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Optimizing Performance of Co-Existing Underlay Secondary Networks

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***Abstract*—In this paper, we analyze sum throughput and (asymptotic) sum ergodic rate performance of two co-existing downlink multiuser underlay secondary networks employing either fixed-rate transmission (FRT) or (channel aware) adaptive rate transmission (ART). In the considered scenario in which two secondary sources may transmit simultaneously, intelligent apportioning the interference temperature limit (ITL) is vital. We consider cases when this ITL apportioning is based on statistical properties of the channels, or on full (or partial) knowledge of the channel gains. For these cases, proper network management (NM) strategies are evolved to maximize sum throughput or sum ergodic rate of the secondary networks. Each NM strategy determines whether both secondary sources should transmit concurrently or not, and also determines their transmit powers. We demonstrate that a channel aware NM (CANM) strategy is superior to an optimal fixed NM (FNM) strategy. With secondary sources employing non-opportunistic user selection, in case of FRT (ART), we demonstrate that there exists a critical target-rate (ITL) below which it is advantageous to operate both secondary networks concurrently. We present closed form expressions of critical parameters that influence sum throughput and sum ergodic rate. Computer simulations are presented to corroborate the derived expressions.**

***Index Terms*—Co-existing networks, underlay, downlink, sum throughput, sum ergodic rate, network management**

I. Introduction

A rapid increase in wireless devices and services in the past decade or so has led to a demand for very high data rates over the wireless medium. With prolific increase in data traffic, mitigating spectrum scarcity by more efficient utilization of the under-utilized spectrum has drawn attention of researchers both in academia and industry. Cognitive radio (CR) devices have shown promise in alleviating these problems of spectrum scarcity and low spectrum utilization efficiencies.

In underlay mode of operation of cognitive radios, both secondary (unlicensed) and primary (licensed) users co-exist and transmit in parallel so that the total secondary interference caused to the primary user is below a predetermined threshold [1] referred to as the interference temperature limit (ITL). This ensures that the primary performance in terms of throughput or outage is maintained at a desired level. Most of the analysis till date in underlay CR literature is confined to one secondary node transmitting with full permissible power and catering to its own set of receivers, while maintaining service quality of the primary network. For such secondary networks, performance improvement is achieved by exploiting diversity techniques [2], [3], intelligent resource allocation [4], increasing the number of hops [5], etc. However, there still remains scope to further exploit spatial reuse using underlay transmissions, that can lead to improved network coverage and result in higher data rates.

A. Related Work:

The idea of concurrent secondary transmissions has been proposed by researchers for enhancing physical layer security [6], or further improving spectrum utilization efficiency [7], [8]. In [7], two or more cognitive femtocells reuse the spectrum of a macrocell either in a overlay, interweave or underlay manner for better spectrum utilization. It is emphasized that [7] uses transmit power control[1], and focuses on estimating the maximum permissible density of femtocells in a macrocell. The authors in [8] solve a multi-objective optimization problem that maximizes secondary sum rate, while minimizing the total interference caused to the primary receiver. Performance study of randomly distributed underlay heterogeneous networks can be found in [9]–[11]. In heterogeneous networks, interferences influence overall performance and also determine feasibility of practical implementation. In situations when secondary networks reuse white spaces of the macro base station, unwanted interferences may arise either due to sensing error or primary user return. While the former arises due to faulty sensing of the primary spectrum, causing excessive interference to the primary receiver from secondary transmitters, the latter depends on primary traffic pattern and can have detrimental effect on network performance. Ways of mitigating such interferences have been addressed in [12]–[14]. However, to implement an *underlay* scheme with concurrent secondary transmissions in a cellular framework, the major issue not only lies in mitigating interferences among other heterogeneous users, but also careful handling of interferences from heterogeneous transmitters to maintain QoS of the macro cell [15]. Thus, radio resource allocation [16], [17] and interference management [18] play key roles in deployment of such heterogeneous networks. Radio environment maps [19] have been proposed to serve as databases for dynamic spectrum access. For the purpose of implementation, their system architectures and models [20], [21] have been a major subject of study over recent years. A comprehensive survey of heterogeneous

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[1]In this paper, we focus exclusively on receive power control, which is more appropriate for small primary coded packets.

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networks, their implementation and future goals is addressed in [22] (and references therein).

B. Motivation and Contribution:

Unlike other works, we consider use of co-existing downlink[2] underlay cognitive radio networks. Further, we analyze the sum throughput and sum ergodic rate (with fixed and adaptive rate transmissions respectively) of the underlay networks with peak interference constraints, and seek to evolve network management cum power control strategies to maximize them[3]. We argue later in this paper that the suggested framework finds application in increasing spectrum utilization efficiency of cellular system with relay stations, or in facilitating simultaneous transmissions by up to two D2D nodes in the same frequency band as that used by the macro base BS. The main contributions of our paper are as follows:

1) Unlike other works on co-existing secondary networks that mainly focus on optimization [8], [24] and game theoretic approaches [25], we present for the first time analytical closed-form sum throughput (for FRT) and sum ergodic rate (for ART) expressions for two co-existing underlay multiuser downlink networks assuming peak interference power control at the secondary nodes. Such analysis give insight on network parameters that affect performance, and enables system designers to perform optimizations.
2) We analyze the case when ITL is apportioned in a statistically optimum manner, and determine the throughput optimal NM strategy. We also consider cases when this apportioning is assisted by knowledge of instantaneous channel gains, and evolve NM strategies. We demonstrate, availability of channel state information (CSI) results in increased throughput.
3) When secondary sources employ non-opportunistic user selection (or when two D2D units transmit simultaneously in the same band as the BS) and use statistical ITL apportioning to accommodate concurrent secondary FRT, the NM strategy to maximize sum throughput is to use concurrent secondary transmissions *only when the target rate is below a certain critical target rate*. We derive analytical closed form expressions for both the critical target rate and the throughput-optimal statistical power allocation parameter. Above this critical target rate, the NM strategy selects one network (the one with larger throughput[4]). For opportunistic user selection with equal number of users in each secondary network, the NM strategy to maximize sum throughput is to use concurrent secondary transmission *only when the outage requirement at both secondary networks is higher than a critical outage.* For such a case, we show that sum throughput is maximized when the same throughput-optimal statistical power allocation is used. For an outage requirement lower than the critical outage, the NM strategy again selects one network (the one offering lower outage).
4) For adaptive rate signaling, when secondary sources employ non-opportunistic user selection and statistical ITL apportioning is used to accommodate concurrent secondary transmission, we establish the NM strategy that maximizes sum ergodic rate for a special case. We show that concurrent transmission by both secondary networks is useful *only when the ITL is below a certain critical value*, whose an analytical expression is derived.
5) Use of opportunistic user selection with more number of users at individual secondary networks is seen to result in improved performance.

The derived expressions and insights are a useful aid to system designers.

C. Organization of the paper:

The rest of the paper is organized as follows. Section II presents a brief description of the system model. Section III presents analytical sum throughput expression for fixed-rate signaling, assuming apportioning of the ITL based on statistical channel knowledge. Using the asymptotic expression for this sum throughput, the optimal NM strategy is evolved. Section IV presents analysis of asymptotic sum throughput when apportioning of the ITL is CSI assisted, and discusses two NM strategies. Analysis of asymptotic sum ergodic rate assuming adaptive rate signaling is presented in Section V, and a NM strategy is evolved for a special case. Section VI presents a summary of all NM strategies along with simulation of the derived results. Finally, concluding remarks are presented in Section VII.

II. System Model and Problem Formulation

We consider two cognitive underlay downlink networks[5], where two secondary transmitters S_1 and S_2 transmit symbols concurrently in the range of a primary network by selecting downlink transmission receivers R_{1i^*} (among R_{1i} receivers, $i \in [1, L]$) and R_{2i^*} (among R_{2i} receivers, $i \in [1, M]$) respectively, from their cluster of registered users (Fig. 1). We consider independent opportunistic user selection in each secondary network, as well as non-opportunistic user selection (based on round robin scheduling for example) as separate cases. We ensure that the total secondary interference caused to the primary receiver R_P is below the ITL by careful apportioning of power between S_1 and S_2. All channels are assumed to be independent, and of quasi-static Rayleigh

[2]It is far easier for secondary base stations or nodes emulating base stations to acquire channel knowledge to the primary receiver than any other secondary node. This makes downlink a favorable choice in an underlay scenario, besides keeping analysis tractable.

[3]A portion of this work limited only to FRT and FNM strategy has been accepted and presented in IEEE PIMRC 2017 [23].

[4]We do not address the issue of fairness here. Any standard approach can be used to introduce fairness. Similarly, providing priority to some network over others may be of practical interest. One approach is to use weighted sum throughput or weighted ergodic rate. These issues are not taken up in this paper, and are topics of future work.

[5]Although primary and secondary networks are often assumed to be licensed and unlicensed users respectively, this need not always be the case. They can indeed be users of the same network transmitting concurrently to increase spectrum utilization efficiency. The same logic extends for two co-existing secondary networks. This eliminates most of the difficulties associated with interference channel estimation, synchronization, etc.

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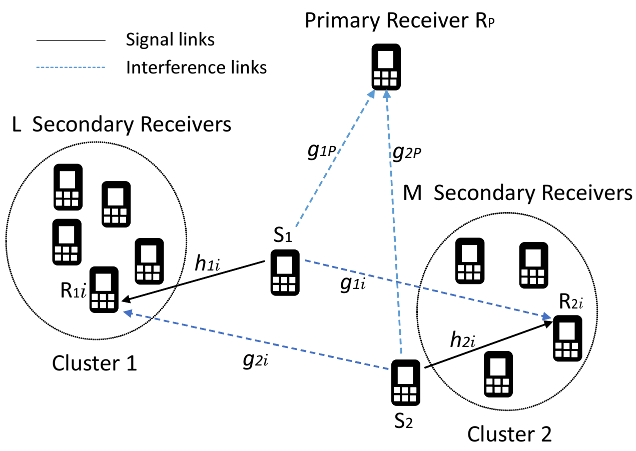

Fig. 1: System model of co-existing underlay CR networks

fading type. The channels between S_1 and R_{1i} are denoted by $h_{1i} \sim \mathcal{CN}\left(0, 1/\lambda_{11}\right)$, $i \in [1, L]$. The channels between S_2 and R_{2i} are denoted by $h_{2i} \sim \mathcal{CN}\left(0, 1/\lambda_{22}\right)$, $i \in [1, M]$. Due to concurrent secondary transmissions, each transmitter interferes with the receivers of the other cluster. The interference channels between S_1 and R_{2i} are denoted by $g_{1i} \sim \mathcal{CN}\left(0, 1/\mu_{12}\right)$, $i \in [1, M]$. The interference channels between S_2 and R_{1i} are denoted by $g_{2i} \sim \mathcal{CN}\left(0, 1/\mu_{21}\right)$, $i \in [1, L]$. Cognitive radios are typically used for short range communication and therefore located in small clusters. Thus, receivers of each secondary network are closely spaced and have roughly equal distances from the transmitter and the secondary nodes of the other co-existing network. The channels to R_P from S_1 and S_2 are denoted by $g_{1P} \sim \mathcal{CN}\left(0, 1/\mu_{1P}\right)$ and $g_{2P} \sim \mathcal{CN}\left(0, 1/\mu_{2P}\right)$ respectively. We neglect primary interference at the secondary nodes assuming the primary transmitter to be located far away from the secondary receivers, which is a common assumption in underlay CR literature, and well justified on information theoretic grounds [26], [27]. Zero-mean additive white Gaussian noise of variance σ_n^2 is assumed at all terminals. As in all underlay networks, it is assumed that S_1 and S_2 can estimate $|g_{1P}|^2$ and $|g_{2P}|^2$ respectively by observing the primary reverse channel, or using pilots transmitted by R_P.

S_1 transmits unit energy symbols x with power $P_{S_1} = \min(\beta P, \alpha I_P/|g_{1P}|^2)$ and S_2 transmits unit energy symbols z with power $P_{S_2} = \min((1-\beta)P, (1-\alpha)I_P/|g_{2P}|^2)$ at every signaling interval. We use βP and $(1-\beta)P$, with $0 \leq \beta \leq 1$ as the secondary peak powers. Clearly, by varying both β and P, any desired set of powers can be chosen. I_P denotes the ITL, and $0 \leq \alpha \leq 1$ denotes the parameter that apportions I_P between S_1 and S_2. We note that with these random source powers, the interference power at the primary receiver is guaranteed to be lower than I_P. The received signals ($y_{R_{1i}}$ and $y_{R_{2i}}$) at R_{1i} and R_{2i} can be written as follows:

$$y_{R_{1i}} = \sqrt{P_{S_1}} h_{1i} x + \sqrt{P_{S_2}} g_{2i} z + n_{R_{1i}}, i \in [1, L]$$
$$y_{R_{2i}} = \sqrt{P_{S_2}} h_{2i} z + \sqrt{P_{S_1}} g_{1i} x + n_{R_{2i}}, i \in [1, M], \quad (1)$$

where $n_{R_{1i}}, n_{R_{2i}} \sim \mathcal{CN}(0, \sigma_n^2)$ are additive white Gaussian noise samples at R_{1i} and R_{2i} respectively. When transmitters S_1 and S_2 select the receivers R_{1i^*} and R_{2i^*}, the instantaneous signal-to-interference-plus-noise ratios (SINRs) Γ_1 and Γ_2 at R_{1i^*} and R_{2i^*} can be written as:

$$\Gamma_1 = \frac{P_{S_1}|h_{1i^*}|^2}{P_{S_2}|g_{2i^*}|^2 + \sigma_n^2}, \quad \Gamma_2 = \frac{P_{S_2}|h_{2i^*}|^2}{P_{S_1}|g_{1i^*}|^2 + \sigma_n^2}, \quad (2)$$

where g_{2i^*} is the interference channel from S_2 to the intended receiver R_{1i^*} and g_{1i^*} is the interference channel from S_1 to the intended receiver R_{2i^*}. With opportunistic selection, $i^* = \arg\max_i[|h_{ki}|^2]$, where $i = 1, 2, \ldots, L$ when $k = 1$, and $i = 1, 2, \ldots, M$ when $k = 2$. As a separate case, when more than two secondary users reuse the primary spectrum and perform concurrent transmissions, the interference temperature needs to be apportioned between all the transmitting users to maintain ITL constraint at R_P. Increasing the number of concurrent transmissions not only reduces the available power per secondary transmitter, but also reduces the downlink SINRs due to increased number of secondary interferences, and is not always useful. Therefore the gains diminish with increase in concurrent secondary transmissions. Moreover, to implement such a scheme, computer-based optimization techniques are required, which is beyond the purview of this present work.

Remark 2.1: Relay stations (RS) are part of existing standards (LTE-A) and their use has been suggested in 5G [28]. If S_1 and S_2 are RSs in a single sector, and R_{1i} and R_{2i} are the cellular users registered with them, it is clear that they can use underlay techniques to manage interference to cellular user R_p. The suggested framework fits in well here, and can allow up to two RSs to transmit simultaneously with the cellular BS. Note that since all users here belong to the same network, many of the licensing and implementation difficulties (synchronization, estimation of channels etc) associated with cognitive radio systems are eliminated.

Remark 2.2: The suggested framework clearly finds application in D2D communication (S_1 and S_2 are D2D nodes).

Remark 2.3: The suggested framework can also be applied to a scenario where femto-cell networks co-exist in the operating range of a macro cell (the primary in this case). However, CANM may not be possible in this case.

FNM and CANM are the two broad NM strategies evolved in this paper. In FNM, the ITL is apportioned statistically between S_1 and S_2 to maximize sum throughput or sum ergodic rate, and does not require knowledge of instantaneous channel estimates. For cases when each secondary transmitter has access to instantaneous signal and interference channel gains to its selected receiver, a CSI assisted CANM strategy is formulated, where α is chosen instantaneously to further improve throughput performance. A suboptimal CANM strategy of choosing α instantaneously is also proposed when instantaneous mutual secondary interference channels are unknown to the transmitters, and only statistical knowledge of such links is available.

III. Performance of Secondary Networks with FRT and FNM

Assuming FRT at rate R by S_1 and S_2, the sum throughput τ_{FR} using α and β chosen statistically is defined as follows:

$$\tau_{FR} = (1 - p_{out1})R + (1 - p_{out2})R. \quad (3)$$

p_{out1} and p_{out2} are outage probabilities of the two secondary user pairs S_1-R_{1i^*} and S_2-R_{2i^*}.

A. Derivation of p_{out1} and p_{out2}:

1) Derivation of p_{out1}: Outage probability p_{out1} is defined as $p_{out1} = \Pr\{\Gamma_1 < \gamma_{th}\}$, where $\gamma_{th} = 2^R - 1$. We note, $P_{S_1} = \beta P$ if $\beta P < \frac{\alpha I_P}{|g_{1P}|^2}$ ($|g_{1P}|^2 < \frac{\alpha I_P}{\beta P} = U_1$), and $\frac{\alpha I_P}{|g_{1P}|^2}$ otherwise. Similarly, $P_{S_2} = (1-\beta)P$ if $(1-\beta)P < \frac{(1-\alpha)I_P}{|g_{2P}|^2}$ ($|g_{2P}|^2 < \frac{(1-\alpha)I_P}{(1-\beta)P} = U_2$), and $\frac{(1-\alpha)I_P}{|g_{2P}|^2}$ otherwise. p_{out1} can be written as a sum of four terms based on different values of P_{S_1} and P_{S_2} as follows:

$$\begin{aligned}
p_{out1} &= \underbrace{\Pr\left\{\frac{\beta P \max\limits_{i\in[1,L]}[|h_{1i}|^2]}{(1-\beta)P|g_{2i^*}|^2 + \sigma_n^2} < \gamma_{th}, \begin{matrix}|g_{1P}|^2 < U_1,\\ |g_{2P}|^2 < U_2\end{matrix}\right\}}_{p_{11}} \\
&+ \underbrace{\Pr\left\{\frac{\alpha I_P \frac{\max\limits_{i\in[1,L]}[|h_{1i}|^2]}{|g_{1P}|^2}}{(1-\alpha)I_P\frac{|g_{2i^*}|^2}{|g_{2P}|^2} + \sigma_n^2} < \gamma_{th}, \begin{matrix}|g_{1P}|^2 \geq U_1,\\ |g_{2P}|^2 \geq U_2\end{matrix}\right\}}_{p_{12}} \\
&+ \underbrace{\Pr\left\{\frac{\alpha I_P \frac{\max\limits_{i\in[1,L]}[|h_{1i}|^2]}{|g_{1P}|^2}}{(1-\beta)P|g_{2i^*}|^2 + \sigma_n^2} < \gamma_{th}, \begin{matrix}|g_{1P}|^2 \geq U_1,\\ |g_{2P}|^2 < U_2\end{matrix}\right\}}_{p_{13}} \\
&+ \underbrace{\Pr\left\{\frac{\beta P \max\limits_{i\in[1,L]}[|h_{1i}|^2]}{(1-\alpha)I_P\frac{|g_{2i^*}|^2}{|g_{2P}|^2} + \sigma_n^2} < \gamma_{th}, \begin{matrix}|g_{1P}|^2 < U_1,\\ |g_{2P}|^2 \geq U_2\end{matrix}\right\}}_{p_{14}}. \quad (4)
\end{aligned}$$

p_{1n}, $n \in \{1,2,3,4\}$ in (4) can be written in a compact manner as:

$$\begin{aligned}
p_{1n} &= \Pr\left\{\max_{i\in[1,L]}[|h_{1i}|^2] < \Theta_n\gamma_{th}|g_{2i^*}|^2 + \rho_n, |g_{1P}|^2 \overset{n}{\gtrless} U_1,\right. \\
&\qquad \left. |g_{2P}|^2 \overset{n}{\gtrless} U_2\right\}, n \in \{1,2,3,4\} \\
&= \mathbb{E}\left[F_X\left(\Theta_n\gamma_{th}|g_{2i^*}|^2 + \rho_n\right)\right] \\
&= \mathbb{E}\left[1 - \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}e^{-\lambda_{11}j(\Theta_n\gamma_{th}|g_{2i^*}|^2+\rho_n)}\right]. \quad (5)
\end{aligned}$$

Here $\Theta_n = \left\{(\frac{1-\beta}{\beta}), (\frac{1-\alpha}{\alpha})\frac{|g_{1P}|^2}{|g_{2P}|^2}, (\frac{1-\beta}{\alpha})\frac{P|g_{1P}|^2}{I_P}, (\frac{1-\alpha}{\beta})\frac{I_P}{P|g_{2P}|^2}\right\}$ and $\rho_n = \left\{\frac{\gamma_{th}\sigma_n^2}{\beta P}, \frac{\gamma_{th}\sigma_n^2}{\alpha I_P}|g_{1P}|^2, \frac{\gamma_{th}\sigma_n^2}{\alpha I_P}|g_{1P}|^2, \frac{\gamma_{th}\sigma_n^2}{\beta P}\right\}$ for $n = \{1,2,3,4\}$ respectively. $\mathbb{E}$ denotes expectation over random variables $|g_{2i^*}|^2$, $|g_{1P}|^2$ and $|g_{2P}|^2$, where $|g_{1P}|^2 < U_1$ for $n = \{1,4\}$ and $|g_{1P}|^2 \geq U_1$ for $n = \{2,3\}$, while $|g_{2P}|^2 < U_2$ for $n = \{1,3\}$ and $|g_{2P}|^2 \geq U_2$ for $n = \{2,4\}$ respectively. For notational convenience, we define random variable $X = \max\limits_{i\in[1,L]}[|h_{1i}|^2]$. Clearly, it has cumulative distribution function (CDF) $F_X(x) = (1 - e^{-\lambda_{11}x})^L$. With successive averaging over random variables $|g_{2i^*}|^2$, $|g_{1P}|^2$ and $|g_{2P}|^2$, p_{1n} $\forall n$ can be evaluated. By change of variables and using standard integrals [29, eq.(3.352.2)] and [29, eq.(5.221.5)], p_{11} to p_{14} can be shown to be as in (6)-(9).

$$\begin{aligned}
p_{11} = &\left[1 - \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{e^{-\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\beta P}}}{1 + \frac{\lambda_{11}}{\mu_{21}}(\frac{1-\beta}{\beta})j\gamma_{th}}\right] \\
&(1 - e^{-\mu_{1P}\frac{\alpha I_P}{\beta P}})(1 - e^{-\mu_{2P}\frac{(1-\alpha)I_P}{(1-\beta)P}}). \quad (6)
\end{aligned}$$

$$\begin{aligned}
p_{13} = &(1 - e^{-\mu_{2P}(\frac{1-\alpha}{1-\beta})\frac{I_P}{P}})\left[e^{-\mu_{1P}\frac{\alpha I_P}{\beta P}} - \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\right. \\
&\frac{\mu_{1P}\mu_{21}\alpha I_P}{\lambda_{11}j(1-\beta)P\gamma_{th}}e^{\frac{\mu_{21}\alpha I_P}{\lambda_{11}j(1-\beta)P\gamma_{th}}(\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P})} \\
&\left. E_1\left[(\mu_{1P} + \lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P})(\frac{\mu_{21}\alpha I_P}{\lambda_{11}j(1-\beta)P\gamma_{th}} + \frac{\alpha I_P}{\beta P})\right]\right]. \quad (8)
\end{aligned}$$

$$\begin{aligned}
p_{14} = &(1 - e^{-\mu_{1P}\frac{\alpha I_P}{\beta P}})\left[e^{-\mu_{2P}(\frac{1-\alpha}{1-\beta})\frac{I_P}{P}} - \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\right. \\
&e^{-\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\beta P}}\left\{e^{-\mu_{2P}(\frac{1-\alpha}{1-\beta})\frac{I_P}{P}} - \mu_{2P}\frac{\lambda_{11}}{\mu_{21}}j\frac{(1-\alpha)I_P\gamma_{th}}{\beta P}\right. \\
&e^{\frac{\mu_{2P}\lambda_{11}}{\mu_{21}}j\frac{(1-\alpha)I_P\gamma_{th}}{\beta P}}E_1\left[\frac{\mu_{2P}\lambda_{11}}{\mu_{21}}j\frac{(1-\alpha)I_P\gamma_{th}}{\beta P}\right. \\
&\left.\left.\left. + \mu_{2P}(\frac{1-\alpha}{1-\beta})\frac{I_P}{P}\right]\right\}\right]. \quad (9)
\end{aligned}$$

Note that $E_1[.]$ in (7)-(9) and in other subsequent equations denotes the exponential integral function, where $E_1[x] = \int\limits_x^\infty \frac{e^{-t}}{t}dt$.

2) Derivation of p_{out2}: The outage probability p_{out2} is defined as $p_{out2} = \Pr\{\Gamma_2 < \gamma_{th}\}$. Due to the identical structure of SINRs Γ_1 and Γ_2, p_{out2} can be derived in the same manner as p_{out1}. Similar to (4), p_{out2} can be expressed as a sum of four terms as $p_{out2} = \sum\limits_{n=1}^{4} p_{2n}$, with closed form expressions for p_{2n} as in (10)-(13).

$$\begin{aligned}
p_{21} = &\left[1 - \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{e^{-\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\beta)P}}}{1 + \frac{\lambda_{22}}{\mu_{12}}(\frac{\beta}{1-\beta})k\gamma_{th}}\right] \\
&(1 - e^{-\mu_{2P}\frac{(1-\alpha)I_P}{(1-\beta)P}})(1 - e^{-\mu_{1P}\frac{\alpha I_P}{\beta P}}). \quad (10)
\end{aligned}$$

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$$
\begin{aligned}
p_{12} = {} & e^{-(\mu_{1P}\frac{\alpha}{\beta}+\mu_{2P}(\frac{1-\alpha}{1-\beta}))\frac{I_P}{P}} - \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\Bigg[\frac{e^{-(\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P})\frac{\alpha I_P}{\beta P}}\, e^{-\mu_{2P}(\frac{1-\alpha}{1-\beta})\frac{I_P}{P}}}{1+\frac{\lambda_{11}}{\mu_{1P}}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P}} - \mu_{1P}\mu_{2P}\frac{\lambda_{11}}{\mu_{21}}j(\frac{1-\alpha}{\alpha}) \\
& \gamma_{th}\Bigg\{\frac{e^{-[\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P}-\mu_{2P}\frac{\lambda_{11}}{\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}]\frac{\alpha I_P}{\beta P}}}{[\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P}-\mu_{2P}\frac{\lambda_{11}}{\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}]^2}\Bigg(1+[\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P}-\mu_{2P}\frac{\lambda_{11}}{\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}]\frac{\alpha I_P}{\beta P}\Bigg) \\
& E_1\Bigg[\frac{\mu_{2P}\lambda_{11}}{\mu_{21}}j(1-\alpha)\gamma_{th}\frac{I_P}{\beta P}+\mu_{2P}(\frac{1-\alpha}{1-\beta})\frac{I_P}{P}\Bigg] - \Bigg(\frac{1}{[\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P}-\mu_{2P}\frac{\lambda_{11}}{\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}]^2} \\
& -\frac{(\frac{\alpha}{1-\beta})\frac{I_P}{P}}{\frac{\lambda_{11}}{\mu_{21}}j\gamma_{th}[\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P}-\mu_{2P}\frac{\lambda_{11}}{\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}]}\Bigg)e^{-\left(1-\frac{\mu_{21}}{\mu_{2P}(1-\alpha)}(\frac{\mu_{1P}\alpha}{\lambda_{11}j\gamma_{th}}+\frac{\sigma_n^2}{I_P})\right)\mu_{2P}(\frac{1-\alpha}{1-\beta})\frac{I_P}{P}} E_1\Bigg[(\mu_{1P}+ \\
& \frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\alpha I_P})(\frac{\alpha I_P}{\beta P}+\frac{\mu_{21}\alpha I_P}{\lambda_{11}j\gamma_{th}(1-\beta)P})\Bigg] - \frac{e^{-(\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P})\frac{\alpha I_P}{\beta P}}\, e^{-\mu_{2P}(\frac{1-\alpha}{1-\beta})\frac{I_P}{P}}/(\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P})}{\Bigg(\mu_{1P}+\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\alpha I_P}-\mu_{2P}\frac{\lambda_{11}}{\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}\Bigg)}\Bigg\}\Bigg]. \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
p_{22} = {} & e^{-(\mu_{2P}(\frac{1-\alpha}{1-\beta})+\mu_{1P}\frac{\alpha}{\beta})\frac{I_P}{P}} - \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\Bigg[\frac{e^{-(\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P})\frac{(1-\alpha)I_P}{(1-\beta)P}}\, e^{-\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}}}{1+\frac{\lambda_{22}}{\mu_{2P}}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P}} - \mu_{2P}\mu_{1P}\frac{\lambda_{22}}{\mu_{12}}k(\frac{\alpha}{1-\alpha}) \\
& \gamma_{th}\Bigg\{\frac{e^{-[\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P}-\mu_{1P}\frac{\lambda_{22}}{\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}]\frac{(1-\alpha)I_P}{(1-\beta)P}}}{[\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P}-\mu_{1P}\frac{\lambda_{22}}{\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}]^2}\Bigg(1+[\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P}-\mu_{1P}\frac{\lambda_{22}}{\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}]\frac{(1-\alpha)I_P}{(1-\beta)P}\Bigg) \\
& E_1\Bigg[\frac{\mu_{1P}\lambda_{22}}{\mu_{12}}k\alpha\gamma_{th}\frac{I_P}{(1-\beta)P}+\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}\Bigg] - \Bigg(\frac{1}{[\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P}-\mu_{1P}\frac{\lambda_{22}}{\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}]^2} \\
& -\frac{\frac{\mu_{12}}{\lambda_{22}}(\frac{1-\alpha}{\beta})\frac{I_P}{P}}{k\gamma_{th}[\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P}-\mu_{1P}\frac{\lambda_{22}}{\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}]}\Bigg)e^{-\left(1-\frac{\mu_{12}}{\mu_{1P}\alpha}(\frac{\mu_{2P}(1-\alpha)}{\lambda_{22}k\gamma_{th}}+\frac{\sigma_n^2}{I_P})\right)\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}} E_1\Bigg[(\mu_{2P}+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{(1-\alpha)I_P}) \\
& \Bigg(\frac{(1-\alpha)I_P}{(1-\beta)P}+\frac{\mu_{12}(1-\alpha)I_P}{\lambda_{22}k\gamma_{th}\beta P}\Bigg)\Bigg] - \frac{e^{-(\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P})\frac{(1-\alpha)I_P}{(1-\beta)P}}\, e^{-\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}}/(\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P})}{\Bigg(\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P}-\mu_{1P}\frac{\lambda_{22}}{\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}\Bigg)}\Bigg\}\Bigg]. \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
p_{23} = {} & (1-e^{-\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}})\Bigg[e^{-\mu_{2P}\frac{(1-\alpha)I_P}{(1-\beta)P}} - \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1} \\
& \frac{\mu_{2P}\mu_{12}(1-\alpha)I_P}{\lambda_{22}k\beta P\gamma_{th}} e^{\frac{\mu_{12}(1-\alpha)I_P}{\lambda_{22}k\beta P\gamma_{th}}(\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P})} \\
& E_1\Bigg[(\mu_{2P}+\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\alpha)I_P})(\frac{\mu_{12}(1-\alpha)I_P}{\lambda_{22}k\beta P\gamma_{th}} \\
& +\frac{(1-\alpha)I_P}{(1-\beta)P})\Bigg]\Bigg]. \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
p_{24} = {} & (1-e^{-\mu_{2P}\frac{(1-\alpha)I_P}{(1-\beta)P}})\Bigg[e^{-\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}} - \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1} \\
& e^{-\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{(1-\beta)P}}\Bigg\{e^{-\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}} - \mu_{1P}\frac{\lambda_{22}}{\mu_{12}}k\frac{\alpha I_P\gamma_{th}}{(1-\beta)P} \\
& e^{\frac{\mu_{1P}\lambda_{22}}{\mu_{12}}k\frac{\alpha I_P\gamma_{th}}{(1-\beta)P}} E_1\Bigg[\frac{\mu_{1P}\lambda_{22}k\alpha I_P\gamma_{th}}{\mu_{12}(1-\beta)P}+\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}\Bigg]\Bigg\}\Bigg]. \qquad (13)
\end{aligned}
$$

B. Asymptotic Performance with FRT and FNM Strategy

1) Asymptotic Sum Throughputs $\tau_{FR}^{P\to\infty}$ and $\tau_{FR}^{I_P\to\infty}$: In this subsection, we evaluate asymptotic sum throughput expressions $\tau_{FR}^{P\to\infty}$ and $\tau_{FR}^{I_P\to\infty}$ for $P\to\infty$ and $I_P\to\infty$ respectively. When $P\to\infty$, the system operates in a fully cognitive regime with terminal outages attaining saturation, and powers limited by interference constraints to R_P. $\tau_{FR}^{P\to\infty}$

can be obtained from p_{out1} and p_{out2} using $P \to \infty$ in (3). It can be observed that with $P \to \infty$, $p_{outi}^{P\to\infty} = \lim_{P\to\infty} p_{i2}, i \in \{1,2\}$[6]. By using $\lim_{x\to 0}\left(\ln(x) + E_1[x]\right) = -\gamma$ [30, eq.(3.2.4)] (where $\gamma \approx 0.57721$ denotes Euler's constant) in (7) and (11), we obtain the following asymptotic expressions of $p_{outi}^{P\to\infty}, i \in \{1,2\}$:

$$p_{out1}^{P\to\infty} = 1 - \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\Bigg[\frac{1}{1+\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}\alpha I_P}} - \frac{\frac{\mu_{2P}\lambda_{11}}{\mu_{1P}\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}\Bigg\{\ln\left(\frac{1+\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}\alpha I_P}}{\frac{\mu_{2P}\lambda_{11}}{\mu_{1P}\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}}\right)}{[1+\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}\alpha I_P} - \frac{\mu_{2P}\lambda_{11}}{\mu_{1P}\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}]^2} + \left(\frac{\frac{\mu_{2P}\lambda_{11}}{\mu_{1P}\mu_{21}}j(\frac{1-\alpha}{\alpha})\gamma_{th}}{1+\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}\alpha I_P}}\right) - 1\Bigg\}\Bigg]. \quad (14)$$

$$p_{out2}^{P\to\infty} = 1 - \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\Bigg[\frac{1}{1+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}(1-\alpha)I_P}} - \frac{\frac{\mu_{1P}\lambda_{22}}{\mu_{2P}\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}\Bigg\{\ln\left(\frac{1+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}(1-\alpha)I_P}}{\frac{\mu_{1P}\lambda_{22}}{\mu_{2P}\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}}\right)}{[1+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}(1-\alpha)I_P} - \frac{\mu_{1P}\lambda_{22}}{\mu_{2P}\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}]^2} + \left(\frac{\frac{\mu_{1P}\lambda_{22}}{\mu_{2P}\mu_{12}}k(\frac{\alpha}{1-\alpha})\gamma_{th}}{1+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}(1-\alpha)I_P}}\right) - 1\Bigg\}\Bigg]. \quad (15)$$

When $I_P \to \infty$, S_1 and S_2 both transmit using their peak powers and are not limited by interference constraints to R_P. It is similar to the situation when secondary sources perform transmit power control and ensure service quality for the primary users (as in [7]). $\tau_{FR}^{I_P\to\infty}$ can be obtained from p_{out1} and p_{out2} using $I_P \to \infty$ in (3). We note that with $I_P \to \infty$, $p_{outi}^{I_P\to\infty} = \lim_{I_P\to\infty} p_{i1}, i \in \{1,2\}$[7], and can be written as:

$$p_{out1}^{I_P\to\infty} = 1 - \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{e^{-\lambda_{11}j\frac{\gamma_{th}\sigma_n^2}{\beta P}}}{1+\frac{\lambda_{11}}{\mu_{21}}j(\frac{1-\beta}{\beta})\gamma_{th}},$$
$$p_{out2}^{I_P\to\infty} = 1 - \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{e^{-\lambda_{22}k\frac{\gamma_{th}\sigma_n^2}{(1-\beta)P}}}{1+\frac{\lambda_{22}}{\mu_{12}}k(\frac{\beta}{1-\beta})\gamma_{th}}. \quad (16)$$

2) Optimization and Critical Target Rate: It is well known that throughput of cognitive radio networks saturate with practical peak powers. It is in this scenario (peak interference region when transmit powers are limited by interference constraints to the primary alone), that cognitive radios perform the best, and should be typically operated. We therefore seek to optimize performance in this peak interference region by maximizing $\tau_{FR}^{P\to\infty}$ through appropriate choice of α, which leads us to the FNM strategy. Clearly, $\alpha^* = \arg\max_{\alpha}(\tau_{FR}^{P\to\infty})$. In a practical network setting when secondary downlink distances are much smaller than the distance to the primary receiver, the variances of interference channels to the primary are much smaller (μ_{1P} and μ_{2P} are larger) than the downlink variances (so that λ_{11} and λ_{22} are smaller), implying that $\lambda_{11} \ll \mu_{1P}I_P$ and $\lambda_{22} \ll \mu_{2P}I_P$ typically. Hence, the terms $\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}\alpha I_P}$ and $\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}(1-\alpha)I_P}$ in (14) and (15) respectively are small quantities for practical values of target rates, and can be ignored. It will become apparent later that computing α^* for high target rates is not required. Thus, $p_{out1}^{P\to\infty}$ and $p_{out2}^{P\to\infty}$ reduce to the following form with $A = \frac{\mu_{2P}\lambda_{11}}{\mu_{1P}\mu_{21}}$, $B = \frac{\mu_{1P}\lambda_{22}}{\mu_{2P}\mu_{12}}$ and $t = \frac{1-\alpha}{\alpha}$:

$$p_{out1}^{P\to\infty} \approx \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{(\gamma_{th}Atj - \ln(\gamma_{th}Atj) - 1)}{\frac{(1-\gamma_{th}Atj)^2}{\gamma_{th}Atj}},$$
$$p_{out2}^{P\to\infty} \approx \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{(\gamma_{th}\frac{B}{t}k - \ln(\gamma_{th}\frac{B}{t}k) - 1)}{\frac{(1-\gamma_{th}\frac{B}{t}k)^2}{\gamma_{th}\frac{B}{t}k}}. \quad (17)$$

Using the first order rational approximation for logarithm [31] $\ln(z) \approx \frac{2(z-1)}{(z+1)}$ in (17), which is close to (or follows) the logarithm function for a large range of z (and used in underlay literature [32]), $\frac{(z-\ln(z)-1)}{\frac{(1-z)^2}{z}} \approx \frac{z}{z+1}$. Hence, $p_{outi}^{P\to\infty}, i \in \{1,2\}$ in (17) can further be approximated as follows:

$$p_{out1}^{P\to\infty} \approx 1 - \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{1}{\gamma_{th}Atj+1},$$
$$p_{out2}^{P\to\infty} \approx 1 - \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{1}{\gamma_{th}\frac{B}{t}k+1}. \quad (18)$$

Obtaining α^* for general L and M is mathematically tedious, and can be evaluated offline by numerical search[8]. However, we present a closed form α^* for a couple of special cases.

Lemma *3.1:* When $L = M = 1$, or for general L and M with non-opportunistic user selection[9], $\alpha^* \approx \frac{1}{1+\frac{\mu_{1P}}{\mu_{2P}}\sqrt{\frac{\lambda_{22}}{\lambda_{11}}\frac{\mu_{21}}{\mu_{12}}}}$ is an extremum point of the function $\tau_{FR}^{P\to\infty}$ when viewed as a function of α in the range $0 \le \alpha \le 1$. Also, when $L = M$, the same α^* is the approximate point of intersection of two outage probabilities $p_{out1}^{P\to\infty}$ and $p_{out2}^{P\to\infty}$, when viewed as functions of α in the range $0 \le \alpha \le 1$ (other system parameters being constant).

Proof: When $L = M = 1$, the first derivative of $\tau_{FR}^{P\to\infty}$ with respect to $t = (1-\alpha)/\alpha$ using $p_{out1}^{P\to\infty}$ and $p_{out2}^{P\to\infty}$ from (18) is:

$$\frac{d}{dt}\tau_{FR}^{P\to\infty} = \left[\frac{B\gamma_{th}}{(B\gamma_{th}+t)^2} - \frac{A\gamma_{th}}{(1+A\gamma_{th}t)^2}\right]R. \quad (19)$$

On equating (19) to zero and solving for t, two roots $\sqrt{B/A}$ and $-\sqrt{B/A}$ can be obtained. Since $\alpha = \frac{1}{1+t}$, the corre-

[6]This is true because when $P \to \infty$ the terms $(1 - e^{\mu_{1P}\frac{\alpha}{\beta}\frac{I_P}{P}})$ in p_{11}, p_{14}, p_{21}, p_{23} and $(1 - e^{\mu_{2P}\frac{(1-\alpha)}{(1-\beta)}\frac{I_P}{P}})$ in p_{11}, p_{13}, p_{21}, p_{24} are zero.

[7]With $I_P \to \infty$, all exponential terms containing I_P as argument in the numerator, tend to zero. A product of such exponential terms with $E_1[.]$ terms containing I_P as argument in the numerator, also tend to zero. This makes all other terms except p_{11} and p_{21} tend to zero.

[8]We note that there is no dependence on instantaneous channel estimates.

[9]When $L = M = 1$, random variables $\max_{i\in[1,L]}[|h_{1i}|^2]$ and $\max_{i\in[1,M]}[|h_{2i}|^2]$ individually follow the exponential distribution. For non-opportunistic user selection (round robin user selection for example) with generalized L and M secondary users, since there is no selection mechanism involved to choose the best possible channel to the receiver, the max operator is not applicable. Here every user gets a chance, with channel gain to the intended receiver being exponentially distributed. A similar situation arises when two D2D units transmit simultaneously in the same band as the BS.

sponding roots in terms of α are $\frac{1}{1+\sqrt{B/A}}$ and $\frac{1}{1-\sqrt{B/A}}$. The latter root becomes negative when $B > A$ and exceeds unity when $B < A$, and is therefore discarded. Clearly, the former root lies within 0 and 1, and on substitution of A and B values, the closed form expression for α^* results. Note that $p_{out1}^{P\to\infty}$ ($p_{out2}^{P\to\infty}$) is a strictly decreasing (strictly increasing) function of α. Thus, $\arg\max_{\alpha}[\min(p_{out1}^{P\to\infty}, p_{out2}^{P\to\infty})]$ is the point of intersection of $p_{out1}^{P\to\infty}$ and $p_{out2}^{P\to\infty}$. Again, in (17), with $L = M$, we have $p_{out1}^{P\to\infty} = p_{out2}^{P\to\infty}$ when $\gamma_{th}At = \gamma_{th}\frac{B}{t}$. On substitution and solving for α, $\alpha = \alpha^*$ is obtained in the range $0 \leq \alpha \leq 1$. ■

In addition we have the following observations from Lemma 3.1: 1) α decreases when the ratio $\frac{\mu_{1P}}{\mu_{2P}}$ increases, or when S_2 is closer to the primary than S_1. This implies throughput can be maximized if a larger fraction of ITL is allocated to S_2, as S_1 has a weaker channel to primary (has more available power) and can meet its outage requirement with less transmit power. 2) α decreases with increase in $\frac{\lambda_{22}}{\lambda_{11}}$. In other words, when S_1-R_{1i^*} channel is better than S_2-R_{2i^*}, S_1 is able to meet its outage requirement with less transmit power, and a larger fraction of ITL needs to be allocated to S_2 to improve performance. 3) α decreases with the ratio $\frac{\mu_{21}}{\mu_{12}}$, or when the interference between S_1 to R_{2i^*} is stronger than the interference between S_2 to R_{1i^*}. Thus, allocating a larger fraction of ITL to S_2 causes less interference to users of S_1, which improves the overall throughput. For symmetric channel conditions, *i.e.* $\lambda_{11} = \lambda_{22}$, $\mu_{12} = \mu_{21}$ and $\mu_{1P} = \mu_{2P}$, $\alpha^* = 0.5$ as expected.

To evolve a network management strategy, it is important to study the concavity/convexity of $\tau_{FR}^{P\to\infty}$. We do so in the following lemma.

Lemma *3.2:* When $L = M = 1$, or for general L and M with non-opportunistic user selection, the optimum FNM strategy is to allow concurrent secondary transmission with $\alpha = \alpha^*$ as obtained in Lemma 3.1 when the target rate R is less than a critical target rate R_c, given by $R_c \approx \log_2\left(1 + \sqrt{\frac{\mu_{12}\mu_{21}}{\lambda_{11}\lambda_{22}}}\right)$. When target rate $R \geq R_c$, the optimum FNM strategy is to either use $\alpha = 0$ or $\alpha = 1$, whichever results in higher throughput (i.e. allow only one network to operate in isolation, whichever offers higher throughput).

Proof: $\tau_{FR}^{P\to\infty}$ will improve with concurrent secondary transmissions when $\frac{d^2}{d\alpha^2}\tau_{FR}^{P\to\infty}$ is negative, or in other words, $\tau_{FR}^{P\to\infty}$ is concave with respect to α. In order to evaluate $\frac{d^2}{d\alpha^2}\tau_{FR}^{P\to\infty}$, we first take a derivative with respect to t in (19) and obtain $\frac{d^2}{dt^2}\tau_{FR}^{P\to\infty} = \left[-\frac{2B\gamma_{th}}{(B\gamma_{th}+t)^3} + \frac{2A^2\gamma_{th}^2}{(1+A\gamma_{th}t)^3}\right]R$. From the chain rule of derivatives, we have, $\frac{d^2}{d\alpha^2}\tau_{FR}^{P\to\infty} = \frac{d^2}{dt^2}\tau_{FR}^{P\to\infty}.(\frac{dt}{d\alpha})^2 + \frac{d}{dt}\tau_{FR}^{P\to\infty}.\frac{d^2t}{d\alpha^2}$. Since $\frac{dt}{d\alpha} = -\frac{1}{\alpha^2}$ and $\frac{d^2t}{d\alpha^2} = \frac{2}{\alpha^3}$, upon substitution, we can rewrite $\frac{d^2}{d\alpha^2}\tau_{FR}^{P\to\infty}$ as follows:

$$\frac{d^2}{d\alpha^2}\tau_{FR}^{P\to\infty} = \left[-\frac{2B\gamma_{th}}{(B\gamma_{th}+t)^3} + \frac{2A^2\gamma_{th}^2}{(1+A\gamma_{th}t)^3}\right]\frac{R}{\alpha^4} + \underbrace{\left[\frac{B\gamma_{th}}{(B\gamma_{th}+t)^2} - \frac{A\gamma_{th}}{(1+A\gamma_{th}t)^2}\right]\frac{2R}{\alpha^3}}_{T}. \quad (20)$$

Clearly, depending on the value of γ_{th}, (20) can either be positive or negative. By substituting $t = \sqrt{B/A}$ and $\alpha = \frac{1}{1+\sqrt{B/A}}$ (as derived from (19)) in (20), and equating to zero, we solve for γ_{th} (or equivalently for R). We note the term T becomes zero upon substitution of $t = \sqrt{B/A}$ and $\alpha = \frac{1}{1+\sqrt{B/A}}$ in (20). $R = 0$ are the first two roots, and $R = \log_2(1 + 1/\sqrt{AB})$ is obtained as the third root. By discarding the first two roots and substituting A and B in the third root, the closed form expression for critical target rate R_c is obtained. When $R > R_c$, $\tau_{FR}^{P\to\infty}$ is convex, and the largest throughput is obtained only at the boundary points. ■

Remark 3.1: For a generalized L and M with opportunistic user selection too, $\tau_{FR}^{P\to\infty}$ is concave w.r.t α for small R, and convex otherwise. Establishing this, and obtaining closed form expressions for this critical rate and optimum α is difficult due to intractable nature of the expressions. However, this has been verified by extensive computer simulations over a wide range of parameter values. Note that R_c and α^* for such a case can be evaluated using offline numerical search[8]. In case of fixed transmit power control by secondary sources, concurrent secondary transmissions can still lead to improved throughput with judicious choice of fixed transmit powers. $\tau_{FR}^{I_P\to\infty}$ is neither concave nor convex w.r.t β. An offline numerical search is required to obtain the optimum β ($= \beta^*$) that maximizes $\tau_{FR}^{I_P\to\infty}$ (Fig. 3). We emphasize, R_c, α^* and β^* depend only on statistical channel parameters and do not require real-time computation.

In situations when there is a given outage requirement at the secondary networks, the throughput-optimal FNM strategy is presented in the following lemma.

Lemma *3.3:* When $L = M$, and the given outage requirement p_o at both secondary networks is higher than a critical outage probability $p_c = p_{out1}^{P\to\infty}(\alpha = \alpha^*) = p_{out2}^{P\to\infty}(\alpha = \alpha^*)$, the optimum FNM strategy is to allow concurrent secondary transmissions with $\alpha = \alpha^*$ as obtained in Lemma 3.1. When p_o is lower than p_c, the optimum FNM strategy is to either use $\alpha = 0$ or $\alpha = 1$, whichever results in lower outage (i.e. allow only one network to operate in isolation, whichever offers lower outage/ higher throughput).

Proof: For $L = M$, when p_o is higher than p_c, outage requirement of p_o is met at both secondary networks if α is chosen such that $\alpha_L \leq \alpha \leq \alpha_R$, where $\alpha_L = p_{out1}^{P\to\infty\ -1}(p_o)$ and $\alpha_R = p_{out2}^{P\to\infty\ -1}(p_o)$. The FNM strategy chooses to use $\alpha = \alpha^*$, since it minimizes both $p_{out1}^{P\to\infty}$ and $p_{out2}^{P\to\infty}$ (and hence maximizes the sum throughput). When p_o is lower than p_c, it is possible to only satisfy the outage requirement of any one of the two secondary networks. This is because, outage requirement of p_o is met only when $\alpha \leq \alpha_{L'}$ or $\alpha \geq \alpha_{R'}$, where $\alpha_{L'} = p_{out2}^{P\to\infty\ -1}(p_o)$ and $\alpha_{R'} = p_{out1}^{P\to\infty\ -1}(p_o)$, implying that only one out of the two secondary networks can be operational. The FNM strategy therefore chooses to operate one secondary network with α chosen at the boundary points (either $\alpha = 0$ or $\alpha = 1$) that yields minimum outage. ■

IV. Asymptotic Sum Throughput with FRT and CANM

In this section, we evaluate the asymptotic sum throughput performance of co-existing secondary networks when the secondary transmitters connect to a controlling node so that α is calculated instantaneously in each signaling interval. In the first case when each secondary transmitter has knowledge of instantaneous signal and interference links to its selected secondary receiver (interference link to the primary receiver is assumed to be always available to perform underlay transmissions), we evaluate an expression for asymptotic sum throughput and devise a CANM strategy that results in maximum achievable sum throughput for a given system setting[10]. We then take up a practical scenario wherein interference channels to selected secondary receivers are not available, devise a CANM strategy, and evaluate the approximate sum throughput.

A. CANM strategy with FRT - Asymptotic performance with CSI:

We now formulate the CANM strategy with FRT when each secondary transmitter has knowledge of instantaneous signal and the interference links to its selected secondary receiver. When secondary transmitters perform opportunistic user selection and transmit powers of secondary networks are limited by interference constraints ($P \to \infty$), SINRs Γ_1 and Γ_2 take their asymptotic forms $\Gamma_1^{P\to\infty}$ and $\Gamma_2^{P\to\infty}$ respectively (random variables $\max\limits_{i\in[1,L]}[|h_{1i}|^2]$ and $\max\limits_{i\in[1,M]}[|h_{2i}|^2]$ have been replaced by X and Y respectively for notational convenience) as:

$$\Gamma_1^{P\to\infty} = \frac{\alpha I_P \frac{X}{|g_{1P}|^2}}{(1-\alpha)I_P\frac{|g_{2i^*}|^2}{|g_{2P}|^2} + \sigma_n^2},$$
$$\Gamma_2^{P\to\infty} = \frac{(1-\alpha)I_P \frac{Y}{|g_{2P}|^2}}{\alpha I_P\frac{|g_{1i^*}|^2}{|g_{1P}|^2} + \sigma_n^2}. \tag{21}$$

Lemma *4.1:* Let: $\alpha_1 = \frac{\frac{\gamma_{th}\sigma_n^2}{I_P} + \gamma_{th}\frac{|g_{2i^*}|^2}{|g_{2P}|^2}}{\frac{X}{|g_{1P}|^2} + \gamma_{th}\frac{|g_{2i^*}|^2}{|g_{2P}|^2}}$ and $\alpha_2 = \frac{-\frac{\gamma_{th}\sigma_n^2}{I_P} + \frac{Y}{|g_{2P}|^2}}{\frac{Y}{|g_{2P}|^2} + \gamma_{th}\frac{|g_{1i^*}|^2}{|g_{1P}|^2}}$. When $\alpha_1 \le \alpha_2$, the optimum CANM strategy with FRT and opportunistic user selection is to allow concurrent secondary transmission with α such that $\alpha_1 \le \alpha \le \alpha_2$. When $\alpha_1 > \alpha_2$, the optimum CANM strategy with FRT is to either operate S_1 with $\alpha > \alpha_1$ or S_2 with $\alpha < \alpha_2$ (both offer the same throughput).

Proof: A detailed proof is presented in Appendix A. ■

In the following lemma, we present an analytical expression for throughput $\tau_{FR}^{CSI(full)}$ that is available using the above CANM strategy.

[10]We establish through extensive simulations in Section VI that availability of channel knowledge results in larger sum throughput.

Lemma *4.2:* The throughput $\tau_{FR}^{CSI(full)}$ for CANM with FRT when CSI of signal and interference links to selected secondary receivers are available is as follows:

$$\tau_{FR}^{CSI(full)} = \Bigg[-\sum_{k=1}^{M}\sum_{j=1}^{L}\binom{M}{k}\binom{L}{j}(-1)^{j+k}\frac{\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2}{\left[1-\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2\right]^2} \left\{\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2 - \ln\left(\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2\right) - 1\right\} - \left\{\sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}I_P}}{1+\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}I_P}}\right\} \left\{\sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}I_P}}{1+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}I_P}}\right\} + 2\Bigg]R. \tag{22}$$

Proof: A detailed proof is presented in Appendix B. ■

B. CANM strategy with FRT - Asymptotic Performance with Partial CSI:

Acquiring all the channel knowledge required can be difficult or even infeasible in practical network implementations. In this subsection, we take up a practical scenario where the instantaneous interference channel to the selected secondary receiver is unknown to the transmitter. Thus, the secondary networks are assumed to only have statistical knowledge of such secondary interference links. When these interferences are weak, we can replace such channels with their average values. The SINRs $\Gamma_1^{P\to\infty}$ and $\Gamma_2^{P\to\infty}$ of (21) now become:

$$\Gamma_1^{P\to\infty} \approx \frac{\alpha I_P\frac{X}{|g_{1P}|^2}}{(1-\alpha)\frac{I_P}{\mu_{21}|g_{2P}|^2} + \sigma_n^2},$$
$$\Gamma_2^{P\to\infty} \approx \frac{(1-\alpha)I_P\frac{Y}{|g_{2P}|^2}}{\alpha\frac{I_P}{\mu_{12}|g_{1P}|^2} + \sigma_n^2}. \tag{23}$$

Lemma *4.3:* Let: $\widehat{\alpha}_1 = \frac{\frac{\gamma_{th}\sigma_n^2}{I_P} + \frac{\gamma_{th}}{\mu_{21}|g_{2P}|^2}}{\frac{X}{|g_{1P}|^2} + \frac{\gamma_{th}}{\mu_{21}|g_{2P}|^2}}$ and $\widehat{\alpha}_2 = \frac{-\frac{\gamma_{th}\sigma_n^2}{I_P} + \frac{Y}{|g_{2P}|^2}}{\frac{Y}{|g_{2P}|^2} + \frac{\gamma_{th}}{\mu_{12}|g_{1P}|^2}}$. With opportunistic user selection at the secondary networks, the CANM strategy when the secondary interference channels have low variances but are unknown, is as follows: When $\widehat{\alpha}_1 \le \widehat{\alpha}_2$, the CANM strategy is to allow both secondary networks to be operational with α such that $\widehat{\alpha}_1 \le \alpha \le \widehat{\alpha}_2$. When $\widehat{\alpha}_1 > \widehat{\alpha}_2$, the CANM strategy is to either operate S_1 with $\alpha > \widehat{\alpha}_1$ or S_2 with $\alpha < \widehat{\alpha}_2$ (both offer the same throughput).

Proof: SINR-s $\Gamma_1^{P\to\infty}$ and $\Gamma_2^{P\to\infty}$ of (23), when equated to threshold γ_{th} ($\gamma_{th} = 2^R - 1$), result in imperfect estimates of α_1 and α_2 (denoted by $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$ respectively). Thus, we have, $\widehat{\alpha}_1 = \frac{\frac{\gamma_{th}\sigma_n^2}{I_P} + \frac{\gamma_{th}}{\mu_{21}|g_{2P}|^2}}{\frac{X}{|g_{1P}|^2} + \frac{\gamma_{th}}{\mu_{21}|g_{2P}|^2}}$ and $\widehat{\alpha}_2 = \frac{-\frac{\gamma_{th}\sigma_n^2}{I_P} + \frac{Y}{|g_{2P}|^2}}{\frac{Y}{|g_{2P}|^2} + \frac{\gamma_{th}}{\mu_{12}|g_{1P}|^2}}$ to frame the decision rule. The rest of the proof is along similar lines as in Lemma 4.1, and is omitted. ■

In the following lemma, we present an approximate expression for throughput $\tau_{FR}^{CSI(partial)}$ with FRT assuming CANM strategy with partial CSI.

Lemma *4.4:* When partial CSI is available (secondary interference channels to selected receivers have low variances but are unknown) as in Lemma 4.3, and the CANM strategy is utilized with FRT, an approximate throughput $\tau_{FR}^{CSI(partial)}$ is as follows:

$$\tau_{FR}^{CSI(partial)} \approx \Bigg[1 + \sum_{k=1}^{M}\sum_{j=1}^{L}\binom{M}{k}\binom{L}{j}(-1)^{j+k} 2\sqrt{\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2}K_1\left(2\sqrt{\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2}\right) - \left\{\sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}I_P}}{1+\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}I_P}}\right\} \left\{\sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}I_P}}{1+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}I_P}}\right\}\Bigg]R. \quad (24)$$

Proof: A detailed proof is presented in Appendix C. ■

V. Asymptotic Performance of Secondary Networks with ART and FNM

In case of ART with opportunistic user selection at the secondary networks, both S_1 and S_2 adjust their transmission rates according to channel conditions. We evaluate asymptotic sum ergodic rate $\tau_{AR}^{P\to\infty}$ with a statistically chosen α (FNM) to characterize performance as follows:

$$\tau_{AR}^{P\to\infty} = \underbrace{\mathbb{E}_{\Gamma_1^{P\to\infty}}\left[\log_2\left(1+\Gamma_1^{P\to\infty}\right)\right]}_{R_1} + \underbrace{\mathbb{E}_{\Gamma_2^{P\to\infty}}\left[\log_2\left(1+\Gamma_2^{P\to\infty}\right)\right]}_{R_2}, \quad (25)$$

where $\Gamma_1^{P\to\infty}$ and $\Gamma_2^{P\to\infty}$ are as defined in (21).

Lemma *5.1:* For ART and FNM strategy with opportunistic user selection at the secondary networks, the asymptotic sum ergodic rate is $\tau_{AR}^{P\to\infty} = R_1 + R_2$, where R_1 and R_2 are given by:

$$R_1 = \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{b_1/j}{(a_1+\frac{b_1}{j}-1)}\left[\log_2(\frac{b_1}{j}) + \frac{a_1}{(1-a_1)}\log_2(a_1)\right] + \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{a_1b_1/j}{(a_1+\frac{b_1}{j}-1)^2}\left[Di_2(0) - Di_2(\frac{a_1+b_1/j-1}{b_1/j}) - Di_2(\frac{a_1+b_1/j-1}{a_1}) + Di_2(\frac{a_1+b_1/j-1}{a_1b_1/j})\right]/\ln(2), \quad (26)$$

$$R_2 = \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{b_2/k}{(a_2+\frac{b_2}{k}-1)}\left[\log_2(\frac{b_2}{k}) + \frac{a_2}{(1-a_2)}\log_2(a_2)\right] + \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{a_2b_2/k}{(a_2+\frac{b_2}{k}-1)^2}\left[Di_2(0) - Di_2(\frac{a_2+b_2/k-1}{b_2/k}) - Di_2(\frac{a_2+b_2/k-1}{a_2}) + Di_2(\frac{a_2+b_2/k-1}{a_2b_2/k})\right]/\ln(2), \quad (27)$$

where $a_1 = \frac{(1-\alpha)I_P\mu_{2P}}{\mu_{21}\sigma_n^2}$, $b_1 = \frac{\alpha I_P\mu_{1P}}{\lambda_{11}\sigma_n^2}$, $a_2 = \frac{\alpha I_P\mu_{1P}}{\mu_{12}\sigma_n^2}$ and $b_2 = \frac{(1-\alpha)I_P\mu_{2P}}{\lambda_{22}\sigma_n^2}$. $Di_2(x) = -\int_1^x \frac{\ln(t)}{t-1}dt$ used in (26) and (27) denote the Dilogarithm function defined in terms of the Spence's integral.

Proof: A detailed derivation is presented in Appendix D. ■

In order to evolve a FNM strategy for ART, we need to analyze the behavior of the sum ergodic rate as a function of α, and determine the optimum $\alpha = \alpha^\dagger$ that maximizes it. Unfortunately, the sum ergodic rate expression is a complicated function comprising of Dilogarithm terms, and difficult to approximate. We therefore analyze a special case in the following lemma.

Lemma *5.2:* Let $\alpha^\dagger = \arg\max_{\alpha}[\tau_{AR}^{P\to\infty}]$. For ART, when $L = M = 1$ and the channels are symmetric ($\lambda_{11} = \lambda_{22} = \lambda, \mu_{12} = \mu_{21} = \mu_S, \mu_{1P} = \mu_{2P} = \mu_P$), *i.e.* secondary nodes in each network are closer to each other than from the other network and also the primary receiver, the best FNM strategy is to allow concurrent secondary transmissions with $\alpha^\dagger = 0.5$, when I_P is below a critical ITL $I_{P_{(c)}}$, as given by $I_{P_{(c)}} \approx \frac{\mu_S\sigma_n^2(\mu_S-2\lambda)}{\lambda\mu_P}$. When $I_P > I_{P_{(c)}}$, only one secondary network should be operational.

Proof: In a practical scenario, the two secondary networks are relatively distant from each other, and thus interference channels between the secondary networks have relatively low variances. The same logic extends to interference channels to the primary receiver, as the primary user is expected to be located far from both the secondary networks as compared to the distance between secondary nodes in the same network. Thus, we approximate all interference channels by their respective mean values in $\Gamma_1^{P\to\infty}$ and $\Gamma_2^{P\to\infty}$ of (21) respectively[11]. The approximated $\tau_{AR}^{P\to\infty}$ expression for $L = M = 1$ can then be written as:

$$\tau_{AR}^{P\to\infty} \approx \mathbb{E}\left[\log_2\left(1+\frac{\alpha I_P\mu_{1P}X}{(1-\alpha)I_P\frac{\mu_{2P}}{\mu_{21}}+\sigma_n^2}\right)\right] + \mathbb{E}\left[\log_2\left(1+\frac{(1-\alpha)I_P\mu_{2P}Y}{\alpha I_P\frac{\mu_{1P}}{\mu_{12}}+\sigma_n^2}\right)\right]. \quad (28)$$

In (28), the first $\mathbb{E}$ denotes expectation over random variable X while the second $\mathbb{E}$ denotes expectation over random variable Y. Using the standard integral [29, eq.(4.337.2)], X and Y in (28) can be averaged, which leads to the following expression

[11] In Section VI, we demonstrate accuracy of the approximation through extensive computer simulations

for $\tau_{AR}^{P\to\infty}$:

$$\tau_{AR}^{P\to\infty} = \Bigg\{ e^{\frac{(1-\alpha)I_P\frac{\mu_{2P}}{\mu_{21}}+\sigma_n^2}{\alpha I_P \frac{\mu_{1P}}{\lambda_{11}}}} E_1\left[\frac{(1-\alpha)I_P\frac{\mu_{2P}}{\mu_{21}}+\sigma_n^2}{\alpha I_P\frac{\mu_{1P}}{\lambda_{11}}}\right] + e^{\frac{\alpha I_P\frac{\mu_{1P}}{\mu_{12}}+\sigma_n^2}{(1-\alpha)I_P\frac{\mu_{2P}}{\lambda_{22}}}} E_1\left[\frac{\alpha I_P\frac{\mu_{1P}}{\mu_{12}}+\sigma_n^2}{(1-\alpha)I_P\frac{\mu_{2P}}{\lambda_{22}}}\right]\Bigg\}/\ln(2). \quad (29)$$

We note that (29) consists of a summation of two terms of the form $e^x E_1[x]$, which can be tightly upper bounded as summation of two $\ln(1+\frac{1}{x})$ terms, when x is positive and not very close to zero [33, ineq.(5.1.20)]. With this approximation, we arrive at:

$$\begin{aligned}\tau_{AR}^{P\to\infty} &\approx \log_2\left(1+\frac{\alpha I_P\frac{\mu_{1P}}{\lambda_{11}}}{(1-\alpha)I_P\frac{\mu_{2P}}{\mu_{21}}+\sigma_n^2}\right) + \log_2\left(1+\frac{(1-\alpha)I_P\frac{\mu_{2P}}{\lambda_{22}}}{\alpha I_P\frac{\mu_{1P}}{\mu_{12}}+\sigma_n^2}\right),\\ &= \log_2\left(\left\{1+\frac{\alpha I_P\frac{\mu_{1P}}{\lambda_{11}}}{(1-\alpha)I_P\frac{\mu_{2P}}{\mu_{21}}+\sigma_n^2}\right\}\left\{1+\frac{(1-\alpha)I_P\frac{\mu_{2P}}{\lambda_{22}}}{\alpha I_P\frac{\mu_{1P}}{\mu_{12}}+\sigma_n^2}\right\}\right). \end{aligned} \quad (30)$$

For symmetric signal and interference links, the above equation reduces to:

$$\tau_{AR,sym}^{P\to\infty} = \log_2\left(\left\{1+\frac{\alpha I_P\frac{\mu_P}{\lambda}}{(1-\alpha)I_P\frac{\mu_P}{\mu_S}+\sigma_n^2}\right\}\left\{1+\frac{(1-\alpha)I_P\frac{\mu_P}{\lambda}}{\alpha I_P\frac{\mu_P}{\mu_S}+\sigma_n^2}\right\}\right). \quad (31)$$

It can be readily established that $\alpha = 0.5$ is an exremum point of $\tau_{AR,sym}^{P\to\infty}$ (proof is omitted). A second derivative of (31) w.r.t α, followed by some algebraic simplification and $\alpha = 0.5$ gives:

$$\frac{d^2}{d\alpha^2}\tau_{AR,sym}^{P\to\infty} = \frac{2I_P^2\mu_s\mu_P^2(\mu_S\sigma_n^2+I_P\mu_P)[(2\lambda-\mu_S)\mu_S\sigma_n^2+Ip\lambda\mu_P]}{\log(2)(\mu_S\sigma_n^2+0.5I_P\mu_P)^2[\lambda\mu_S\sigma_n^2+0.5I_P(\lambda+\mu_S)\mu_P]^2}. \quad (32)$$

$\tau_{AR,sym}^{P\to\infty}$ is concave when (32) is negative, and convex otherwise. Equating (32) to zero and solving for I_P, results in a closed form expression for the critical ITL. Clearly, the roots are $I_P = \{0, 0, -\frac{\mu_S\sigma_n^2}{\mu_P}, \frac{\mu_S\sigma_n^2(\mu_S-2\lambda)}{\lambda\mu_P}\}$. The first three roots are either zero or negative, and are thus discarded. Practically, since $\mu_S \gg \lambda$, the fourth root is a positive quantity, which is the critical ITL expression with symmetric main and interference channels. ■

In Lemma 5.2, for an extreme case when $\mu_S \to \infty$, $I_{P_{(c)}} \to \infty$, implying concavity of $\tau_{AR}^{P\to\infty}$ w.r.t α. $I_{P_{(c)}}$ decreases with increase in μ_P, or when the primary user is distant. This further motivates frequency reuse with co-existing underlay cognitive radio networks. Furthermore, $I_{P_{(c)}}$ also decreases with λ, $i.e.$ when secondary downlink channels become poor. This is expected, as mutual secondary interferences overwhelm the main signal links, and switching to single secondary transmission becomes a better choice.

Remark 5.1: Although the proof is provided for a special case, the arguments do hold in general, and the optimum α and FNM can be found by numerical methods.
Remark 5.2: The sum ergodic rate expressions are not strong functions of α, except when α is close to zero or one. Use of $\alpha = 0.5$ results in little rate loss in practically all cases (Fig. 7).
We summarize all NM strategies evolved in this paper in Table I[12].

VI. SIMULATION RESULTS

In this section, we present simulation results to validate the derived expressions and bring out useful insights. Let $\mathbb{E}[|h_{ij}|^2] \propto d_{ii}^{-\phi}$ and $\mathbb{E}[|g_{ij}|^2] \propto r_{ij}^{-\phi}$, where ϕ is the path-loss exponent (assumed to be 3 in this paper). d_{ii} is the normalized distance between the transmitter and the j^{th} receiver in cluster i, where $j = 1, 2, \ldots, L$ when $i = 1$, and $j = 1, 2, \ldots, M$ when $i = 2$. r_{ij} is the normalized distance between the transmitter of cluster i to j^{th} receiver of the other cluster. Also, $\mathbb{E}[|g_{iP}|^2] \propto r_{iP}^{-\phi}$, where r_{iP} denotes distance between the transmitter of cluster i, $i \in \{1,2\}$ to the primary receiver.

In Fig. 2 we plot $\tau_{FR}^{P\to\infty}$ vs α for different target rates. The system parameters chosen are as follows: $d_{11} = 2$ units, $d_{22} = 1$ unit, $r_{1P} = r_{2P} = 3$ units, $r_{12} = 4$ units, $r_{21} = 3$ units, $L = M = 1$, and $I_P = 20dB$. When target rates are below R_c (Lemma 3.2), there is an improvement in sum throughput of the order of 1 bpcu when optimum α is chosen using concurrent transmission. If R exceeds $R_c(= 3.9724)$, switching to single transmission is the best. This happens because at high target rates, both user pairs suffer link outages, and mutual interferences further degrades performance. Switching to a single network not only increases transmit power, but also nulls the interference from the other network, cumulatively improving outage and throughput performance.

In Fig. 3 we plot $\tau_{FR}^{P\to\infty}$ vs α for $L = M = 1$. The system parameters chosen are as follows: $d_{11} = 1$ unit, $d_{22} = 2$ units, $r_{1P} = 4$ units, $r_{2P} = 3$ units, $r_{12} = 3$ units, $r_{21} = 3.5$ units, and $I_P = 10dB$. $R = 1$ is assumed to ensure that $R < R_c = 3.7037$ (so that concurrent transmission is advantageous). Clearly $\alpha^* = 0.1058$ as derived in Lemma 3.1, maximizes the sum throughput. We also plot $\tau_{FR}^{I_P\to\infty}$ vs β in Fig. 3, for $L = 4$ and $M = 2$. The plot clearly shows that use of concurrent transmission can improve sum throughput when secondary networks use peak transmit power control. β^* as shown in the figure, is computed numerically and is seen to be accurate. The system parameters chosen are as follows: $d_{11} = 2$ unit, $d_{22} = 1$ units, $r_{1P} = 4$ units, $r_{2P} = 3$ units, $r_{12} = 3$ units and $r_{21} = 4$ units, $P = 10dB$ and $R = 1$.

In Fig. 4, we plot τ_{FR} vs R and $\tau_{FR}^{P\to\infty}$ vs R, and show the effect of opportunistic user selection in the two secondary networks on sum throughput performance with concurrent transmissions. We choose parameters as follows: $d_{11} = d_{22} = 1$ unit, $r_{1P} = r_{2P} = 3$ units, $r_{12} = r_{21} = 3$

[12]Please note, in Table I, R_c and $I_{P_{(c)}}$ refers to generalized critical target rate and generalized critical ITL respectively, and not confined to the special case of $L = M = 1$. Such quantities can be computed using offline numerical methods.

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TABLE I: A Summary of Network Management Strategies

		Network Selection Criteria		Choice			Available Throughput
		Target Rate/Outage	Throughput	α	S_1	S_2	
Fixed Rate Trans-mission (FRT)	Fixed Network Management (FNM)	$R < R_c$ OR $(p_o > p_c)$	N.A.	$\arg\max_{\alpha}(\tau_{FR}^{P\to\infty})$. OR $\arg\max_{\alpha}[\min(p_{out1}^{P\to\infty}, p_{out2}^{P\to\infty})]$	ON	ON	$\tau_{FR}^{P\to\infty}$
		$R > R_c$ OR $(p_o < p_c)$	$\tau_{FR}^{P\to\infty}(\alpha = 1) >$ $\tau_{FR}^{P\to\infty}(\alpha = 0)$	1	ON	OFF	$\tau_{FR}^{P\to\infty}(\alpha = 1)$
		$R > R_c$ OR $(p_o < p_c)$	$\tau_{FR}^{P\to\infty}(\alpha = 1) <$ $\tau_{FR}^{P\to\infty}(\alpha = 0)$	0	OFF	ON	$\tau_{FR}^{P\to\infty}(\alpha = 0)$
	Channel Aware Network Management (CANM)		Network Selection Criteria	Choice			Available Throughput
				α	S_1	S_2	
		With CSI of Signal and Interference Links to Selected Receivers	$\alpha_1 \leq \alpha_2$	Choose α between α_1 and α_2	ON	ON	$2R.\Pr\{\alpha_1 \leq \alpha_2\}$
			$\alpha_1 > \alpha_2$	Choose $\alpha > \alpha_1$ OR Choose $\alpha < \alpha_2$	ON / OFF	OFF / ON	$R.\Big[\Pr\{\alpha_1 > \alpha_2\}$ - $\Pr\{\alpha_1 > 1, \alpha_2 < 0\}\Big]$
		With CSI of Signal Links to Selected Receivers	$\widehat{\alpha}_1 \leq \widehat{\alpha}_2$	Choose α between $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$	ON	ON	$2R.\Pr\{\widehat{\alpha}_1 \leq \widehat{\alpha}_2\}$
			$\widehat{\alpha}_1 > \widehat{\alpha}_2$	Choose $\alpha > \widehat{\alpha}_1$ OR Choose $\alpha < \widehat{\alpha}_2$	ON / OFF	OFF / ON	$R.\Big[\Pr\{\widehat{\alpha}_1 > \widehat{\alpha}_2\}$ - $\Pr\{\widehat{\alpha}_1 > 1, \widehat{\alpha}_2 < 0\}\Big]$
Adaptive Rate Transmission (ART)	With Symmetric Secondary Node Positions	Network Selection Criteria		Choice			Available Ergodic Rate
		I_P		α	S_1	S_2	
		$I_P < I_{P_{(c)}}$		0.5	ON	ON	$\tau_{AR}^{P\to\infty}$
		$I_P > I_{P_{(c)}}$		Choose $\alpha = 1$ OR Choose $\alpha = 0$	ON / OFF	OFF / ON	$\tau_{AR}^{P\to\infty}(\alpha = 1)$ OR $\tau_{AR}^{P\to\infty}(\alpha = 0)$

units, $\alpha = \beta = 0.5$ and $I_P = 10dB$. $P = 10dB$ is chosen for plotting τ_{FR}. Clearly, τ_{FR} and $\tau_{FR}^{P\to\infty}$ both increase with L and M. The plot also shows sum throughput is higher when the secondary networks are operated in the peak interference region. From Lemma 3.2, it is clear that R_c increases with user selection (this R_c refers to the network having generalized L and M users, which needs to be determined numerically. However, intuitively it is clear that user selection statistically improve the main channels, thereby increasing R_c), which causes rightward shift in the peaks of τ_{FR} and $\tau_{FR}^{P\to\infty}$. As evident from earlier discussions, sum throughput first increase and then decreases after a certain critical rate as both S_1-R_{1i^*} and S_2-R_{2i^*} links then have higher outages, thereby decreasing the overall performance with concurrent transmissions.

In Fig. 5, we present a comparative study of asymptotic sum throughput performances $\tau_{FR}^{P\to\infty}$ (with statistically optimum α), $\tau_{FR}^{CSI(full)}$ and $\tau_{FR}^{CSI(partial)}$. We have chosen system parameters as: $d_{11} = d_{22} = 1$ unit, $r_{1P} = 3$ units, $r_{2P} = 4$ units, $r_{12} = 4$ units, $r_{21} = 3$ units, $L = M = 1$, and $I_P = 20dB$. Clearly, at lower target rates (when $R < R_c$ for $\tau_{FR}^{P\to\infty}$, $\alpha_1 < \alpha_2$ for $\tau_{FR}^{CSI(full)}$, and $\widehat{\alpha}_1 < \widehat{\alpha}_2$ for $\tau_{FR}^{CSI(partial)}$), concurrent transmission takes place. When the target rate increases beyond a point when the system cannot accommodate concurrent transmissions, it switches to a single secondary transmission mode (the secondary network offering higher throughput in case of FNM, and any one of the two secondary networks in case of CANM, is chosen). This is the reason why sum throughput curves in Fig. 5 are not smooth functions when plotted against target rates. Moreover, as expected, sum throughput improves with channel knowledge, which causes $\tau_{FR}^{CSI(full)}$ to perform the best, and $\tau_{FR}^{P\to\infty}$ to perform the poorest. However, acquiring and passing on instantaneous channel information is difficult in many situations. When target rates are low, a statistical ITL apportioning is good enough to satisfy outage requirement of both secondary networks and all three schemes offer almost the same throughput.

Similar to τ_{FR}, $\tau_{FR}^{CSI(full)}$ also improves with opportunistic user selection at the two secondary networks (Fig. 6). From (35), $\Pr\{\alpha_1 \leq \alpha_2\} = \Pr\{XY > \gamma_{th}^2 |g_{1i^*}|^2 |g_{2i^*}|^2\}$ increases with L and M. Moreover (38), which indicates the joint probability of both secondary networks being in outage with individual maximum available powers and no cross interferences, decreases with user selection. This causes an overall increase in $\tau_{FR}^{CSI(full)}$ with increase in L and M, as seen in (34). The same logic extends to the case of $\tau_{FR}^{CSI(partial)}$, which also improves with user selection at the secondary networks, and is not plotted due to space limitation. The system parameters chosen for plotting Fig. 6 are the same as those used in Fig. 5.

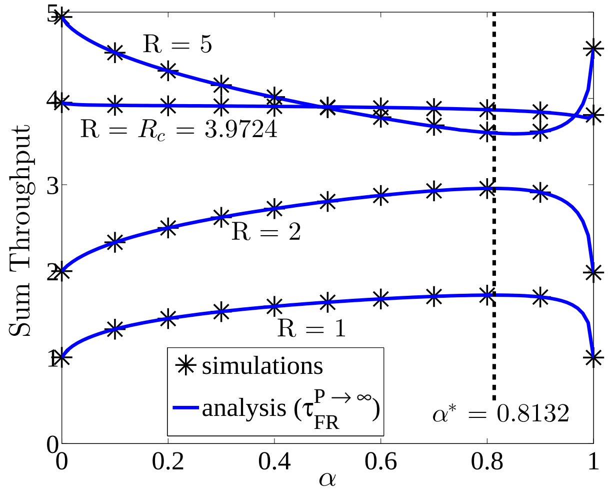

Fig. 2: $\tau_{FR}^{P\to\infty}$ vs α for $R = 1, 2, R_c, 5$.

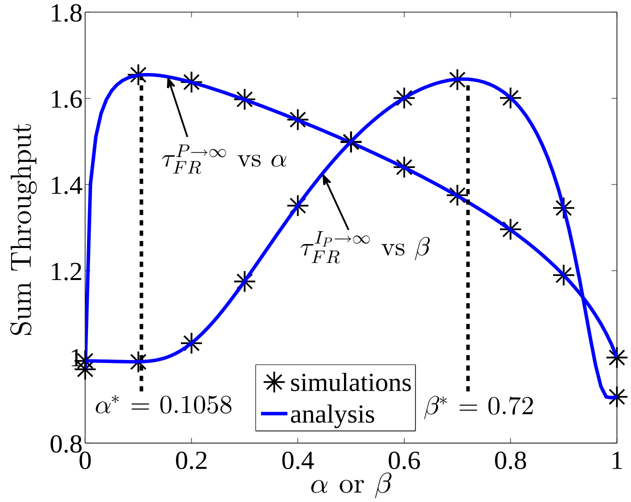

Fig. 3: $\tau_{FR}^{P\to\infty}$ vs α and $\tau_{FR}^{I_P\to\infty}$ vs β with α^* and β^*.

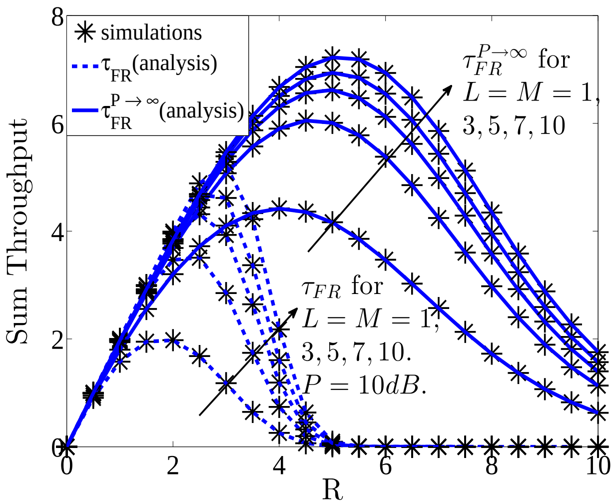

Fig. 4: τ_{FR} vs R and $\tau_{FR}^{P\to\infty}$ vs R with $L = M = 1, 3, 5, 7, 10$.

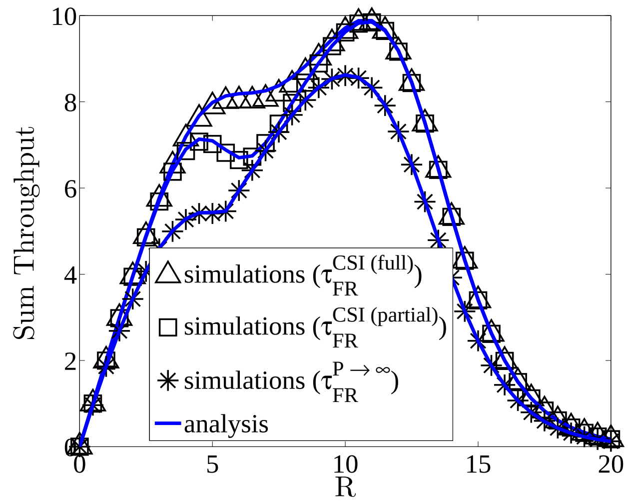

Fig. 5: $\tau_{FR}^{P\to\infty}$ (with α^*), $\tau_{FR}^{CSI(partial)}$ and $\tau_{FR}^{CSI(full)}$ vs R.

In Fig. 7, we demonstrate the behavior of asymptotic sum ergodic rate $\tau_{AR}^{P\to\infty}$ w.r.t α for different values of I_P, when the two secondary networks are symmetrically placed. We choose $\lambda = 1$ unit, $\mu_S = 3.5$ units, $\mu_P = 3$ units and $L = M = 1$ for plotting. As discussed in Sec. V, the plot indicates a weak dependence of asymptotic sum ergodic rate on α, as $\tau_{AR}^{P\to\infty}$ is almost flat over the entire range of α, except close to 0 and 1. When I_P is less than $I_{P_{(c)}}$, concurrent secondary transmission is found to be useful as it offers a performance improvement of more than 1 bpcu. However, with high I_P values (I_P is higher than $I_{P_{(c)}}$), switching to single secondary transmission is the best. The critical I_P value $I_{P_{(c)}}$ (of Lemma 5.2) is also plotted. It is seen to be accurate for practical distances between secondary and primary networks.

VII. Conclusion

In this paper we analyze performance of two co-existing underlay multiuser secondary downlink networks. We demonstrate that accommodating two secondary networks instead of one, and operating them judiciously by following certain network management (NM) guidelines, can lead to improved spectrum utilization and performance. For fixed (adaptive) rate transmission, when the target rate (interference temperature limit (ITL)) is below a critical value, we show that apportioning the ITL and operating both secondary networks concurrently is the key to improve throughput (ergodic rate) performance. We study the apportioning of ITL when channel state information (CSI) is available. We also study apportioning of ITL based on partial CSI, and the extreme case when only statistical knowledge of channels is available.

Appendix A: Proof of Lemma 4.1

When CSI of signal and interference links to selected secondary receivers (and interference links to the primary receiver) is perfectly available, α can exactly be calculated when $\Gamma_1^{P\to\infty}$ and $\Gamma_2^{P\to\infty}$ of (21) are compared to a threshold

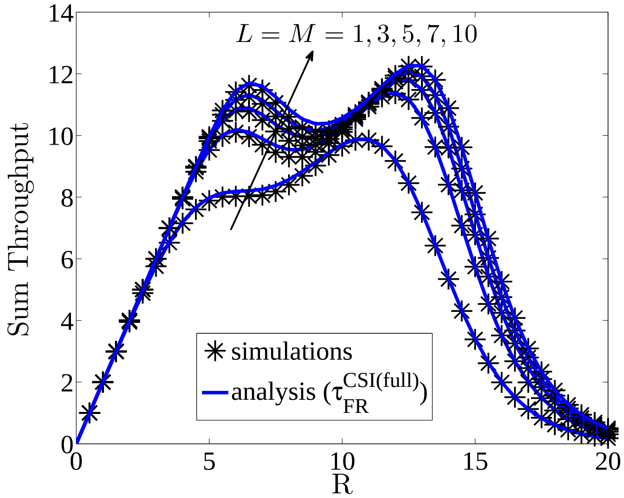

Fig. 6: $\tau_{FR}^{CSI(full)}$ vs R with $L = M = 1, 3, 5, 7, 10$.

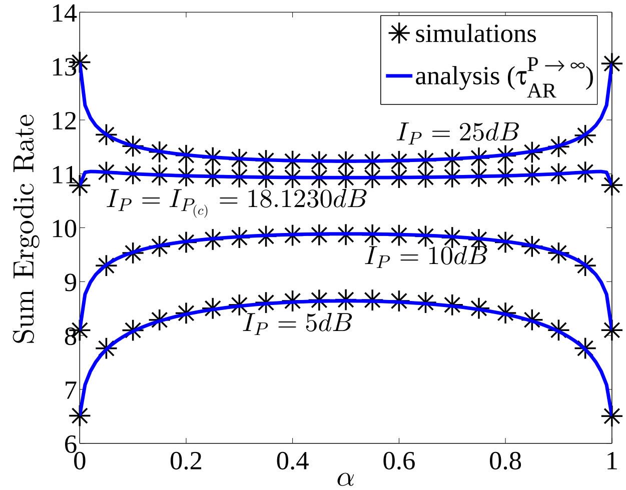

Fig. 7: $\tau_{AR}^{P\to\infty}$ vs α for $I_P = 5dB, 10dB, I_{P_{(c)}}, 25dB$.

γ_{th} ($\gamma_{th} = 2^R - 1$). Clearly, when $\Gamma_1^{P\to\infty} = \gamma_{th}$, on equating, $\alpha = \alpha_1 = \frac{\frac{\gamma_{th}\sigma_n^2}{I_P} + \gamma_{th}\frac{|g_{2i^*}|^2}{|g_{2P}|^2}}{\frac{X}{|g_{1P}|^2} + \gamma_{th}\frac{|g_{2i^*}|^2}{|g_{2P}|^2}}$. When $\Gamma_2^{P\to\infty} = \gamma_{th}$, on equating, $\alpha = \alpha_2 = \frac{-\frac{\gamma_{th}\sigma_n^2}{I_P} + \frac{Y}{|g_{2P}|^2}}{\frac{Y}{|g_{2P}|^2} + \gamma_{th}\frac{|g_{1i^*}|^2}{|g_{1P}|^2}}$. Thus, α_1 and α_2 are random quantities.

Case 1: In a given realization when $\alpha_1 \leq \alpha_2$, and if α is chosen such that it lies between α_1 and α_2, then both secondary networks meet their outage requirements ($\Gamma_1^{P\to\infty} \geq \gamma_{th}$ and $\Gamma_2^{P\to\infty} \geq \gamma_{th}$) and they can be operated simultaneously. Whereas if α is chosen to be less than α_1 or greater than α_2, only one of the secondary networks meet its outage requirement and one at most can be operational. Since α_1 and α_2 are perfectly known, the CANM chooses an α that would always lie between α_1 and α_2, making both secondary networks operational together.

Case 2: In a given realization when $\alpha_1 > \alpha_2$, and if α is chosen such that it lies between α_1 and α_2, both secondary networks are in outage, as $\Gamma_1^{P\to\infty} < \gamma_{th}$ and $\Gamma_2^{P\to\infty} < \gamma_{th}$ events occur together. Whereas if α is chosen to be less than α_2, then *only* S_2 becomes operational ($\Gamma_2^{P\to\infty} > \gamma_{th}$), while *only* S_1 becomes operational ($\Gamma_1^{P\to\infty} > \gamma_{th}$) when α is chosen to be greater than α_1. Both α_1 and α_2 do not consistently lie in between 0 and 1. For $\gamma_{th} > 0$, α_1 is always positive and has a tendency to exceed 1 for high target rates. Similarly α_2 is always less than 1 and has a tendency to become negative for high target rates. Thus with such an event (both $\alpha_1 > 1$ and $\alpha_2 < 0$), an outage is said to have occurred. From Case 2 it is clear that when $\alpha_1 > \alpha_2$, either one of the users can be operational, or there can be an outage event with $\alpha_1 > 1$ and $\alpha_2 < 0$ occurring together. Hence, when $\alpha_1 > \alpha_2$, the CANM strategy either chooses α less than α_2 or α greater than α_1, where both secondary networks offer the same throughput with FRT.

APPENDIX B: PROOF OF LEMMA 4.2

The sum throughput expression $\tau_{FR}^{CSI(full)}$ can be defined as follows:

$$\tau_{FR}^{CSI(full)} = \Pr\{\alpha_1 \leq \alpha_2\}2R + \Big[\Pr\{\alpha_1 > \alpha_2\} - \Pr\{\alpha_1 > \alpha_2, \alpha_1 > 1, \alpha_2 < 0\}\Big]R. \quad (33)$$

Since $\alpha_1 > \alpha_2$ is always true with $\alpha_1 > 1$ and $\alpha_2 < 0$ (see Appendix A), and $\Pr\{\alpha_1 > \alpha_2\} = 1 - \Pr\{\alpha_1 \leq \alpha_2\}$, the final expression for $\tau_{FR}^{CSI(full)}$ simplifies to:

$$\tau_{FR}^{CSI(full)} = \Big[1 + \Pr\{\alpha_1 \leq \alpha_2\} - \Pr\{\alpha_1 > 1, \alpha_2 < 0\}\Big]R. \quad (34)$$

B-1) Evaluation of $\Pr\{\alpha_1 \leq \alpha_2\}$*:* With $\frac{\gamma_{th}\sigma_n^2}{I_P}$ assumed to be reasonably small (see α_1 and α_2 defined in Appendix A) for practical target rates, $\Pr\{\alpha_1 \leq \alpha_2\}$ is evaluated as:

$$\Pr\{\alpha_1 \leq \alpha_2\} \approx \Pr\left\{\left(1 + \frac{X|g_{2P}|^2}{|g_{1P}|^2|g_{2i^*}|^2\gamma_{th}}\right)^{-1} \leq \left(1 + \frac{\gamma_{th}|g_{2P}|^2|g_{1i^*}|^2}{Y|g_{1P}|^2}\right)^{-1}\right\} = \Pr\{XY \geq \gamma_{th}^2|g_{1i^*}|^2|g_{2i^*}|^2\}. \quad (35)$$

Clearly, the CDF-s of X and Y are $F_X(x) = (1 - e^{-\lambda_{11}x})^L$ and $F_Y(y) = (1 - e^{-\lambda_{22}y})^M$, which after binomial expansion and differentiation, yield probability density functions (PDF-s) $f_X(x) = \sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\lambda_{11}je^{-\lambda_{11}jx}$ and $f_Y(y) = \sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\lambda_{22}ke^{-\lambda_{22}ky}$ respectively. Thus, (35) can

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be written in terms of the complementary CDF of X as:

$$\Pr\{\alpha_1 \le \alpha_2\} = \mathbb{E}\left[\sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}e^{-\lambda_{11}j\gamma_{th}^2\frac{|g_{1i^*}|^2|g_{2i^*}|^2}{Y}}\right], \tag{36}$$

where $\mathbb{E}$ denotes expectation over random variables $|g_{1i^*}|^2$, Y and $|g_{2i^*}|^2$. By successive averaging using standard integrals [29, eq.(3.352.4)] and [29, eq.(5.221.5)], (36) is evaluated finally as:

$$\Pr\{\alpha_1 \le \alpha_2\} = 1 - \sum_{k=1}^{M}\sum_{j=1}^{L}\binom{M}{k}\binom{L}{j}(-1)^{j+k} \frac{\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2}{\left[1-\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2\right]^2}\left\{\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2 - \ln\left(\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2\right)-1\right\}. \tag{37}$$

B-2) Evaluation of $\Pr\{\alpha_1 > 1, \alpha_2 < 0\}$*:* Upon substitution of α_1 and α_2 (from Appendix A) in $\Pr\{\alpha_1 > 1, \alpha_2 < 0\}$, we get,

$$\begin{aligned}\Pr\{\alpha_1 > 1, \alpha_2 < 0\} &= \Pr\left\{\frac{I_P X}{\sigma_n^2|g_{1P}|^2} < \gamma_{th}, \frac{I_P Y}{\sigma_n^2|g_{2P}|^2} < \gamma_{th}\right\} \\ &= \left[\sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}I_P}}{1+\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}I_P}}\right] \left[\sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}I_P}}{1+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}I_P}}\right].\end{aligned} \tag{38}$$

By substituting (37) and (38) in (34), the final expression of $\tau_{FR}^{CSI(full)}$ results, as shown in (22).

APPENDIX C: PROOF OF LEMMA 4.4

Similar to (34), we formulate $\tau_{FR}^{CSI(partial)}$ as follows:

$$\tau_{FR}^{CSI(partial)} \approx \left[1+\Pr\{\widehat{\alpha}_1 \le \widehat{\alpha}_2\} - \Pr\{\widehat{\alpha}_1 > 1, \widehat{\alpha}_2 < 0\}\right]R. \tag{39}$$

C-1) Evaluation of $\Pr\{\widehat{\alpha}_1 \le \widehat{\alpha}_2\}$*:* With $\frac{\gamma_{th}\sigma_n^2}{I_P}$ assumed to be reasonably small for practical target rates (see $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$ expressions in the proof of Lemma 4.3), $\Pr\{\widehat{\alpha}_1 \le \widehat{\alpha}_2\}$ is evaluated as:

$$\begin{aligned}\Pr\{\widehat{\alpha}_1 \le \widehat{\alpha}_2\} &\approx \Pr\left\{\left(1+\frac{X|g_{2P}|^2\mu_{21}}{|g_{1P}|^2\gamma_{th}}\right)^{-1} \le \left(1+\frac{\gamma_{th}|g_{2P}|^2}{Y|g_{1P}|^2\mu_{12}}\right)^{-1}\right\} \\ &= \Pr\left\{XY \ge \frac{\gamma_{th}^2}{\mu_{12}\mu_{21}}\right\}.\end{aligned} \tag{40}$$

Rewriting (40) in terms of the complementary CDF of X, we get,

$$\Pr\{\widehat{\alpha}_1 \le \widehat{\alpha}_2\} = \mathbb{E}\left[\sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}e^{-\frac{\lambda_{11}j\gamma_{th}^2}{\mu_{12}\mu_{21}Y}}\right]. \tag{41}$$

$\mathbb{E}$ denotes expectation over random variable Y. Using the standard integral [29, eq.(3.324.1)], the final expression of $\Pr\{\widehat{\alpha}_1 \le \widehat{\alpha}_2\}$ can be evaluated and finally written as:

$$\Pr\{\widehat{\alpha}_1 \le \widehat{\alpha}_2\} = \sum_{k=1}^{M}\sum_{j=1}^{L}\binom{M}{k}\binom{L}{j}(-1)^{j+k} 2\sqrt{\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2}K_1\left(2\sqrt{\frac{\lambda_{11}\lambda_{22}}{\mu_{12}\mu_{21}}jk\gamma_{th}^2}\right), \tag{42}$$

where $K_1(.)$ is the modified Bessel's function of the second kind.

C-2) Evaluation of $\Pr\{\widehat{\alpha}_1 > 1, \widehat{\alpha}_2 < 0\}$*:* Substituting expressions of $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$ in $\Pr\{\widehat{\alpha}_1 > 1, \widehat{\alpha}_2 < 0\}$ results in the same probability expression $\Pr\{\alpha_1 > 1, \alpha_2 < 0\}$ as in (38). Thus,

$$\begin{aligned}\Pr\{\widehat{\alpha}_1 > 1, \widehat{\alpha}_2 < 0\} &= \Pr\left\{\frac{I_P X}{\sigma_n^2|g_{1P}|^2} < \gamma_{th}, \frac{I_P Y}{\sigma_n^2|g_{2P}|^2} < \gamma_{th}\right\} \\ &= \left[\sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}I_P}}{1+\frac{\lambda_{11}j\gamma_{th}\sigma_n^2}{\mu_{1P}I_P}}\right] \left[\sum_{k=1}^{M}\binom{M}{k}(-1)^{k+1}\frac{\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}I_P}}{1+\frac{\lambda_{22}k\gamma_{th}\sigma_n^2}{\mu_{2P}I_P}}\right].\end{aligned} \tag{43}$$

By substituting (42) and (43) in (39), a final expression of $\tau_{FR}^{CSI(partial)}$ results, as shown in (24).

APPENDIX D: PROOF OF LEMMA 5.1

D-1) Derivation of R_1*:* From (25), we rewrite R_1 as:

$$R_1 = \mathbb{E}_{U,V}\left[\log_2\left(1+\frac{U}{V+1}\right)\right], \tag{44}$$

where, $U = \frac{\alpha I_P}{\sigma_n^2}\max_{i\in[1,L]}[|h_{1i}|^2]/|g_{1P}|^2$ and $V = \frac{(1-\alpha)I_P}{\sigma_n^2}|g_{2i^*}|^2/|g_{2P}|^2$. Their PDF-s can be evaluated and written as follows:

$$\begin{aligned} f_U(u) &= \frac{\alpha I_P}{\sigma_n^2}\sum_{j=1}^{L}\binom{L}{j}(-1)^{j+1}\frac{\frac{\mu_{1P}}{\lambda_{11}j}}{(u+\frac{\alpha I_P}{\sigma_n^2}\frac{\mu_{1P}}{\lambda_{11}j})^2}, \\ f_V(v) &= \frac{(1-\alpha)I_P}{\sigma_n^2}\frac{\frac{\mu_{2P}}{\mu_{21}}}{(v+\frac{(1-\alpha)I_P}{\sigma_n^2}\frac{\mu_{2P}}{\mu_{21}})^2}.\end{aligned} \tag{45}$$

In [34], it was shown that a generalized ergodic rate of the form $\mathbb{E}_{U,V}\left[\log_2\left(1+\frac{U}{V+1}\right)\right]$ can be evaluated using the following moment generating function (MGF) approach:

$$\mathbb{E}_{U,V}\left[\log_2\left(1+\frac{U}{V+1}\right)\right] = \int_0^\infty \frac{M_V(z)-M_{U,V}(z)}{\ln(2)z}e^{-z}dz, \tag{46}$$

where $M_V(z)$ is the MGF of V and $M_{U,V}(z)$ is the joint MGF of U and V. The two MGF-s can be evaluated as $M_V(z) = \mathbb{E}_V\left[e^{-zV}\right]$ and $M_{U,V}(z) = \mathbb{E}_{U,V}\left[e^{-z(U+V)}\right]$ respectively. By defining constants $a_1 = \frac{(1-\alpha)I_P\mu_{2P}}{\mu_{21}\sigma_n^2}$, $b_1 = \frac{\alpha I_P \mu_{1P}}{\lambda_{11}\sigma_n^2}$ in (45) and using [29, eq.(3.353.3)], we derive $M_V(z)$ and $M_{U,V}(z)$ as:

$$M_V(z) = a_1 \int_0^\infty \frac{e^{-zv}}{(v+a_1)^2} dv = 1 - a_1 z e^{a_1 z} E_1[a_1 z], \quad (47)$$

$$\begin{aligned} M_{U,V}(z) &= \int_0^\infty \int_0^\infty \sum_{j=1}^{L} \binom{L}{j} (-1)^{j+1} \frac{b_1}{j} a_1 \frac{e^{-z(u+v)} du dv}{(u+\frac{b_1}{j})^2 (v+a_1)^2} \\ &= \left\{ 1 - a_1 z e^{a_1 z} E_1[a_1 z] \right\} \\ &\quad \left\{ 1 - \sum_{j=1}^{L} \binom{L}{j} (-1)^{j+1} \frac{b_1}{j} z e^{\frac{b_1}{j} z} E_1\left[\frac{b_1}{j} z\right] \right\}. \quad (48) \end{aligned}$$

By substituting (47) and (48) in (46) and rearranging terms, R_1 can be rewritten as:

$$\begin{aligned} \mathbb{E}_{U,V}\left[\log_2\left(1+\frac{U}{V+1}\right)\right] &= \frac{1}{\ln(2)} \sum_{j=1}^{L} \binom{L}{j} (-1)^{j+1} \\ &\quad \Bigg[\underbrace{\int_0^\infty \left\{ \frac{b_1}{j} e^{(\frac{b_1}{j}-1)z} E_1\left[\frac{b_1}{j} z\right] \right\} dz}_{I_1} \\ &\quad \underbrace{- \int_0^\infty \left\{ \frac{a_1 b_1}{j} z e^{-(1-a_1-\frac{b_1}{j})z} E_1[a_1 z] E_1\left[\frac{b_1}{j} z\right] \right\} dz}_{I_2} \Bigg]. \quad (49) \end{aligned}$$

Using integrals [30, eq.(4.2.4)] and [30, eq.(4.6.25)], I_1 and I_2 in (49) are evaluated as follows:

$$\begin{aligned} I_{11} &= \frac{b_1}{j-b_1} \ln(j/b_1), \\ I_{12} &= \frac{a_1 b_1/j}{\left(1-a_1-\frac{b_1}{j}\right)^2} \left[\frac{\pi^2}{6} - Li_2(a_1) - Li_2\left(\frac{b_1}{j}\right) \right. \\ &\quad + \ln(a_1) \ln\left(\frac{b_1}{j}\right) - \left\{ \ln\left(1-\frac{b_1}{j}\right) + \frac{a_1}{1-\frac{b_1}{j}} - 1 \right\} \\ &\quad \left. \ln\left(\frac{b_1}{j}\right) - \left\{ \ln(1-a_1) + \frac{b_1/j}{1-a_1} - 1 \right\} \ln(a_1) \right], \quad (50) \end{aligned}$$

where $Li_2(x) = -\int_0^x \frac{\ln(1-t)}{t} dt$ denotes the Euler-Dilogarithm function. The Euler-Dilogarithm function can also be expressed in terms of Spence's Integral, $Di_2(x) = -\int_1^x \frac{\ln(t)}{t-1} dt$, where $Di_2(1-x) = Li_2(x)$. We shall use both interchangeably in the analysis. Using (50), (49) in (44) and rearranging terms, we write R_1 as a difference of two summation terms:

$$\begin{aligned} R_1 &= \sum_{j=1}^{L} \binom{L}{j} (-1)^{j+1} \left[\frac{b_1}{j-b_1} \log_2(j/b_1) + \frac{a_1 b_1/j}{(1-a_1-\frac{b_1}{j})^2} \right. \\ &\quad \left. \left\{ \left(\frac{a_1}{1-\frac{b_1}{j}} - 1 \right) \log_2(\frac{b_1}{j}) + \left(\frac{b_1/j}{1-a_1} - 1 \right) \log_2(a_1) \right\} \right] \\ &\quad - \sum_{j=1}^{L} \binom{L}{j} (-1)^{j+1} \frac{a_1 b_1/j}{(1-a_1-\frac{b_1}{j})^2} \left[\frac{\pi^2}{6} - Di_2(1-a_1) \right. \\ &\quad - Di_2(1-\frac{b_1}{j}) + \ln(a_1)\ln(\frac{b_1}{j}) - \ln(1-\frac{b_1}{j})\ln(\frac{b_1}{j}) \\ &\quad \left. - \ln(1-a_1)\ln(a_1) \right] / \ln(2). \quad (51) \end{aligned}$$

The first summation term in (51) can further be simplified algebraically. Using identities [35, eq.(12)] and [35, eq.(6)], the second summation term can be expressed in terms of four Dilogarithm terms. This results in the final analytical expression of R_1 as shown in (26). With $\Gamma_1^{P\to\infty}$ and $\Gamma_2^{P\to\infty}$ having identical forms, R_2 is derived similarly as R_1 using constants $a_2 = \frac{\alpha I_P \mu_{1P}}{\mu_{12}\sigma_n^2}$ and $b_2 = \frac{(1-\alpha) I_P \mu_{2P}}{\lambda_{22}\sigma_n^2}$ in place of a_1 and b_1, whose final expression is presented in (27).

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