waves can shatter fragile objects, an example being the shattering of a wine glass when a soprano hits a high note.

(c) Buildings that vibrate at natural frequencies close to the frequency of seismic waves face the possibility of collapse during earthquakes.
Chapter 9: Thermal Physics
- Internal energy
- Temperature scales
- Specific heat capacity
- Specific latent heat
- First law of thermodynamics
- The ideal gas equation
- Kinetic energy of a molecule

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>Show an understanding that internal energy is determined by the state of the system and that it can be expressed as the sum of a random distribution of kinetic and potential energies associated with the molecules of a system.</td>
</tr>
<tr>
<td><strong>Internal Energy:</strong></td>
<td>is the sum of the kinetic energy of the molecules due to its random motion &amp; the pe of the molecules due to the intermolecular forces.</td>
</tr>
<tr>
<td><strong>“Internal energy is determined by the state of the system.”</strong></td>
<td>Explain what this means.</td>
</tr>
<tr>
<td>Internal energy is determined by the values of the current state and is independent of how the state is arrived at. Thus if a system undergoes a series of changes from one state A to another state B, its change in internal energy is the same, regardless of which path (the changes in the p &amp; V) it has taken to get from A to B.</td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>Relate a rise in temperature of a body to an increase in its internal energy.</td>
</tr>
<tr>
<td>Since Kinetic Energy proportional to temp, and internal energy of the system = sum of its Kinetic Energy and Potential Energy, a rise in temperature will cause a rise in Kinetic Energy and thus an increase in internal energy.</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>Show an understanding that regions of equal temperature are in thermal equilibrium.</td>
</tr>
<tr>
<td>If two bodies are in thermal equilibrium, there is no net flow of heat energy between them and they have the same temperature. (NB: this does not imply they must have the same internal energy as internal energy depends also on the number of molecules in the 2 bodies, which is unknown here)</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td>Show an understanding that there is an absolute scale of temperature which does not depend on the property of any particular substance, i.e. the thermodynamic scale.</td>
</tr>
<tr>
<td>e.</td>
<td>Apply the concept that, on the thermodynamic (Kelvin) scale, absolute zero is the temperature at which all substances have a minimum internal energy.</td>
</tr>
<tr>
<td><strong>Thermodynamic (Kelvin) scale:</strong></td>
<td>of temperature: theoretical scale that is independent of the properties of any particular substance.</td>
</tr>
<tr>
<td>An absolute scale of temp is a temp scale which does not depend on the property of any particular substance (ie the thermodynamic scale)</td>
<td></td>
</tr>
<tr>
<td><strong>Absolute zero:</strong></td>
<td>Temperature at which all substances have a minimum internal energy (NOT: zero internal energy.)</td>
</tr>
<tr>
<td>f.</td>
<td>Convert temperatures measured in Kelvin to degrees Celsius: ( T/\text{K} = T/\text{°C} + 273.15 ).</td>
</tr>
<tr>
<td><strong>T/K = T/°C + 273.15,</strong> by definition of the Celsius scale.</td>
<td></td>
</tr>
<tr>
<td>g.</td>
<td>Define and use the concept of specific heat capacity, and identify the main principles of its determination by electrical methods.</td>
</tr>
<tr>
<td><strong>Specific heat capacity</strong> is defined as the amount of heat energy needed to produce unit temperature change (NOT: by 1 K) for unit mass (NOT: 1 kg) of a substance, without causing a change in state.</td>
<td></td>
</tr>
<tr>
<td>i.e. ( c = \frac{Q}{m\Delta T} )</td>
<td></td>
</tr>
<tr>
<td><strong>ELECTRICAL METHODS</strong></td>
<td></td>
</tr>
<tr>
<td>h.</td>
<td>Define and use the concept of specific latent heat, and identify the main principles of its determination by electrical methods.</td>
</tr>
</tbody>
</table>
Specific latent heat of vaporisation is defined as the amount of heat energy needed to change unit mass of a substance from liquid phase to gaseous phase without a change of temperature.

Specific latent heat of fusion is defined as the amount of heat energy needed to change unit mass of a substance from solid phase to liquid phase without a change of temperature.

\[ L = \frac{Q}{m} \quad \text{for both cases of vaporisation & melting} \]

The specific latent heat of vaporisation is greater than the specific latent heat of fusion for a given substance (N06P2Q2)

- During vaporisation, there is a greater increase in volume than in fusion;
- Thus more work is done against atmospheric pressure during vaporisation.
- The increase in vol also means the increase in the (molecular) potential energy, & hence, internal energy, during vaporisation more than that during melting.
- Hence by 1st Law of Thermodynamics, heat supplied during vaporisation more than that during melting; hence \( l_v > l_f \) (since \( Q = ml = \Delta U - W \))

\[ \text{Note:} \]
1. the use of comparative terms: greater, more, and >
2. the increase in internal energy is due to an increase in the PE, NOT KE of molecules
3. the system here is NOT to be considered as an ideal gas system

Similarly, you need to explain why, when a liq is boiling, thermal energy is being supplied, and yet, the temp of the liq does not change. (N97P3Q5, [4 m])

i. Explain using a simple kinetic model for matter why

<table>
<thead>
<tr>
<th>Melting</th>
<th>Boiling</th>
<th>Evaporation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occurrence</td>
<td>Throughout the substance, at fixed temperature and pressure</td>
<td>On the surface, at all temperatures</td>
</tr>
<tr>
<td>Spacing(vol) &amp; PE of molecules</td>
<td>Increase slightly</td>
<td>Increase significantly</td>
</tr>
<tr>
<td>Temperature &amp; hence KE of molecules</td>
<td>Remains constant during process</td>
<td>Decrease for remaining liquid</td>
</tr>
</tbody>
</table>

j. Recall and use the first law of thermodynamics expressed in terms of the change in internal energy, the heating of the system and the work done on the system.

First Law of Thermodynamics:
The increase in internal energy of a system is equal to the sum of the heat supplied to the system and the work done on the system.

\[ \Delta U = W + Q \]

\[ \Delta U: \text{Increase in internal energy of the system} \]

\[ Q: \text{Heat supplied to the system} \]

\[ W: \text{work done on the system} \]

\{Need to recall the sign convention for all 3 terms\}

Work is done by a gas when it expands; work is done on a gas when it is compressed.

\[ W = \text{area under pressure-volume graph} \]

For constant pressure (isobaric process), \( \text{Work done} = \text{pressure} \times \Delta \text{Volume} \)

Isothermal process: a process where \( T = \text{const} \) (\( \Rightarrow \Delta U = 0 \) for ideal gas)
ΔU for a cycle = 0  {since U ∝ T,  &  ΔT = 0 for a cycle }

k. Recall and use the ideal gas equation pV = nRT where n is the amount of gas in moles.

Equation of state for an ideal gas:

- p V = n R T,  where T is in Kelvin {NOT: °C}, n: no. of moles.
- p V = N k T,  where N: no. of molecules, k:Boltzmann const

Ideal Gas: a gas which obeys the ideal gas equation pV = nRT FOR ALL VALUES OF P, V & T

l. Show an understanding of the significance of the Avogadro constant as the number of atoms in 0.012 kg of carbon-12.

Avogadro constant: defined as the number of atoms in 12 g of carbon-12. It is thus the number of particles (atoms or molecules) in one mole of substance.

m. Use molar quantities where one mole of any substance is the amount containing a number of particles equal to the Avogadro constant.

n. Recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature to new situations or to solve related problems.

For an ideal gas, internal energy U = Sum of the KE of the molecules only {since PE = 0 for ideal gas}

ie  \[ U = N \times \frac{1}{2} \ m \ <c^2> = N \times \frac{3}{2}kT \]  {for monatomic gas}

- U depends on T AND number of molecules N.
- U ∝ T  for a given number of molecules

Ave KE of a molecule, \( \frac{1}{2} \ m \ <c^2> \propto T \)  { T in K: not °C }
SECTION IV

WAVES
Chapter 10: Wave Motion
- Progressive Waves
- Transverse and Longitudinal Waves
- Polarisation
- Determination of frequency and wavelength

a. Show an understanding and use the terms displacement, amplitude, phase difference, period, frequency, wavelength and speed.

(a) Displacement \((y)\): Position of an oscillating particle from its equilibrium position.

(b) Amplitude \((y_0\) or \(A\)): The maximum magnitude of the displacement of an oscillating particle from its equilibrium position.

(c) Period \((T)\): Time taken for a particle to undergo one complete cycle of oscillation.

(d) Frequency \((f)\): Number of oscillations performed by a particle per unit time.

(e) Wavelength \((\lambda)\): For a progressive wave, it is the distance between any two successive particles that are in phase, e.g. it is the distance between 2 consecutive crests or 2 troughs.

(f) Wave speed \((v)\): The speed at which the waveform travels in the direction of the propagation of the wave.

(g) Wave front: A line or surface joining points which are at the same state of oscillation, i.e. in phase, e.g. a line joining crest to crest in a wave.

(h) Ray: The path taken by the wave. This is used to indicate the direction of wave propagation. Rays are always at right angles to the wave fronts (i.e. wave fronts are always perpendicular to the direction of propagation).

b. Deduce, from the definitions of speed, frequency and wavelength, the equation \(v = f\lambda\).

From the definition of speed, \(\text{Speed} = \frac{\text{Distance}}{\text{Time}}\)

A wave travels a distance of one wavelength, \(\lambda\), in a time interval of one period, \(T\).

The frequency, \(f\), of a wave is equal to \(\frac{1}{T}\).

Therefore, speed, \(v = \frac{\lambda}{\frac{1}{f}} = (f\lambda)\)

Hence, \(v = f\lambda\)

c. Recall and use the equation \(v = f\lambda\)

Example 10C1
A wave travelling in the positive \(x\) direction is showed in the figure. Find the amplitude, wavelength, period, and speed of the wave if it has a frequency of 8.0 Hz.
Amplitude \( (A) = 0.15 \text{ m} \)

Wavelength \( (\lambda) = 0.40 \text{ m} \)

Period \( (T) = \frac{1}{f} = \frac{1}{8.0} \approx 0.125 \text{ s} \)

Speed \( (v) = f \lambda = 8.0 \times 0.40 = 3.20 \text{ m s}^{-1} \)

d. Show an understanding that energy is transferred due to a progressive wave.

A wave which results in a net transfer of energy from one place to another is known as a **progressive wave**.

e. Recall and use the relationship, intensity \( \propto (\text{amplitude})^2 \)

**Intensity** (of a wave): is defined as the rate of energy flow per unit time (power) per unit cross-sectional area perpendicular to the direction of wave propagation.

\[
\text{Intensity} = \frac{\text{Power}}{\text{Area}} = \frac{\text{Energy}}{\text{Time} \times \text{Area}}
\]

For a point source (which would emit spherical wavefronts),

\[
\text{Intensity} = \frac{1}{2 \pi \omega x_0^2} \quad \text{where } x_0: \text{amplitude} \& \ r: \text{distance from the point source.}
\]

\[\therefore \text{I} \propto \frac{x_0^2}{r^2} \quad \text{(Pt Source)}\]

For all wave sources, \( \text{I} \propto (\text{Amplitude})^2 \)

f. Analyse and interpret graphical representations of transverse and longitudinal waves.

**Transverse wave:** A wave in which the oscillations of the wave particles **NOT**: movement are perpendicular to the direction of the propagation of the wave.

**Longitudinal wave:** A wave in which the oscillations of the wave particles are parallel to the direction of the propagation of the wave.
g. Show an understanding that polarisation is a phenomenon associated with transverse waves.

Polarisation is said to occur when oscillations are in one direction in a plane, normal to the direction of propagation.

{Only transverse waves can be polarized; longitudinal waves can't.}

h. Determine the frequency of sound using a calibrated cathode ray oscilloscope.

**Example 10H1**

The following stationary wave pattern is obtained using a C.R.O. whose screen is graduated in centimetre squares. Given that the time-base is adjusted such that 1 unit on the horizontal axis of the screen corresponds to a time of 1.0 ms, find the period and frequency of the wave.

\[ \text{Period, } T = (4 \text{ units}) \times 1.0 \]
\[ = 4.0 \text{ ms} \]
\[ = 4.0 \times 10^{-3} \text{ s} \]

\[ f = \frac{1}{T} = \frac{1}{4 \times 10^{-3}} = 250 \text{ Hz} \]

i. Determine the wavelength of sound using stationary waves.

See Chapter 11
Chapter 11: Superposition
- Stationary Waves
- Diffraction
- Interference
- Two-source interference patterns
- Diffraction grating

### a. Explain and use the principle of superposition in simple applications.

**Principle of Superposition:** When two or more waves of the same type meet at a point, the resultant displacement of the waves is equal to the vector sum of their individual displacements at that point.

### b. Show an understanding of experiments which demonstrate stationary waves using microwaves, stretched strings and air columns.

**Stretched String**
A horizontal rope with one end fixed and another attached to a vertical oscillator. Stationary waves will be produced by the direct and reflected waves in the string.

![Stretched String Diagram](image)

Or we can have the string stopped at one end with a pulley as shown below.

![Pulley Diagram](image)

**Microwaves**
A microwave emitter placed a distance away from a metal plate that reflects the emitted wave. By moving a detector along the path of the wave, the nodes and antinodes could be detected.

![Microwaves Diagram](image)
Air column
A tuning fork held at the mouth of an open tube projects a sound wave into the column of air in the tube. The length of the tube can be changed by varying the water level. At certain lengths of the tube, the air column resonates with the tuning fork. This is due to the formation of stationary waves by the incident and reflected sound waves at the water surface.

Stationary (Standing) Wave is one
- whose waveform/wave profile does not advance (move),
- where there is no net transport of energy, and
- where the positions of antinodes and nodes do not change (with time).

A stationary wave is formed when two progressive waves of the same frequency, amplitude and speed, travelling in opposite directions are superposed. (Assume boundary conditions are met)

<table>
<thead>
<tr>
<th>Stationary Waves</th>
<th>Progressive Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Amplitude</strong></td>
<td>Varies from maximum at the anti-nodes to zero at the nodes.</td>
</tr>
<tr>
<td><strong>Wavelength</strong></td>
<td>Twice the distance between a pair of adjacent nodes or anti-nodes.</td>
</tr>
<tr>
<td><strong>Phase</strong></td>
<td>Particles in the same segment between 2 adjacent nodes, are in phase. Particles in adjacent segments are in anti-phase.</td>
</tr>
<tr>
<td><strong>Wave Profile</strong></td>
<td>The wave profile does not advance.</td>
</tr>
<tr>
<td><strong>Energy</strong></td>
<td>No energy is transported by the wave.</td>
</tr>
</tbody>
</table>

Node is a region of destructive superposition where the waves always meet out of phase by $\pi$ radians. Hence displacement here is permanently zero (or minimum).

Antinode is a region of constructive superposition where the waves always meet in phase. Hence a particle here vibrates with maximum amplitude (but it is NOT a pt with a permanent large displacement!)

Dist between 2 successive nodes/antinodes = $\frac{\lambda}{2}$

Max pressure change occurs at the nodes (NOT the antinodes) because every node changes from being a pt of compression to become a pt of rarefaction (half a period later)

d. j. Explain the meaning of the term diffraction.
Recall and solve problems by using the formula $dsin\theta = n\lambda$ and describe the use of a diffraction grating to determine the wavelength of light. (The structure and use of the spectrometer is not required.)

Diffraction: refers to the spreading (or bending) of waves when they pass through an opening (gap), or round an obstacle (into the “shadow” region). (Illustrate with diag)

For significant diffraction to occur, the size of the gap = $\lambda$ of the wave
For a diffraction grating, \( d \sin \theta = n \lambda \), \( d \) = dist between successive slits (grating spacing) = reciprocal of number of lines per metre

When a “white light” passes through a diffraction grating, for each order of diffraction, a longer wavelength {red} diffracts more than a shorter wavelength {violet} (as \( \sin \theta \propto \lambda \)).

e. Show an understanding of experiments which demonstrate diffraction including the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap.

Diffraction refers to the spreading of waves as they pass through a narrow slit or near an obstacle.

For diffraction to occur, the size of the gap should approximately be equal to the wavelength of the wave.

f. Show an understanding of the terms interference and coherence.

Coherent waves: Waves having a constant phase difference {not: zero phase difference/in phase}

Interference may be described as the superposition of waves from 2 coherent sources.

For an observable/well-defined interference pattern, the waves must be coherent, have about the amplitude, be unpolarised or polarised in the same direction, & be of the same type.

g. Show an understanding of experiments which demonstrate two-source interference using water, light and microwaves.

Water Waves

Interference patterns could be observed when two dippers are attached to the vibrator of the ripple tank. The ripples produce constructive and destructive interference. The dippers are coherent sources because they are fixed to the same vibrator.
Microwaves

Microwave emitted from a transmitter through 2 slits on a metal plate would also produce interference patterns. By moving a detector on the opposite side of the metal plate, a series of rise and fall in amplitude of the wave would be registered.

Light Waves (Young’s double slit experiment)

Since light is emitted from a bulb randomly, the way to obtain two coherent light sources is by splitting light from a single slit.

The 2 beams from the double slit would then interfere with each other, creating a pattern of alternate bright and dark fringes (or high and low intensities) at regular intervals, which is also known as our interference pattern.

h. Show an understanding of the conditions required if two-source interference fringes are to be observed.
Condition for **Constructive Interference** at a pt P:

*phase difference* of the 2 waves at P = 0  \{or 2π, 4π, etc\}

Thus, with 2 *in-phase sources*, * implies *path difference* = nλ;  with 2 *antiphase sources*: path difference = (n + ½)λ

Condition for **Destructive Interference** at a pt P:

*phase difference* of the 2 waves at P = π  \{ or 3π, 5π, etc \}

With 2 *in-phase sources*, + implies *path difference* = (n + ½)λ, with 2 *antiphase sources*: path difference = nλ

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<td><em>phase difference</em> of the 2 waves at P = 0  {or 2π, 4π, etc}</td>
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1. Recall and solve problems using the equation $\lambda = \frac{\lambda D}{a}$ for double-slit interference using light.

**Fringe separation** $x = \lambda D \frac{a}{a}$, if $a << D$ \{applies only to Young’s Double Slit interference of light.\}

Ie, NOT for microwaves, sound waves, water waves

Phase difference $\Delta \phi$ betw the 2 waves at any pt X (betw the central & 1st maxima) is (approx) proportional to the dist of X from the central maxima. \{N01 & N06\}

Using 2 sources of equal amplitude $x_0$, the resultant amplitude of a bright fringe would be doubled \{2$x_0$\}, & the resultant intensity increases by 4 times \{not 2 times\}. \{\text{Resultant} \propto (2x_0)^2\}
Chapter 12: Electric Fields
- Concept of an electric field
- Force between point charges
- Electric field of a point charge
- Uniform electric field
- Electric potential

a. Show an understanding of the concept of an electric field as an example of a field of force and define electric field strength as force per unit positive charge.

**Electric field strength/intensity** at a point is defined as the force per unit positive charge acting at that point (a vector; Unit: N C\(^{-1}\) or V m\(^{-1}\))

\[
E = \frac{F}{q} \rightarrow F = qE
\]

- The electric force on a positive charge in an electric field is in the direction of E, while
- The electric force on a negative charge is opposite to the direction of E.
- Hence a +ve charge placed in an electric field will accelerate in the direction of E and gain KE (& simultaneously lose EPE); while a negative charge caused to move (projected) in the direction of E will decelerate, i.e. lose KE, (& gain EPE).

b. Represent an electric field by means of field lines.

- **Uniform electric field between two parallel metal plates**
- **Like charges of equal magnitude**
- **Unlike charges (positive charge of larger magnitude)**
- **Unlike charges of equal magnitude**
c. Recall and use Coulomb's law in the form \( F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 r^2} \) for the force between two point charges in free space or air.

**Coulomb's law:**

The (mutual) electric force \( F \) acting between 2 point charges \( Q_1 \) and \( Q_2 \) separated by a distance \( r \) is given by:

\[
F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 r^2}
\]

where \( \varepsilon_0 \): permittivity of free space

or, the (mutual) electric force between two point charges is proportional to the product of their charges inversely proportional to the square of their separation.

**EXAMPLE 12C1**

Two positive charges, each 4.18 \( \mu \text{C} \), and a negative charge, -6.36 \( \mu \text{C} \), are fixed at the vertices of an equilateral triangle of side 13.0 cm. Find the electrostatic force on the negative charge.

\[
F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 r^2}
\]

\[
= \left( 8.99 \times 10^9 \right) \left( 4.18 \times 10^{-6} \right) \left( 6.36 \times 10^{-6} \right) \left( 13.0 \times 10^{-2} \right)^2
\]

\[= 14.1 \text{ N}\]

(Note: negative sign for -6.36 \( \mu \text{C} \) has been ignored in the calculation)

\[
F_R = 2 \times F \cos 30^\circ
\]

\[= 24.4 \text{ N, vertically upwards}\]

---

d. Recall and use \( E = \frac{Q}{4\pi \varepsilon_0 r^2} \) for the field strength of a point charge in free space or air.

**Electric field strength due to a Point Charge Q:**

\[
E = \frac{Q}{4\pi \varepsilon_0 r^2}
\]

(NB: Do NOT substitute a negative \( Q \) with its negative sign in calculations!)

**EXAMPLE 12D1**

In the figure below, determine the point (other than at infinity) at which the total electric field strength is zero.

From the diagram, it can be observed that the point where \( E \) is zero lies on a straight line where the charges lie, to the left of the -2.5 \( \mu \text{C} \) charge.

Let this point be a distance \( r \) from the left charge.

Since the total electric field strength is zero,

\[
E_{6\mu} = E_{2\mu}
\]

\[
\frac{6\mu}{(1 + r)^2} = \frac{2.5\mu}{r^2}
\]

(Note: negative sign for -2.5 \( \mu \text{C} \) has been ignored here)

\[
\frac{6}{(1 + r)^2} = \frac{2.5}{r^2}
\]

\[
\frac{\sqrt{6r}}{r} = \sqrt{2.5} \left( 1 + r \right)
\]

\[= 1.82 \text{ m}\]

The point lies on a straight line where the charges lie, 1.82 m to the left of the -2.5 \( \mu \text{C} \) charge.
e. Calculate the field strength of the uniform field between charged parallel plates in terms of potential difference and separation.

f. Calculate the forces on charges in uniform electric fields.

Uniform electric field between 2 Charged Parallel Plates: \( E = \frac{V}{d} \)

d: perpendicular dist between the plates, 
V: potential difference between plates

Path of charge moving at 90° to electric field: parabolic. 
Beyond the pt where it exits the field, the path is a straight line, at a tangent to the parabola at exit.

EXAMPLE 12E1
An electron (\( m = 9.11 \times 10^{-31} \) kg; \( q = -1.6 \times 10^{-19} \) C) moving with a speed of 1.5 x 10⁷ m s⁻¹, enters a region between 2 parallel plates, which are 20 mm apart and 60 mm long. The top plate is at a potential of 80 V relative to the lower plate. Determine the angle through which the electron has been deflected as a result of passing through the plates.

Time taken for the electron to travel 60 mm horizontally = \( \frac{\text{Distance}}{\text{Speed}} = \frac{60 \times 10^{-3}}{1.5 \times 10^7} = 4 \times 10^{-9} \) s

\( E = \frac{V}{d} = \frac{80}{20 \times 10^{-3}} = 4000 \text{ V m}^{-1} \)

\( a = \frac{F}{m} = \frac{eE}{m} = \frac{(1.6 \times 10^{-19})(4000)}{(9.1 \times 10^{-31})} = 7.0 \times 10^{14} \text{ m s}^{-2} \)

\( v_y = u_y + at = 0 + (7.0 \times 10^{14})(4 \times 10^{-9}) = 2.8 \times 10^6 \text{ m s}^{-1} \)

\( \tan \theta = \frac{v_y}{v_x} = \frac{2.8 \times 10^6}{1.5 \times 10^7} = 0.187 \)
\( \therefore \theta = 10.6° \)

g. Describe the effect of a uniform electric field on the motion of charged particles.

- Equipotential surface: a surface where the electric potential is constant
- Potential gradient = 0, ie \( E \) along surface = 0
- Hence no work is done when a charge is moved along this surface. \( W=QV, V=0 \)
- Electric field lines must meet this surface at right angles.
- (If the field lines are not at 90° to it, it would imply that there is a non-zero component of \( E \) along the surface. This would contradict the fact that \( E \) along an equipotential = 0.)

h. Define potential at a point in terms of the work done in bringing unit positive charge from infinity to the point.

Electric potential at a point: is defined as the work done in moving a unit positive charge from infinity to that point, \( \{ \text{a scalar; unit: V} \} \) ie \( V = \frac{W}{Q} \)

The electric potential at infinity is defined as zero. At any other point, it may be positive or negative depending on the sign of \( Q \) that sets up the field. (Contrast gravitational potential.)

i. State that the field strength of the field at a point is numerically equal to the potential gradient at that point
Relation between $E$ and $V$: $E = -\frac{dV}{dr}$

i.e. The electric field strength at a pt is numerically equal to the potential gradient at that pt.

**NB:** Electric field lines point in direction of decreasing potential (i.e. from high to low pot).

## j.

Use the equation $V = \frac{Q}{4\pi\varepsilon_0 r}$ for the potential in the field of a point charge.

### Electric potential energy $U$ of a charge $Q$ at a pt where the potential is $V$:

$U = QV$

**→ Work done** $W$ on a charge $Q$ in moving it across a pd $\Delta V$: $W = Q \Delta V$

### Electric Potential due to a point charge $Q$:

$V = \frac{Q}{4\pi\varepsilon_0 r}$ (in List of Formulae)

(NB: Substitute $Q$ with its sign)

## k.

Recognise the analogy between certain qualitative and quantitative aspects of electric field and gravitational fields.

See 7h
### Chapter 13: Current of Electricity

- Electric current
- Potential difference
- Resistance and Resistivity
- Sources of electromotive force

#### a. Show an understanding that electric current is the rate of flow of charged particles.

**Electric current** is the rate of flow of charge. (NOT: charged particles)

#### b. Define charge and coulomb.

Electric charge $Q$ passing a point is defined as the product of the (steady) current at that point and the time for which the current flows, i.e. $Q = I \times t$

**One coulomb** is defined as the charge flowing per second pass a point at which the current is one ampere.

#### c. Recall and solve problems using the equation $Q = It$.

**EXAMPLE 13C1**

An ion beam of singly-charged Na$^+$ and K$^+$ ions is passing through vacuum. If the beam current is $20 \, \mu\text{A}$, calculate the total number of ions passing any fixed point in the beam per second. (The charge on each ion is $1.6 \times 10^{-19} \text{C}$.)

\[
\text{Current, } I = \frac{Q}{t} = \frac{Ne}{t}, \text{ where } N \text{ is the no. of ions and } e \text{ is the charge on one ion.}
\]

No. of ions per second
\[
= \frac{N}{I} = \frac{1}{e} = \frac{20 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{14}
\]

#### d. Define potential difference and the volt.

**Potential difference** is defined as the energy transferred from electrical energy to other forms of energy when unit charge passes through an electrical device, i.e. \( V = \frac{W}{Q} \)

P. D. = Energy Transferred / Charge = Power / Current or, is the ratio of the power supplied to the device to the current flowing, i.e. \( V = \frac{P}{I} \)

**The volt**: is defined as the potential difference between 2 pts in a circuit in which one joule of energy is converted from electrical to non-electrical energy when one coulomb passes from 1 pt to the other, i.e. 1 volt = One joule per coulomb

**Difference between Potential and Potential Difference (PD):**

The potential at a point of the circuit is due to the amount of charge present along with the energy of the charges. Thus, the potential along circuit drops from the positive terminal to negative terminal, and potential differs from points to points.

Potential Difference refers to the difference in potential between any given two points.

For example, if the potential of point A is 1 V and the potential at point B is 5 V, the PD across \( \text{AB, or } V_{\text{AB}} \), is 4 V. In addition, when there is no energy loss between two points of the circuit, the potential of these points is same and thus the PD across is 0 V.

#### e. Recall and solve problems by using $V = \frac{W}{Q}$

**EXAMPLE 13E1**

A current of 5 mA passes through a bulb for 1 minute. The potential difference across the bulb is 4 V.
Calculate

(a) The amount of charge passing through the bulb in 1 minute.

\[ Q \ = \ I \cdot t \]
\[ = \ 5 \times 10^{-3} \times 60 \]
\[ = \ 0.3 \ \text{C} \]

(b) The work done to operate the bulb for 1 minute.

Potential difference across the bulb = \( \frac{W}{Q} \)
\[ = \ \frac{4}{0.3} \]
Work done to operate the bulb for 1 minute = \( 0.3 \times 4 \)
\[ = \ 1.2 \ \text{J} \]

f. Recall and solve problems by using \( P = VI, P = I^2R \).

Electrical Power, \( P = VI = I^2R = \frac{V^2}{R} \)

{Brightness of a lamp is determined by the power dissipated, NOT: by V, or I or R alone}

EXAMPLE 13F1

A high-voltage transmission line with a resistance of 0.4 \( \Omega \) km\(^{-1} \) carries a current of 500 A. The line is at a potential of 1200 kV at the power station and carries the current to a city located 160 km from the power station. Calculate

(a) the power loss in the line.

The power loss in the line \( P = I^2R \)
\[ = 500^2 \times 0.4 \times 160 \]
\[ = 16 \ \text{MW} \]

(b) the fraction of the transmitted power that is lost.

The total power transmitted \( = IV \)
\[ = 500 \times 1200 \times 10^3 \]
\[ = 600 \ \text{MW} \]

The fraction of power loss \[ = \frac{16}{600} \]
\[ = 0.267 \]

g. Define resistance and the ohm.

Resistance is defined as the ratio of the potential difference across a component to the current flowing through it, ie \( R = \frac{V}{I} \)

{It is NOT defined as the gradient of a V-I graph; however for an ohmic conductor, its resistance equals the gradient of its V-I graph as this graph is a straight line which passes through the origin}

The Ohm: is the resistance of a resistor if there is a current of 1 A flowing through it when the pd across it is 1 V, ie, 1 \( \Omega \) = One volt per ampere

h. Recall and solve problems by using \( V = IR \).

EXAMPLE 13H1

In the circuit below, the voltmeter reading is 8.00 V and the ammeter reading is 2.00 A. Calculate the resistance of \( R \).
Resistance of $R = \frac{V}{I} = \frac{8}{2} = 4.0 \, \Omega$

<table>
<thead>
<tr>
<th>I.</th>
<th>Sketch and explain the $I \sim V$ characteristics of a metallic conductor at constant temperature, a semiconductor and a filament lamp. Sketch the temperature characteristics of thermistors.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Metallic conductor (at constant temperature)</th>
<th>Filament lamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I / A$</td>
<td>$I / A$</td>
</tr>
<tr>
<td>$V / V$</td>
<td>$V / V$</td>
</tr>
<tr>
<td>The resistance (i.e. the ratio $\frac{V}{I}$) is constant because metallic conductors at constant temperature obey Ohm’s Law.</td>
<td>As $V$ increases, the temperature increases, resulting in an increase in the amplitude of vibration of ions and the collision frequency of electrons with the lattice ions. Hence the resistance of the filament increases with $V$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thermistor</th>
<th>Semi-conductor diode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I / A$</td>
<td>$I / A$</td>
</tr>
<tr>
<td>$V / V$</td>
<td>$V / V$</td>
</tr>
<tr>
<td>A thermistor is made from semi-conductors. As $V$ increases, temperature increases. This releases more charge carriers (electrons and holes) from the lattice, thus reducing the resistance of the thermistor. Hence, resistance decreases as temperature increases.</td>
<td>In forward bias, a diode has low resistance. In reverse bias, the diode has high resistance until the breakdown voltage is reached.</td>
</tr>
</tbody>
</table>

k. State Ohm’s law.

**Ohm’s law:** The current in a component is proportional to the potential difference across it provided physical conditions (eg temp) stay constant.

l. Recall and solve problems by using $R = \frac{\rho L}{A}$.

$R = \frac{\rho L}{A}$ \{for a conductor of length $l$, uniform x-sect area $A$ and resistivity $\rho$\}

**Resistivity** is defined as the resistance of a material of unit cross-sectional area and unit length.

(From $R = \frac{\rho}{A}$, $\rho = \frac{RA}{L}$)
EXAMPLE 13L1

Calculate the resistance of a nichrome wire of length 500 mm and diameter 1.0 mm, given that the resistivity of nichrome is $1.1 \times 10^{-6} \, \Omega \cdot m$.

Resistance, $R = \frac{\rho l}{A}$

$$= \frac{(1.1 \times 10^{-6}) (500 \times 10^{-3})}{\pi \left(1 \times 10^{-3}\right)^2}$$

$$= 0.70 \, \Omega$$

m. Define EMF in terms of the energy transferred by a source in driving unit charge round a complete circuit.

Electromotive force EMF is defined as the energy transferred/converted from non-electrical forms of energy into electrical energy when unit charge is moved round a complete circuit.

ie $\text{EMF} = \text{Energy Transferred per unit charge}$,

ie $E = \frac{W}{Q}$

n. Distinguish between EMF and P.D. in terms of energy considerations.

EMF refers to the electrical energy generated from non-electrical energy forms, whereas PD refers to electrical energy being changed into non-electrical energy.

For example,

<table>
<thead>
<tr>
<th>EMF Sources</th>
<th>Energy Change</th>
<th>PC across</th>
<th>Energy Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical Cell</td>
<td>Chem $\rightarrow$ Elec</td>
<td>Bulb</td>
<td>Elec $\rightarrow$ Light</td>
</tr>
<tr>
<td>Generator</td>
<td>Mech $\rightarrow$ Elec</td>
<td>Fan</td>
<td>Elec $\rightarrow$ Mech</td>
</tr>
<tr>
<td>Thermocouple</td>
<td>Thermal $\rightarrow$ Elec</td>
<td>Door Bell</td>
<td>Elec $\rightarrow$ Sound</td>
</tr>
<tr>
<td>Solar Cell</td>
<td>Solar $\rightarrow$ Elec</td>
<td>Heating element</td>
<td>Elec $\rightarrow$ Thermal</td>
</tr>
</tbody>
</table>

o. Show an understanding of the effects of the internal resistance of a source of EMF on the terminal potential difference and output power.

Internal resistance is the resistance to current flow within the power source. It reduces the potential difference (not EMF) across the terminal of the power supply when it is delivering a current.

Consider the circuit below:

The voltage across the resistor, $V = IR$.

The voltage lost to internal resistance $= Ir$.

Thus, the EMF of the cell, $E = IR + Ir$

Therefore, if $I = 0 \, A$ or if $r = 0 \, \Omega$, $V = E$. 

www.youtube.com/megalecture
Chapter 14: D.C. Circuits

- Practical Circuits
- Series and parallel arrangements
- Potential divider
- Balanced potentials

a. Recall and use appropriate circuit symbols as set out in SI Units, Signs, Symbols and Abbreviations (ASE, 1981) and Signs, Symbols and Systematics (ASE, 1995).

b. Draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, and/or any other type of component referred to in the syllabus.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="cell-battery.png" alt="Symbol" /></td>
<td>Cell/ Battery</td>
<td><img src="thermistor.png" alt="Symbol" /></td>
<td>Thermistor</td>
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<tr>
<td><img src="powersupply.png" alt="Symbol" /></td>
<td>Power Supply</td>
<td><img src="diode.png" alt="Symbol" /></td>
<td>Diode</td>
</tr>
<tr>
<td><img src="switch.png" alt="Symbol" /></td>
<td>Switch</td>
<td><img src="potentialdivider.png" alt="Symbol" /></td>
<td>Potential Divider</td>
</tr>
<tr>
<td><img src="ammeter.png" alt="Symbol" /></td>
<td>Ammeter</td>
<td><img src="earth.png" alt="Symbol" /></td>
<td>Earth</td>
</tr>
<tr>
<td><img src="voltmeter.png" alt="Symbol" /></td>
<td>Voltmeter</td>
<td><img src="aerial.png" alt="Symbol" /></td>
<td>Aerial/ Antenna</td>
</tr>
<tr>
<td><img src="galvanometer.png" alt="Symbol" /></td>
<td>Galvanometer</td>
<td><img src="capacitor.png" alt="Symbol" /></td>
<td>Capacitor</td>
</tr>
<tr>
<td><img src="filamentlamp.png" alt="Symbol" /></td>
<td>Filament Lamp</td>
<td><img src="inductor.png" alt="Symbol" /></td>
<td>Inductor</td>
</tr>
<tr>
<td><img src="resistor.png" alt="Symbol" /></td>
<td>Resistor</td>
<td><img src="wiresnoc.png" alt="Symbol" /></td>
<td>Wires crossing with no connection</td>
</tr>
<tr>
<td><img src="variableresistor.png" alt="Symbol" /></td>
<td>Variable Resistor</td>
<td><img src="wiresconn.png" alt="Symbol" /></td>
<td>Wires crossing with connection</td>
</tr>
<tr>
<td><img src="lightdependentresistor.png" alt="Symbol" /></td>
<td>Light-Dependent Resistor</td>
<td><img src="loudspeaker.png" alt="Symbol" /></td>
<td>Loudspeaker</td>
</tr>
</tbody>
</table>

c. Solve problems using the formula for the combined resistance of two or more resistors in series.

d. Solve problems using the formula for the combined resistance of two or more resistors in parallel.

Resistors in Series: \[ R = R_1 + R_2 + \ldots \]

Resistors in Parallel: \[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \]

**EXAMPLE 14CD1**

Three resistors of resistance 2 Ω, 3 Ω and 4 Ω respectively are used to make the combinations X, Y and Z shown in the diagrams. List the combinations in order of increasing resistance.

Resistance for X = \( \left( \frac{1}{2} + \frac{1}{4+3} \right)^{-1} \) = 1.56 Ω

Resistance for Y = 2 + \( \frac{1}{4 + \frac{1}{3}} \) = 3.71 Ω
Resistance for \( Z = \left( \frac{1}{3} + \frac{1}{2} + \frac{1}{4} \right)^{-1} \) = 0.923 Ω

Therefore, the combination of resistors in order of increasing resistance is \( Z \times Y \).

e. **Solve problems involving series and parallel circuits for one source of e.m.f.**

**EXAMPLE 14E1**

E.g. 4 Referring to the circuit drawn, determine the value of \( I_1 \), \( I_2 \) and \( I \), the combined resistance in the circuit.

\[
E = I_1 (160) = I_2 (4000) = I_3 (32000)
\]

\[
I_1 = \frac{2}{160} = 0.0125 \text{ A}
\]

\[
I_2 = \frac{2}{4000} = 5 \times 10^{-4} \text{ A}
\]

\[
I_3 = \frac{2}{32000} = 6.25 \times 10^{-5} \text{ A}
\]

Since \( I = I_1 + I_2 + I_3 \), \( I = 13.1 \text{ mA} \)

Applying Ohm’s Law, \( R = \frac{13.1 \times 10^{-3}}{2} = 153 \Omega \)

**EXAMPLE 14E2**

A battery with an EMF of 20 V and an internal resistance of 2.0 Ω is connected to resistors \( R_1 \) and \( R_2 \) as shown in the diagram. A total current of 4.0 A is supplied by the battery and \( R_2 \) has a resistance of 12 Ω. Calculate the resistance of \( R_1 \) and the power supplied to each circuit component.

\[
E - I r = I_2 R_2
\]

\[
20 - 4 (2) = I_2 (12)
\]

\( I_2 = 1 \text{ A} \)

Therefore, \( I_1 = 4 - 1 = 3 \text{ A} \)

\[
E - I r = I_1 R_1
\]

\[
12 = 3 R_1
\]

Therefore, \( R_1 = 4 \)

Power supplied to \( R_1 \) = \( (I_1)^2 R_1 \)

= 36 W

Power supplied to \( R_2 \) = \( (I_2)^2 R_2 \)

= 12 W

f. **Show an understanding of the use of a potential divider circuit as a source of variable p.d.**

For **potential divider** with 2 resistors in series,

Potential drop across \( R_1 \), \( V_1 = \frac{R_1}{R_1 + R_2} \times \text{PD across } R_1 \) & \( R_2 \)

Potential drop across \( R_2 \), \( V_1 = \frac{R_2}{R_1 + R_2} \times \text{PD across } R_1 \) & \( R_2 \)

**EXAMPLE 14F1**

Two resistors, of resistance 300 kΩ and 500 kΩ respectively, form a potential divider with outer junctions maintained at potentials of +3 V and −15 V.
g. Explain the use of thermistors and light-dependent resistors in potential dividers to provide a potential difference which is dependent on temperature and illumination respectively.

**USAGE OF A THERMISTOR**

A thermistor is a resistor whose resistance varies greatly with temperature. Its resistance decreases with increasing temperature. It can be used in potential divider circuits to monitor and control temperatures.

**EXAMPLE 14G1**

In the figure below, the thermistor has a resistance of 800 Ω when hot, and a resistance of 5000 Ω when cold. Determine the potential at W when the temperature is hot.

When thermistor is hot, potential difference across it

\[
\frac{800}{800 + 1700} \times (7 - 2) = 1.6 \text{ V}
\]

The potential at W = 2 + 1.6 = 3.6 V

**USAGE OF A LIGHT-DEPENDENT RESISTOR**

An LDR is a resistor whose resistance varies with the intensity of light falling on it. Its resistance decreases with increasing light intensity. It can be used in a potential divider circuit to monitor light intensity.

**EXAMPLE 14G2**

In the figure below, the resistance of the LDR is 6.0 MΩ in the dark but then drops to 2.0 kΩ in the light. Determine the potential at point P when the LDR is in the light.

In the light the potential difference across the LDR

\[
\frac{2k}{3k + 2k} \times (18 - 3) = 6 \text{ V}
\]

The potential at P = 18 – 6 = 12 V

h. Recall and solve problems using the principle of the potentiometer as a means of comparing potential differences.

The potential difference along the wire is proportional to the length of the wire. The sliding contact will move along wire AB until it finds a point along the wire such that the galvanometer shows a zero reading.
When the galvanometer shows a zero reading, the current through the galvanometer (and the device that is being tested) is zero and the potentiometer is said to be “balanced”.

If the cell has negligible internal resistance, and if the potentiometer is balanced,

\[ V = \frac{L_1}{L_1 + L_2} \times E \]

**EXAMPLE 14H1**

In the circuit shown, the potentiometer wire has a resistance of 60 Ω. Determine the EMF of the unknown cell if the balanced point is at B.

**Resistance of wire AB**

\[ R_{AB} = \frac{0.65}{0.65 + 0.35} \times 60 = 39 \, \Omega \]

**EMF of the test cell**

\[ E = \frac{39}{60 + 20} \times 12 = 5.85 \, V \]
Chapter 15: Electromagnetism
- Force on a current-carrying conductor
- Force on a moving charge
- Magnetic fields due to currents
- Force between current-carrying conductors

a. Show an appreciation that a force might act on a current-carrying conductor placed in a magnetic field.

b. Recall and solve problems using the equation \( F = BIL\sin\theta \) with directions as interpreted by Fleming’s left-hand rule.

When a conductor carrying a current is placed in a magnetic field, it experiences a magnetic force.

\[ F = BIL\sin\theta \]

(F is directed perpendicularly out of the paper)

Figure 15 shows a wire of length \( L \) carrying a current \( I \) and lying in a magnetic field of flux density \( B \). Suppose the angle between the current \( I \) and the field \( B \) is \( \theta \), the magnitude of the force \( F \) on the conductor is given by

\[ F = BIL\sin\theta \]

The direction of the force can be found using Fleming’s Left Hand Rule (Fig. 16). Note that the force is always perpendicular to the plane containing both the current \( I \) and the magnetic field \( B \).

- If the wire is parallel to the field lines, then \( \theta = 0^\circ \), and \( F = 0 \). (No magnetic force acts on the wire)
- If the wire is at right angles to the field lines, then \( \theta = 90^\circ \), and the magnetic force acting on the wire would be maximum (\( F = BIL \))

**EXAMPLE 15B1**
The 3 diagrams below each show a magnetic field of flux density 2 T that lies in the plane of the page. In each case, a current \( I \) of 10 A is directed as shown. Use Fleming’s Left Hand Rule to predict the directions of the forces and work out the magnitude of the forces on a 0.5 m length of wire that carries the current.

(Assume the horizontal is the current)

\[ F = BIL\sin\theta \]

\[ F = BIL\sin60^\circ \]

\[ F = BIL\sin90^\circ \]

\[ F = 2 \times 10 \times 0.5 \times \sin60^\circ \]

\[ = 8.66 \text{ N} \]

\[ F = 2 \times 10 \times 0.5 \times \sin90^\circ \]

\[ = 10 \text{ N} \]

\[ F = 2 \times 10 \times 0.5 \times \sin180^\circ \]

\[ = 0 \text{ N} \]

c. Define magnetic flux density and the tesla.

**Magnetic flux density** \( B \) is defined as the force acting per unit current in a wire of unit length at right-angles to the field,

\[ B = \frac{F}{IL\sin\theta} \]

\[ \Rightarrow F = BIL\sin\theta \]

\( \theta \): Angle between the \( B \) and \( L \)

(NB: write down the above defining equation & define each symbol if you’re not able to give the “statement form”.)
Direction of the magnetic force is always perpendicular to the plane containing the current I and B (even if \( \theta \neq 0 \)).

The Tesla is defined as the magnetic flux density of a magnetic field that causes a force of one newton to act on a current of one ampere in a wire of length one metre which is perpendicular to the magnetic field.

d. Show an understanding of how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance.

\[
\begin{align*}
\text{mg} \cdot x & = \text{F} \cdot y \\
B & = \frac{\text{mg} \cdot x}{ILy}
\end{align*}
\]

**EXAMPLE 15D1**

A 100-turn rectangular coil 6.0 cm by 4.0 cm is pivoted about a horizontal axis as shown below. A horizontal uniform magnetic field of direction perpendicular to the axis of the coil passes through the coil.

Initially, no mass is placed on the pan and the arm is kept horizontal by adjusting the counter-weight. When a current of 0.50 A flows through the coil, equilibrium is restored by placing a 50 mg mass on the pan, 8.0 cm from the pivot. Determine the magnitude of the magnetic flux density and the direction of the current in the coil.

Taking moments about the pivot, sum of Anti-clockwise moments = Clockwise moment

\[
\begin{align*}
(2 \times n)(F_B) \times P & = W \times Q \\
(2 \times n)(BIL) \times P & = mg \times Q, \text{ where } n: \text{no. of wires on each side of the coil.} \\
(2 \times 100)(B \times 0.5 \times 0.06) \times 0.02 & = 50 \times 10^{-6} \times 9.81 \times 0.08 \\
B & = 3.27 \times 10^{-4} \text{ T}
\end{align*}
\]

e. Predict the direction of the force on a charge moving in a magnetic field.

Force acting on a moving charge: \( F = BQ \sin \theta \)  \( \{\theta: \text{Angle between } B \text{ and } v\} \)
The direction of this force may be found by using Fleming's left hand rule. The angle θ determines the type of path the charged particle will take when moving through a uniform magnetic field:

- If θ = 0°, the charged particle takes a straight path since it is not deflected (F = 0)
- If θ = 90°, the charged particle takes a circular path since the force at every point in the path is perpendicular to the motion of the charged particle.

Since F is always be perpendicular to v (even if θ ≠ 0), the magnetic force can provide the centripetal force, \[ Bqv = \frac{mv^2}{r} \]

Recall and solve problems using \[ F = Bqv \sin \theta \].

**EXAMPLE 15F1**

An electron moves in a circular path in vacuum under the influence of a magnetic field.

The radius of the path is 0.010 m and the flux density is 0.010 T. Given that the mass of the electron is \(9.11 \times 10^{-31}\) kg and the charge on the electron is \(-1.6 \times 10^{-19}\) C, determine

(i) whether the motion is clockwise or anticlockwise;

The magnetic force on the electron points towards the centre of the circular path; hence using Fleming’s left hand rule, we deduce that the current I points to the left. The electron must be moving clockwise.

(ii) the velocity of the electron.

\[ Bqv = \frac{mv^2}{r} \]

\[ v = \frac{Bqr}{m} = \frac{(0.010)(1.6 \times 10^{-19})(0.010)}{9.11 \times 10^{-31}} = 1.76 \times 10^7 \text{ m s}^{-1} \]

g. Describe and analyse deflections of beams of charged particles by uniform electric and uniform magnetic fields.

Use Fleming’s Left Hand Rule to analyse, then apply Parabolic Motion to analyse.

h. Explain how electric and magnetic fields can be used in velocity selection for charged particles.

**Crossed-Fields in Velocity Selector:**

A setup whereby an E-field and a B-field are perpendicular to each other such that they exert equal & opposite forces on a moving charge (if the velocity is "a certain value")

I.e., if \[ \text{Magnetic Force} = \text{Electric Force} \]

\[ Bqv = qE \]

\[ v = \frac{E}{B} \]

Only particles with speed \(v = \frac{E}{B}\) emerge from the cross-fields undeflected.

For particles with speed \(v > \frac{E}{B}\), Magnetic Force > Electric Force

For particles with speed \(v < \frac{E}{B}\), Magnetic Force < Electric Force

i. Sketch flux patterns due to a long straight wire, a flat circular coil and a long solenoid.
j. **Show an understanding that the field due to a solenoid may be influenced by the presence of ferrous core.**

The strength of the generated magnetic field can be increased (by about 1000 times) by adding a ferrous (iron) core inside the solenoid. This is because a ferrous material has a higher permeability than air. Another explanation is that iron, being a ferromagnetic material, becomes magnetised when placed into the solenoid, thus contributing to the overall magnetic field strength of the solenoid. [Permeability, \( \mu \), is a constant relating the strength of the magnetic field at a point to the current in the wire. (Not in syllabus)]

k. **Explain the forces between current-carrying conductors and predict the direction of the forces.**

Each conductor should thus experience a force \( F = BIL \sin \theta \), either of attraction or repulsion, depending on the direction of the currents. Whether the forces are attractive or repulsive can be predicted using the Right hand grip rule and Fleming’s left-hand rule.

<table>
<thead>
<tr>
<th>CURRENTS FLOWING IN SAME DIRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis of force acting on conductor 1</td>
</tr>
<tr>
<td>( I_1 )</td>
</tr>
<tr>
<td>( B_2 )</td>
</tr>
</tbody>
</table>

| Analysis of force acting on conductor 2 |
|\( I_1 \) | \( I_2 \) |
| \( B_1 \) | \( F_2 \) |

| CURRENTS FLOWING IN OPPOSITE DIRECTIONS |
EXAMPLE 15K1

A long length of aluminium foil ABC is hung over a wooden rod as shown below. A large current is momentarily passed through the foil in the direction ABC, and the foil moves.

(i) Draw arrows to indicate the directions in which AB and BC move.
Since currents in AB and BC are 'unlike' currents (they are flowing in opposite directions), the two foil sections AB and BC will repel each other.

(ii) Explain why the foil moves in this way.
The current in the left foil AB produces a magnetic field in the other (BC). According to the Right Hand Grip Rule & Fleming’s Left Hand Rule, the force on BC is away from and perpendicular to AB. By a similar consideration, the force on AB is also away from BC. Thus the forces between the foils are repulsive.
**Chapter 16: Electromagnetic Induction**

- Magnetic flux
- Laws of electromagnetic induction

**a. Define magnetic flux and the weber.**

**Electromagnetic induction** refers to the phenomenon where an emf is induced when the magnetic flux linking a conductor changes.

**Magnetic Flux** is defined as the product of the magnetic flux density and the area normal to the field through which the field is passing. It is a scalar quantity and its S.I. unit is the weber (Wb).

\[ \phi = B A \]

**The Weber** is defined as the magnetic flux if a flux density of one tesla passes perpendicularly through an area of one square metre.

**b. Recall and solve problems using** \( \phi = B A \).

**EXAMPLE 16B1**

A magnetic field of flux density 20 T passes down through a coil of wire, making an angle of 60° to the plane of the coil as shown. The coil has 500 turns and an area of 25 cm². Determine:

1. **the magnetic flux through the coil**
   \[ \phi = B A = 20 \sin 60° \times 25 \times 10^{-4} = 0.0433 \text{ Wb} \]

2. **the flux linkage through the coil**
   \[ \Phi = N \phi = 500 \times 0.0433 = 21.65 \text{ Wb} \]

**c. Define magnetic flux linkage.**

**Magnetic Flux Linkage** is the product of the magnetic flux passing through a coil and the number of turns of the coil.

\[ \Phi = N \phi = N B A \]

**d. Infer from appropriate experiments on electromagnetic induction:**

i. **That a changing magnetic flux can induce an e.m.f. in a circuit,**

In the set up shown above, when the switch S connected to coil A is closed, the galvanometer needle connected to coil B moves to 1 side momentarily.

And when the switch S is opened, the galvanometer needle moves to the other side momentarily.

At the instant when switch S is either opened or closed, there is a change in magnetic flux in coil A.

The movement in the needle of the galvanometer indicates that when there is a change in magnetic flux in coil A, a current passes through coil B momentarily. This suggests that an EMF is generated in
coil B momentarily.

ii. **That the direction of the induced e.m.f. opposes the change producing it.**

See below

iii. **The factors affecting the magnitude of the induced e.m.f.**

When a magnet is pushed into a coil as shown, the galvanometer deflects in one direction momentarily.

When the magnet is not moving, the galvanometer shows no reading.

When the magnet is withdrawn from the coil, the galvanometer deflects in the opposite direction momentarily.

When the magnet is moved, its field lines are being “cut” by the coil. This generates an induced EMF in the coil that produces an induced current that flows in the coil, causing the deflection in the ammeter.

The magnitude of the deflection depends on the magnetic field density $B$, the speed of motion $v$ of the magnet, and the number of turns $N$ in the coil.

e. **Recall and solve problems using Faraday’s law of electromagnetic induction and Lenz’s law.**

**Faraday’s Law**
The magnitude of induced EMF is directly proportional/equal to the rate of change of magnetic flux-linkage.

$|E| = \frac{dNBA}{dt}$

**Lenz’s Law**
The direction of the induced EMF is such that its effects oppose the change which causes it, or The induced current in a closed loop must flow in such a direction that its effects opposes the flux change (or change) that produces it

**EXAMPLE 16E1**
Explain how Lenz’s Law is an example of the law of conservation of energy:
(Illustrate with diagram of a coil “in a complete circuit”, bar magnet held in hand of a person (= external agent))

- As the ext agent causes the magnet to approach the coil, by Lenz’s law, a current is induced in such a direction that the coil repels the approaching magnet.
- Consequently, work has to be done by the external agent to overcome this opposition, and
- It is this work done which is the source of the electrical energy (Not: induced emf)

For a straight conductor “cutting across” a $B$-field: $E = B L v \sin \theta$

For a coil rotating in a $B$-field with angular frequency $\omega$: 
\[ E = N B A \cos \omega t, \quad \text{if} \quad \phi = B A \sin \omega t \]
\[ \& \quad E = N B A \sin \omega t, \quad \text{if} \quad \phi = B A \cos \omega t \]

(Whether \( \phi = B A \sin \omega t \), or \( \phi = B A \cos \omega t \), would depend on the initial condition)

The induced EMF is the negative of the gradient of the \( \phi \sim t \) graph \( \text{since} \quad E = -\frac{d\phi}{dt} \)

\[ \Rightarrow \text{the graphs of} \quad E \sim t \quad \& \quad \phi \sim t, \quad \text{for the rotating coil have a phase difference of} \quad 90^\circ. \]

**f.** Explain simple applications of electromagnetic induction.

**Background Knowledge**

**Eddy Currents**

Eddy currents are currents induced in metals moving in a magnetic field or metals that are exposed to a changing magnetic field.

Consider a solid metallic cylinder rotating in a \( B \)-field as shown.

(a) A force resisting the rotation would be generated as shown.

(b) Heat would be generated by the induced current in cylinder.

To reduce eddy currents, the solid cylinder could be replaced with a stack of “coins” with insulation between one another. The insulation between the coins increases resistance and reduces eddy current, thus reducing friction or heating.

**Applications of Eddy Currents**

1. **Induction Cooker**

   Changing magnetic fields in the stove generate eddy currents in the base of the metal pot placed on it, thus producing heat.

   1. The element’s electronics power a coil that produces a high-frequency electromagnetic field.
   2. The field penetrates the metal of the ferrous (magnetic-material) cooking vessel and sets up a circulating eddy current, which generates heat.
   3. The heat generated in the cooking vessel is transferred to the vessel’s contents.
   4. Nothing outside the vessel is affected by the field—as soon as the vessel is removed from the element, or the element turned off, heat generation stops.

   (Note: the process described at #2 above is called an “eddy current”; in fact, most of the heating is from “hysteresis”, which means the resistance of the ferrous material to rapid changes in magnetization—but the general idea is the same: the heat is generated in the cookware).

2. **Metal detectors**

   A pulsing current is applied to the coil, which then induces a magnetic field shown. When the magnetic field of the coil moves across metal, such as the coin in this illustration, the field induces electric currents (called eddy currents) in the coin. The eddy currents induce their own magnetic field, which generates an opposite current in the coil, which induces a signal indicating the presence of metal.
3 Electric guitars

Electric guitars use electromagnetic pickups in which an induced emf is generated in a coil of wire by a vibrating string. Most guitars have at least two pickup coils located below each string. Each pickup is sensitive to different harmonics produced by the vibrating string. The string is made from a magnetisable metal, and the pickup consists of a coil of wire within which a permanent magnet is located. The permanent magnet produces a magnetic field that penetrates the guitar string, causing it to become magnetized with north and south poles. When the string is plucked, it oscillates, thereby changing the magnetic flux that passes through the coil. The changing magnetic flux induces an emf in the coil, and the polarity of this emf alternates with the vibratory motion of the string. A string vibrating at 440 Hz, for example, induces a 440-Hz ac emf in the coil. This signal, after being amplified, is sent to the speakers, which produce a 440-Hz sound wave.

4 Earth-leakage circuit breaker (ELCB)

ELCB is used in electrical circuits in the home to protect the users from electric shocks when there is a fault in the electrical appliance. The ELCB can be triggered to stop a current, depending on whether an induced voltage appears across a sensing coil.

The sensing coil is wrapped around an iron ring, through which the current carrying wire passes. Under normal operating conditions, the current flowing in the coils A and B are equal but opposite directions. The magnetic field produced by both coils will always be equal and opposite, hence cancelling out each other at the secondary or sensing coil. The situation changes when there is a leakage in the electrical appliance, where the current returning is smaller than the current going into the appliance. Under this condition, the net
magnetic field through the secondary coil is no longer zero and changes with time, since the current is ac. The changing magnetic flux causes an induced voltage to appear in the secondary coil, which triggers the circuit breaker to stop the current. ELCB works very fast (in less than a millisecond) and turn off the current before it reaches a dangerous level.

5 Eddy current brake

An eddy current brake, like a conventional friction brake, is responsible for slowing an object, such as a train or a roller coaster. Unlike friction brakes, which apply pressure on two separate objects, eddy current brakes slow an object by creating eddy currents through electromagnetic induction which create resistance, and in turn either heat or electricity. Consider a metal disk rotating clockwise through a perpendicular magnetic field but confined to a limited portion of the disk area. (Compare this with the Faraday's disk earlier)

Sector Oa and Oc are not in the field, but they provide return conducting path, for charges displaced along Ob to return from b to O. The result is a circulation of eddy current in the disk. The current experiences a magnetic force that opposes the rotation of the disk, so this force must be to the right. The return currents lie outside the field, so they do not experience magnetic forces. The interaction between the eddy currents and the field causes a braking action on the disk.
Chapter 17: Alternating Currents

- Characteristics of alternating currents
- The transformer
- Rectification with a diode

a. Show an understanding and use the terms period, frequency, peak value and root-mean square value as applied to an alternating current or voltage.

![Graph showing alternating current](image)

**Peak current, I₀** = 3 A

**Peak-to-peak current, I_{p-p}** = 6 A

**Period, T** = 20 ms

**Frequency, f = \frac{1}{T}** = 50 Hz

(This is the frequency of the mains supply in Singapore.)

**Angular Frequency, ω = 2πf** = 314 rad s⁻¹

**Instantaneous current:** the current at a particular instant.

Since this A.C. signal can be described by the equation:

\[ I = I_0 \sin (\omega t) \]

or

\[ V = V_0 \sin (\omega t) \]

the instantaneous current I or voltage V at time t is given by \( I_0 \sin (\omega t) \) or \( V_0 \sin (\omega t) \).

**Note:** Both the period and amplitude of a sinusoidal A.C should be constant.

**Root-mean-square current of an alternating current** is defined as that steady (NOT direct) current that produces the same heating effect \((i.e \ I^2 R)\) as the alternating current in a given resistor.

b. Deduce that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.

\[
\begin{align*}
\text{(Instantaneous) sinusoidal current: } & I = I_0 \sin \omega t, \quad \text{Similarly, } V = V_0 \sin \omega t \\
I_{rms} & = \frac{I_0}{\sqrt{2}}, \quad V_{rms} = \frac{V_0}{\sqrt{2}}, \quad \text{for sinusoidal ac only}
\end{align*}
\]

**Relationship between Peak, & RMS values of PD & Current:** \( V_0 = I_0 R \), \( V_{rms} = I_{rms} R \)

**Mean/Ave Power, P_{ave} = \frac{I_{rms}^2 R}{2} = \frac{V_{rms}^2 R}{2} = I_{rms} V_{rms} \**

\[ = \frac{1}{2} \times \text{Maximum Instantaneous Power} = \frac{1}{2} I_0 V_0 \quad \text{(for sinusoidal AC)} \]

**Max (Instantaneous) Power, P_{max} = I_0 V_0 = I_0^2 R \**

C. Represent an alternating current or an alternating voltage by an equation of the form \( x = x_0 \sin \omega t \).

For sinusoidal current
d. Distinguish between r.m.s. and peak values and recall and solve problems using the relationship $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ for the sinusoidal case.

The root-mean-square (R.M.S.) value, $I_{\text{rms}}$, of an A.C. is the magnitude of the direct current that produces the same average heating effect as the alternating current in a given resistance whereas peak value is the maximum current of an AC.

e. Show an understanding of the principle of operation of a simple iron-cored transformer and recall and solve problems using $\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}$ for an ideal transformer.

$$\text{Ideal transformer: } V_P I_P = V_S I_S \rightarrow \frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}$$

{[Mean power in the primary coil = Mean power in the secondary coil ]}

{[Values of I & V may be either R.M.S. or peak but not instantaneous values; $\frac{N_S}{N_P}$: turns ratio]}

Power Loss during Transmission of Electrical Power

Power Generated at power station $P_{\text{gen}} = V_i I$, where $I$: current in the transmission, $V_i$: Voltage at which power is transmitted

i.e., $I = \frac{P_{\text{gen}}}{V_i}$

Power Loss in Transmission Cables, $P_L = I^2 R_C = \left(\frac{P_{\text{gen}}}{V_i}\right)^2 R_C$ $R_C = $ cable resistance

Thus to reduce power loss, for a given amt of power generated, electricity is transmitted at high voltage $V_i$ (ie low current). ($V_i$ is NOT the pd across the cables)

f. Explain the use of a single diode for the half-wave rectification of an alternating current.

If a single diode is connected to an A.C. circuit as shown, a half-wave rectification occurs.

The graphs for the input and output voltages, and the output current, are shown below.

In the regions A and C, the diode is forward biased, allowing current to flow. When the input voltage becomes negative, the diode prevents the current flow, because it is reverse biased.
### Chapter 18: Quantum Physics
- Energy of a photon
- The photoelectric effect
- Wave-particle duality
- Energy levels in atoms
- Line spectra
- X-ray spectra
- The uncertainty principle
- Schrödinger model
- Barrier tunnelling

#### a. Show an appreciation of the particulate nature of electromagnetic radiation.

A photon is a discrete packet (or quantum) of energy of an electromagnetic radiation/wave.

#### b. Recall and use \( E = hf \)

**Energy of a photon,** \( E = hf = \frac{hc}{\lambda} \)

Where:
- \( h \): Planck’s constant
- \( \lambda \): wavelength

\( \lambda_{violet} \approx 4 \times 10^{-7} \text{ m}, \lambda_{red} \approx 7 \times 10^{-7} \text{ m} \)

(N07P1Q34: need to recall these values)

**Power of electromagnetic radiation,** \( P = \text{Rate of incidence of photon} \times \text{Energy of a photon} = \left( \frac{N}{t} \right) \frac{hc}{\lambda} \)

#### c. Show an understanding that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature.

**Photoelectric effect** refers to the emission of electrons from a cold metal surface when electromagnetic radiation of sufficiently high frequency falls on it.

**4 Major Observations:**

(a) No electrons are emitted if the frequency of the light is below a minimum frequency (called the threshold frequency), regardless of the intensity of light.

(b) Rate of electron emission (i.e., photoelectric current) is proportional to the light intensity.

(c) Emitted electrons have a range of kinetic energy, ranging from zero to a certain maximum value. Increasing the freq increases the kinetic energies of the emitted electrons and in particular, increases the maximum kinetic energy. This maximum kinetic energy depends only on the frequency and the metal used (\( \phi \)); the intensity has no effect on the kinetic energy of the electrons.

(d) Emission of electrons begins instantaneously (i.e., no time lag between emission & illumination) even if the intensity is very low.

NB: (a), (c) & (d) cannot be explained by Wave Theory of Light; instead they provide evidence for the particulate/particle nature of electromagnetic radiation.

**Explanation for how photoelectric effect provides evidence for the particulate nature of em radiation:** (N07P3)

(Consider the observations (a), (c) & (d). Use **any 2** observations above to describe how they provide evidence that em radiation has a particle nature.)

- According to the “Particle Theory of Light”, em radiation consists of a stream of particles/phots/discrete energy packets, **each of energy** \( hf \). Also, **no more than one electron can absorb the energy of one photon (“All-or-Nothing Law”)**.

- Thus if the energy of a photon \( hf < \) the minimum energy required for emission (\( \phi \)), no emission can take place no matter how intense the light may be. (Explains observation (a))

- This also explains why, **even at very low intensities**, as long as \( hf > \phi \), emission takes place without a time delay between illumination of the metal & ejection of electrons. (Explains observation (d))
d. Recall the significance of threshold frequency.

Threshold frequency is the minimum frequency of the electromagnetic radiation required to eject an electron from a metal surface. (This is because the electrons are held back by the attractive forces of the positive nuclei in the metal.)

e. Recall and use the equation $\frac{1}{2}mv_{\text{max}}^2 = eV_s$ where $V_s$ is the stopping potential.

Work function of a metal is the minimum energy required to eject an electron from a metal surface

$$\phi = hf_0 = \frac{hc}{\lambda_0}$$

$f_0$ = threshold frequency

$\lambda_0$ = threshold wavelength

Maximum KE of electrons, $\frac{1}{2}m_e v_{\text{max}}^2 = eV_s$ (in magnitude), $V_s$: stopping potential

$$hf = \phi + eV_s$$

g. Explain why the maximum photoelectric energy is independent of intensity whereas the photoelectric current is proportional to intensity.

From $eV_s = hf - \phi$,

- If only intensity doubles, the saturation current doubles ($V_s$: no change)
- If only frequency increases, magnitude of $V_s$ also increases, thus no change to saturation current.

Intensity = Incident Power
Illuminated Area = \( \frac{N}{t} \) \( \frac{hc}{\lambda} \)

\( \Rightarrow \) Intensity \( \propto \) Rate of incidence of photons, \( N/t \) (for a given \( \lambda \))

Photocurrent \( I = \left( \frac{n}{t} \right) e \), where \( \left( \frac{n}{t} \right) \) = rate of emission of electrons

Why rate of emission of electrons \( \ll \) rate of incidence of photons (for \( f > f_0 \)):

- Not every photon would collide with an electron; most are reflected by the metal or miss hitting any electron.
- On the way out to the metal surface, an electron may lose its kinetic energy to ions and other electrons it encounters along the way. This energy loss prevents it from overcoming the work function.

1 eV = \( (1.6 \times 10^{-19} \text{ C}) \times (1 \text{ V}) \) = \( 1.6 \times 10^{-19} \text{ J} \)

1 nanometre (nm) = \( 1 \times 10^{-9} \text{ m} \)

h. Recall, use and explain the significance of $hf = \phi + \frac{1}{2}mv_{\text{max}}^2$.

Photoelectric equation: Energy of photon = Work function (energy) + Max. KE of electrons

$$hf = \phi + \frac{1}{2}m_e v_{\text{max}}^2$$
Wave-Particle Duality Concept

- Refers to the idea that light and matter (such as electrons) have both wave & particle properties.
- The wavelength of an object is given by $\lambda = \frac{\hbar}{p}$ (p: momentum of the particle.)
- Interference and diffraction provide evidence for the wave nature of E.M. radiation.
- Photoelectric effect provides evidence for the particulate nature of E.M. radiation.
- These evidences led to the concept of the wave-particle duality of light.

Electron diffraction provides evidence that matter/particles have also a wave nature & thus, have a dual nature.

de Broglie wavelength of a particle ("matter waves"). $\lambda = \frac{\hbar}{p}$

Show an understanding of the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to spectral lines.

Recall and solve problems using the relation $hf = E_1 - E_2$.

Energy Levels of Isolated Atom:

- Are discrete (i.e. can only have certain energy values.)
- Difference between successive energy levels $\Delta E$: decreases as we move from ground state upwards.

The energy level of the ground state gives the ionization energy, i.e. the energy needed to completely remove an electron initially in the ground state from the atom (i.e. to the energy level $n=\infty$, where $E_\infty = 0$).

2 common ways to cause Excitation of an atom:

- When bombarded by an incident electron where $KE$ of incident electron $> \Delta E$
  i.e. $\left(\frac{1}{2} m_e u^2\right)_{\text{before collision}} = \Delta E + \left(\frac{1}{2} m_e v^2\right)_{\text{after collision}}$
- Absorbing an incident photon of frequency $f$ where $hf$ must $= \Delta E$ exactly

Distinguish between emission and absorption line spectra.

Emission line spectrum: A series of discrete/separate bright lines on a dark background, produced by electron transitions within an atom from higher to lower energy levels and emitting photons.

An excited atom during a downward transition emits a photon of frequency $f$, such that $E_i - E_f = hf$
Absorption line spectrum: A continuous bright spectrum crossed by “dark” lines. It is produced when “white light” passes through a cool gas. Atoms/electrons of the cool gas absorb photons of certain frequencies and get excited to higher energy levels which are then quickly re-emitted in all directions.

Explain the origins of the features of a typical X-ray spectrum using quantum theory.

Characteristic X-rays: produced when an electron is knocked out of an inner shell of a target metal atom, allowing another electron from a higher energy level to drop down to fill the vacancy. The x-rays emitted have specific wavelengths, determined by the discrete energy levels which are characteristic of the target atom.

Continuous X-ray Spectrum (Braking Radiation (Bremsstrahung)): produced when electrons are suddenly decelerated upon collision with atoms of the metal target.

Minimum \( \lambda \) of cont. spectrum \( \lambda_{\text{min}} \): given by \( \frac{hc}{\lambda_{\text{min}}} = eV_a \), \( V_a \): accelerating pd of x-ray tube

Show an understanding of and apply the Heisenberg position-momentum and time-energy uncertainty principles in new situations or to solve related problems.

Heisenberg Uncertainty Principles: If a measurement of the position of a particle is made with uncertainty \( \Delta x \) and a simultaneous measurement of its momentum is made with uncertainty \( \Delta p \), the product of these 2 uncertainties can never be smaller than \( \frac{h}{4\pi} \)

i.e. \( \Delta x \Delta p \geq \frac{h}{4\pi} \)

Similarly \( \Delta E \Delta t \geq \frac{h}{4\pi} \) where \( E \) is the energy of a particle at time \( t \)

Show an understanding that an electron can be described by a wave function \( \psi \) where the square of the amplitude of wave function \( |\psi|^2 \) gives the probability of finding the electron at a point. (No mathematical treatment is required.)

A particle can be described by a wave function \( \Psi \) where the square of the amplitude of wave function, \( |\Psi|^2 \), is proportional to the probability of finding the particle at a point.

Show an understanding of the concept of a potential barrier and explain qualitatively the phenomenon of quantum tunnelling of an electron across such a barrier.

Potential barrier
A region of electric field that prevents an atomic particle like an electron on one side of the barrier from passing through to the other side.

OR

- A region where the potential energy of a particle, if it is placed there, is greater than the total energy of the particle.
- Hence the particle would experience an opposing force if it tries to enter into the potential barrier

Describe the application of quantum tunnelling to the probing tip of a scanning tunnelling microscope (STM) and how this is used to obtain atomic-scale images of surfaces. (Details of the structure and operation of a scanning tunnelling microscope are not required.)

Quantum tunnelling:
A quantum-mechanical process whereby a particle penetrates a classically forbidden region of space, i.e. the particle goes through a potential barrier even though it does not have enough energy to overcome it. Due to the wave nature of a particle, there is a non-zero probability that the particle is able to penetrate the potential barrier.

Scanning tunnelling microscope:
Involves passing electrons from the tip of a probe through a potential barrier to a material that is to be
Quantum tunnelling allows electrons to overcome the potential barrier between tip & material.
- Magnitude of tunnelling current is dependent on the dist between the tip and the surface.
- There are two methods to obtain images of the surface of the material:
  1. Maintain the tip at constant height and measure the tunnelling current.
  2. Maintain a constant tunnelling current and measure the (vertical) position of the tip.

(A feedback device adjusts the vertical height of the tip to keep the tunnelling current const as the tip is scanned over the surface (Method 2)). The output of the device provides an image of the surface contour of the material.

**s. Apply the relationship transmission coefficient \( T = \exp(-2kd) \) for the STM in related situations or to solve problems. (Recall of the equation is not required.)**

**Transmission coefficient \( T \):** measures the probability of a particle tunnelling through a barrier.

\[
T = e^{-2kd}
\]

\[
k = \sqrt{\frac{8\pi^2 m (U - E)}{\hbar^2}} \quad \text{given in Formula List}
\]

d: the thickness of the barrier in metres
m: mass of the tunnelling particle in kg
U: the “height” of the potential barrier in J (NOT: eV)
E: the energy of the electron in J
h: The Planck’s constant

**t. Recall and use the relationship \( R + T = 1 \) where \( R \) is the reflection coefficient and \( T \) is the transmission coefficient, in related situations or to solve problems.**

**Reflection coefficient \( R \):** measures the probability that a particle gets reflected by a barrier.

\[
T + R = 1
\]
a. Recall and use the terms spontaneous emission, stimulated emission and population inversion in related situations.

**Spontaneous emission:** A process whereby a photon is emitted when an electron in an excited atom falls naturally to a lower energy level, i.e. without requiring an external event to trigger it.

**Stimulated emission:** A process whereby an incoming photon causes/induces another photon of the same frequency & phase (& direction) to be emitted from an excited atom.

**Laser:** A monochromatic, coherent, parallel beam of high intensity light.

**Meta stable state:** An excited state whose lifetime is much longer than the typical ($10^{-8}$ s) lifetime of excited states.

**Population inversion:** A condition whereby there are more atoms in an excited state than in the ground state.

(A meta stable state is essential for laser production because it is required for population inversion to be achieved, which, in turn, increases the probability of stimulated emissions.)

b. Explain the action of a laser in terms of population inversion and stimulated emission. (Details of the structure and operation of a laser are not required.)

**Conditions to achieve Laser action:**

- Atoms of the laser medium must have a meta-stable state.
- The medium must be in a state of population inversion.
- The emitted photons must be confined in the system long enough to allow them to cause a chain reaction of stimulated emissions from other excited atoms.

c. Describe the formation of energy bands in a solid.

**Formation of Energy Bands in a Solid/Band theory for solids:**

- Unlike the case of an isolated atom, in a solid, the atoms are very much closer to each other.
- This allows the electrons from neighbouring atoms to interact with each other.
- As a result of this interaction, each discrete energy level that is associated with an isolated atom is split into many sub-levels.
  (This is in accordance to Pauli Exclusion Principle which states that: no 2 electrons can be in the same energy state)
- These sub-levels are extremely close to one another such that they form an energy band.
  (In other words, an energy band consists of a very large number of energy levels which are very close together.)

d. Distinguish between conduction band and valence band.

**Valence Band:** The highest energy band that is completely filled with electrons.

**Conduction Band:** The next higher band:
  - For some metals/ good conductors, it is partially-filled;
  - For other metals, the VB & CB overlap (hence it is also partially-filled)

**Energy Gap**

- A region where no energy state can exist;
- It is the energy difference between the CB & VB
Use band theory to account for the electrical properties of metals, insulators and intrinsic semiconductors, with reference to conduction electrons and holes.

Properties of Conductors, Insulators and Semi-conductors at 0 K (“low temp”):

<table>
<thead>
<tr>
<th></th>
<th>Conductors</th>
<th>Insulators</th>
<th>Semi-conductors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conduction Band</td>
<td>Partially filled</td>
<td>Empty</td>
<td></td>
</tr>
<tr>
<td>Valence Band</td>
<td>Completely Occupied</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy gap between the bands</td>
<td>NA</td>
<td>Large (≈10 eV)</td>
<td>Small (≈1 eV)</td>
</tr>
<tr>
<td>Charge Carriers</td>
<td>free electrons</td>
<td>-</td>
<td>free electrons &amp; holes</td>
</tr>
</tbody>
</table>

How band theory explains the relative conducting ability of a metal, intrinsic semiconductor & insulator:

- For a (good) conductor (i.e. a metal), when an electric field is applied, electrons in the **partially-filled conduction band** can very easily gain energy from the field to “jump” to unfilled energy states since they are **nearby**.

- The ease at which these electrons may move to a nearby unfilled/unoccupied energy state, plus the fact that there is a high number density of free electrons make metals very good electrical conductors.

- For an insulator, the conduction band is **completely unoccupied** by electrons; the valence band is **completely occupied** by electrons; and the **energy gap** between the two bands is **very large**.

- Since the conduction band is **completely empty**, it requires a lot of energy to excite the electrons from the valence band to the conduction band across the **wide energy gap**.

- When an electric field is applied, no conduction of electricity occurs. (Thus, insulators make poor conductors of electricity.)

- For intrinsic semi-conductors, the **energy gap** between the two bands is **relatively small** (compared to insulator)

- As such even at room temp, some electrons in the valence band gain enough energy by thermal excitation to jump to the unfilled energy states in the conduction band, leaving vacant energy states in the valence band known as holes.

- When an electric field is applied, the electrons which have jumped into the conduction band and holes (in the valence band) act as **negative** and **positive** charge carriers respectively and conduct electricity.

- (Thus, for intrinsic semiconductors, the ability to conduct vary with temperature (or even light), as light can cause photo-excitation).
f. Analyse qualitatively how n- and p-type doping change the conduction properties of semiconductors.

Doping:
- Refers to the addition of impurity atoms to an intrinsic semiconductor to modify the number and type of charge carriers.
- n-type doping increases the no. of free (NOT: valence) electrons; p-type doping increases the no. of holes.
- Note that, even with a very small increase in the dopants, the electrical resistivity of an extrinsic semiconductor decreases significantly because the number of charge carriers of the intrinsic semiconductor is typically very small.

Explain why electrical resistance of an intrinsic semiconductor material decreases as its temperature rises. (N08P2Q5, 4 m)
(Based on the band theory, a semiconductor has a completely filled valence band and an empty conduction band with a small energy gap in between. Hence there are no charge carriers and the electrical resistance is high.)

1. When temperature is low, electrons in the valence band do not have sufficient energy to jump across the energy gap to get into the conduction band.
2. When temperature rises, electrons in the valence band receive thermal energy to enter into the conduction band leaving holes in the valence band.
3. Electrons in the conduction band & holes in the valence band are mobile charge carriers and can contribute to current.
4. Increasing the number of charge carriers means lower resistance.

2 Differences between p-type silicon & n-type silicon:
- In n-type Si, the majority charge carrier is the electron, its minority charge carrier is the hole. For p-type Si, the situation is reversed.
- In n-type Si, the dopants are typically pentavalent atoms (having 5 valence electrons); In p-type Si, the dopants are typically trivalent atoms (valency = 3)

2. Discuss qualitatively the origin of the depletion region at a p-n junction and use this to explain how a p-n junction can act as a rectifier.

Origin of Depletion Region

How a p-n junction can act as a rectifier
- When a p-n junction diode is connected in reverse bias in a circuit, the negative terminal of the battery pulls holes from the p-type semiconductor leaving behind more negatively-charged acceptor ions. At the same time the positive terminal pulls electrons from the n-type semiconductor leaving behind more positively-charged donor ions.
- This results in the widening of the depletion region and an increase in the height of the potential barrier, and so no current flows.
- When a p-n junction diode is connected in a forward-bias connection in a circuit, the externally applied pd opposes the contact pd across the depletion region.
- If the externally applied pd is great enough, it supplies energy to the holes and electrons to overcome the potential barrier and, so a current will flow. (In general, a forward-bias connection narrows the depletion region and reduces the height of the potential barrier.)

(Thus a p-n junction (diode) allows current to flow in one direction only (when the p-n junction is in forward bias) and so, it can be used as a rectifier to rectify an ac to dc)
a. Infer from the results of the α-particle scattering experiment the existence and small size of the nucleus.

Describe the experimental evidence for a small charged nucleus in an atom. (N08P3Q7a, 4m)
Results of an experiment where a beam of alpha particles is fired at a thin gold foil, where n= no of alpha particles incident per unit time.

Most of the α-particles passed through the metal foil were deflected by very small angles.

A very small proportion was deflected by more than 90°, some of these approaching 180°.

From these 2 observations it can be deduced that: the nucleus occupies only a small proportion of the available space (ie the atom is mostly empty space) & that it is positively charged since the positively-charged alpha particles are repelled/deflected.

b. Distinguish between nucleon number (mass number) and proton number (atomic number).

Nucleon: A particle within the nucleus; can be either a proton or a neutron

Nuclide: An atom with a particular number of protons and a particular number of neutrons

Proton number Z (old name: atomic number): No. of protons in an atom

Nucleon number N (mass number): Sum of number of protons and neutrons in an atom

c. Show an understanding that an element can exist in various isotopic forms each with a different number of neutrons.
<table>
<thead>
<tr>
<th></th>
<th>Isotopes: atoms with the same proton number, but different nucleon number (or different no of neutrons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d.</td>
<td>Use the usual notation for the representation of nuclides and represent simple nuclear reactions by nuclear equations of the form [^{14}_7\text{N} + ^{4}_2\text{He} \rightarrow ^{17}_8\text{O} + ^{1}_1\text{H}.]</td>
</tr>
<tr>
<td></td>
<td>Self-Explanatory</td>
</tr>
<tr>
<td>e.</td>
<td>Show an understanding of the concept of mass defect. Recall and apply the equivalence relationship between energy and mass as represented by [E = mc^2] in problem solving.</td>
</tr>
<tr>
<td>f.</td>
<td>Show an understanding of the concept of binding energy and its relation to mass defect. Explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission.</td>
</tr>
<tr>
<td>g.</td>
<td>Energy &amp; Mass are Equivalent: [E = mc^2 \rightarrow \Delta E = (\Delta m)c^2]</td>
</tr>
</tbody>
</table>
| i. | **Nuclear Binding Energy:**  
|   | - Energy that must be supplied to completely separate the nucleus into its individual nucleons/particles.  
|   | OR  
|   | - The energy released (not lost) when a nucleus is formed from its constituent nucleons.  
| g. | **B.E. per nucleon** is a measure of the stability of the nucleus. |
| g. | **Mass Defect:** The difference in mass between a nucleus and the total mass of its individual nucleons = \[Zm_p + (A-Z)m_n - \text{Mass of Nucleus}\]  
| g. | Thus, **Binding Energy** = Mass Defect \(\times c^2\)  
| g. | In both nuclear fusion and fission, products have **higher B.E. per nucleon** (due to shape of BE per nucleon-nucleon graph), energy is released (not lost) and hence products are more stable.  
| g. | **Energy released** = **Total B.E. after reaction (of products)** - **Total B.E. before reaction (ie of reactants)**  
| g. | **Nuclear fission:** The disintegration of a heavy nucleus into 2 lighter nuclei. Typically, the fission fragments have approximately the same mass and neutrons are emitted. |
| h. | Sketch the variation of binding energy per nucleon with nucleon number.  
| h. | Fig below shows the variation of BE per nucleon plotted against the nucleon no. |
Warning!!! Graph is NOT symmetrical.

j. State and apply to problem solving the concept that nucleon number, proton number, energy and mass are all conserved in nuclear processes.

Principle of Conservation of Energy-Mass:

Total energy-mass before reaction = Total energy-mass after reaction

\[ \sum (m c^2 + \frac{1}{2} m v^2)_{\text{reactants}} = \sum (m c^2 + \frac{1}{2} m v^2)_{\text{products}} + h f \text{ [if } \gamma \text{-photon emitted]} \]

Energy released in nuclear reaction = \( \Delta m c^2 \)

= (Total rest mass before reaction − Total rest mass after reaction) \( \times c^2 \)

k. Show an understanding of the spontaneous and random nature of nuclear decay.
Infer the random nature of radioactive decay from the fluctuations in count rate.

Radioactivity is the spontaneous and random decay of an unstable nucleus, with the emission of an alpha or beta particle, and is usually accompanied by the emission of a gamma ray photon.

Spontaneous: The emission is not affected by factors outside the nucleus

Random: It cannot be predicted when the next emission will occur {Evidence in fluctuation in count-rate}

Decay law:

\[ \frac{dN}{dt} = -\lambda N, \text{ where } N= \text{ No. of undecayed (active) nuclei at that instant;} \]

\[ \rightarrow N = N_0 e^{-\lambda t}; \quad A = A_0 e^{-\lambda t}; \quad C = C_0 e^{-\lambda t}; \quad \text{ [in List of Formulae]} \]

m. Show an understanding of the origin and significance of background radiation.

Background radiation refers to radiation from sources other than the source of interest.

\[ \rightarrow \text{True count rate} = \text{Measured count rate} − \text{Background count rate} \]
n. Show an understanding of the nature of $\alpha$, $\beta$ and $\gamma$ radiations.

<table>
<thead>
<tr>
<th>Nature of $\alpha$, $\beta$ &amp; $\gamma$ (J2008P2Q7 4 m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Alpha Particles</strong></td>
</tr>
<tr>
<td><strong>Notation</strong></td>
</tr>
<tr>
<td><strong>Charge</strong></td>
</tr>
<tr>
<td><strong>Mass</strong></td>
</tr>
<tr>
<td><strong>Nature</strong></td>
</tr>
<tr>
<td><strong>Speed</strong></td>
</tr>
</tbody>
</table>

o. Define the terms activity and decay constant and recall and solve problems using $A = \lambda N$.

Decay constant $\lambda$ is defined as the probability of decay of a nucleus per unit time
(or the fraction of the total no. of undecayed nuclei which will decay per unit time.)

Activity is defined as the rate at which the nuclei are disintegrating.

$$A = \frac{dN}{dt} = \lambda N$$

$$\rightarrow A_0 = \lambda N_0$$

p. Infer and sketch the exponential nature of radioactive decay and solve problems using the relationship $x = x_0e^{-\lambda t}$ where $x$ could represent activity, number of undecayed particles and received count rate.

Number of undecayed nuclei $\propto$ Mass of sample

$$\rightarrow \text{Number of nuclei in sample} = \frac{\text{Sample Mass}}{\text{Mass of 1 mol}} \times N_A$$

where, Mass of 1 mol of nuclide= Nucleon No (or relative atomic mass) expressed in grams \{NOT: in kg!!\}

(Thus for eg, mass of 1 mole of U-235 = 235 g = 235 x $10^{-3}$ kg, NOT: 235 kg)

Application of PCM to radioactive decay (N08P3Q7b(iv))

It is useful to remember that when a stationary nucleus emits a single particle, by PCM, after the decay, the ratio of their KE = ratio of their speeds, which in turn, = reciprocal of the ratio of their masses

q. Define half-life.

Half-life is defined as the average time taken for half the number (not: mass or amount) of undecayed nuclei in the sample to disintegrate,

or, the average time taken for the activity to be halved.

$$t_{1/2} = \frac{\ln 2}{\lambda}$$ \{in List of Formulae\}

r. Solve problems using the relation $\lambda = \frac{0.693}{t_{1/2}}$.

EXAMPLE 20R1

Antimony-124 has a half-life of 60 days. If a sample of antimony-124 has an initial activity of $6.5 \times 10^6$ Bq, what will its activity be after 1 year (365 days)?

Using $A = A_0 e^{-\lambda t}$ eqn (4) & $t_{1/2} = \frac{\ln 2}{\lambda}$

$$\rightarrow A = 9.6 \times 10^4 \text{ Bq}$$
Discuss qualitatively the effects, both direct and indirect, of ionising radiation on living tissues and cells.

<table>
<thead>
<tr>
<th><strong>s.</strong></th>
<th>Radiation damage to biological organisms is often categorized as: <strong>somatic</strong> and <strong>genetic.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Somatic damage</strong> refers to any part of the body except the reproductive organs.</td>
<td>Somatic damage harms that particular organism directly. Some somatic effects include radiation sickness (nausea, fatigue, and loss of body hair) and burns, reddening of the skin, ulceration, cataracts in the eye, skin cancer, leukaemia, reduction of white blood cells, death, etc.</td>
</tr>
<tr>
<td><strong>Genetic damage</strong> refers to damage to reproductive organs.</td>
<td>Genetic effects cause mutations in the reproductive cells and so affect future generations – hence, their effects are indirect. (Such mutations may contribute to the formation of a cancer.)</td>
</tr>
<tr>
<td>Alternatively,</td>
<td></td>
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<tr>
<td>- Ionising radiation may damage living tissues and cells <strong>directly.</strong></td>
<td></td>
</tr>
<tr>
<td>- It may also occur <strong>indirectly</strong> through chemical changes in the surrounding medium, which is mainly water. For example, the ionization of water molecules produces OH free radicals which may react to produce H₂O₂, the powerful oxidizing agent hydrogen peroxide, which can then attack the molecules which form the chromosomes in the nucleus of each cell.</td>
<td></td>
</tr>
</tbody>
</table>