

## Leaning Tower of Lire

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NOTES AND DISCUSSION

Leaning Tower of Lire

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EVERY miser knows that a stack of pennies can be "leaned" slightly off vertical without falling. How far can the top penny be from its position in a vertical stack?

Let us number the coins in a stack of  $(n+1)$  coins from the top down. The stack will not fall if the downward projection of the center of gravity of the top  $r$  coins taken as a group falls on or inside the rim of the  $(r+1)$ st coin. This must be true for  $r=1,2,\dots,n$ . Let the shift of the center of the  $i$ th coin, relative to the center of the  $(i+1)$ st coin be  $d_i$  (see Fig. 1). It is evident that the centers of the coins must lie in a plane for the maximum shift of the top coin. Hence the shift of the  $i$ th coin relative to the center of the  $(r+1)$ st coin is

$$d(i, r+1) = d_i + d_{i+1} + \dots + d_{r-1} + d_r. \tag{1}$$

Let the center of gravity of the top  $r$  coins be shifted a distance  $E_r$  relative to the center of the  $(r+1)$ st coin. Then, if we assume the coins to have unit mass and unit radius,  $E_r$  satisfies the equations

$$rE_r = d(1, r+1) + d(2, r+1) + \dots + d(r, r+1) \tag{2}$$

and

$$rE_r = d_1 + 2d_2 + 3d_3 + \dots + rd_r \text{ for } r=1,2,\dots,n. \tag{3}$$

We could solve Eqs. (3<sub>r</sub>) for the unknown shifts  $d_r$ . However, we are mainly interested in  $d(1, n+1)$ , the total shift of the top coin from its position in a vertical stack. We seek to solve Eqs. (3<sub>r</sub>) for  $d(1, n+1)$  by multiplying each of Eqs. (3<sub>r</sub>) by a suitable constant and adding the products. It turns out that we need to multiply Eqs. (3<sub>r</sub>) by  $1/r(r+1)$  for  $r=1,2,\dots,n-1$  and multiply Eq. (3<sub>n</sub>) by  $1/n$ . Adding the resulting products gives

$$d(1, n+1) = E_n + E_n/2 + E_n/3 + E_n/4 + \dots + E_n/n. \tag{4}$$

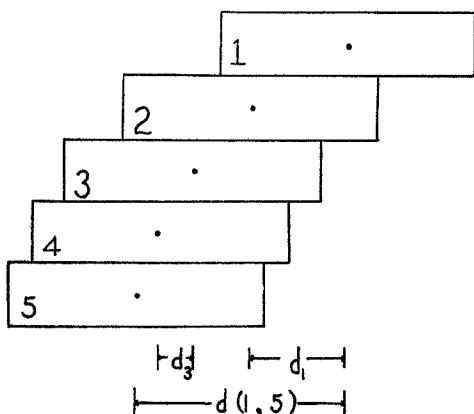


FIG. 1. The notation used in describing the stack of coins.

But  $E_r$  must not be greater than unity, the radius of the coin. Hence if the coins are stacked with maximum shift at each stage, all  $E_r=1$  and

$$d(1, n+1) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + 1/n. \tag{5}$$

This is the famous harmonic series whose sum can be made as large as we please by choosing  $n$  sufficiently large. Hence we can shift the top coin as far as we please from its position in a vertical stack by properly stacking enough coins.

We note that no possible alternative stacking will yield a shift greater than that given by Eq. (5).

Diffusion Cloud Chamber for Research at Small Colleges\*

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ONE of the primary reasons why research is not often attempted at smaller schools is the lack of funds to purchase the necessary, and usually expensive, equipment. This laboratory has undertaken the project<sup>1</sup> of investigating the usefulness of the diffusion cloud chamber<sup>2</sup> as a basic research tool for small colleges. The idea was to construct a chamber as cheaply as possible and yet one which is sufficiently accurate to carry on fundamental research. This letter reports the progress achieved during the past year. Construction details and procedure will be reported in a later paper.

It was felt that a good experiment to carry out the purpose of the investigation was obtaining of a beta spectrum and comparing it was the spectrum from the more elaborate and expensive magnetic-beta spectroscopes. Accordingly, some lead-chloride crystals (obtained from Central Scientific Company) which contained radioactive  $Pb^{210}$  were prepared as a sample.  $Pb^{210}$  has a beta end-point energy of 0.025 Mev and, by giving off a beta ray, becomes  $Bi^{210}$  with an end-point energy of 1.17 Mev. Therefore, the sample used was a mixture of  $Pb^{210}-Bi^{210}$ . With this sample 430 cloud-chamber pictures were taken using a Contax II camera and 35-mm film. Of the 430 pictures, 326 had 510 measurable beta tracks. The radius of curvature and the magnetic field for each track were measured. Using a gauss-centimeter interval of 100, a beta spectrum curve of  $Bi^{210}$  was drawn (Fig. 1). There seem to be two definite peaks in the spectrum. The first peak is at exactly the end-point energy of  $Pb^{210}$ . The second peak corresponds to the types usually associated with internal conversion of a gamma ray.

With the help of the *Tables for Analysis of Beta Spectrum*<sup>3</sup> the end-point energy of the spectrum was determined by two methods. For the first method, in which  $\sqrt{(N/n^2)}$  was plotted against  $\epsilon$ , the end-point energy was 1.05 Mev with an error of 9.8 percent (considering 1.17 as maximum energy of  $Bi^{210}$ ). Here  $N$  is the number of tracks observed,