Total Marks: 15
Due Date: October 8, 2019,
Tuesday 5 pm.

PHY 300/500
October 2, 2019

Homework 3

Problem 1

Levenberg-Marquardt scheme

1. In the first part of this question, I would like you to generate an exponentially decaying data,
$$5\,e^{-0.2t}, \tag{1}$$
where $t \in [0, 10]$. Add random Gaussian noise so that you obtain a noisy decaying exponential. Plot this decay.

2. We would like to fit this fabricated data to an exponential decay,
$$x_1 e^{-x_2 t}, \tag{2}$$
where the unknown parameters are $\vec{\mathbf{x}} = (x_1, x_2)$. Now I would like you to write a computer program employing the Levenberg-Marquardt scheme, which specifies that the parameters are updated according to
$$\mathbf{x}_{k+1} = \mathbf{x}_k - \big(H(x_k) + \beta\ \mathrm{diag}\,(H(x_k))\big)^{-1}\vec{\nabla}\chi^2. \tag{3}$$
Find χ^2, $\vec{\nabla}\chi^2$, and the Hessian $H(\vec{x})$, and where diag(H) is a fixed diagonal matrix with the same elements as H.
Note that in the Levenberg-Marquardt algorithm, the value of β is also changed iteratively. Implement the following scheme.
   (a) Pick an initial guess for $\mathbf{x}$, and $\beta = 0.01$ (say).
   (b) Find $\chi^2(\vec{\mathbf{x}})$.
   (c) Compute $\delta\vec{\mathbf{x}} = -(H + \beta\ \mathrm{diag}(H))^{-1}\vec{\nabla}\chi^2$.
   (d) Find $\chi^2(\vec{\mathbf{x}} + \delta\vec{\mathbf{x}})$.
   (e) If $\chi^2(\vec{\mathbf{x}} + \delta\vec{\mathbf{x}}) \geq \chi^2(\vec{\mathbf{x}})$, increase β by a factor of 10 and go back to step (c).
   (f) If $\chi^2(\vec{\mathbf{x}}+\delta\vec{\mathbf{x}}) < \chi^2(\vec{\mathbf{x}})$, decrease β by a factor of 10 and update the trial $\vec{\mathbf{x}} \to \vec{\mathbf{x}}+\delta\vec{\mathbf{x}}$ and go to step (c).
   (g) Terminate if χ^2 does not change by a value greater than some threshold, e.g. 10^{-10} etc.

3. What is your best estimate of the parameter $\vec{\mathbf{x}}$?

Problem 2

Question 7.5 from Hughes, "Measurements and their Uncertainties".

Homework 3

Problem 3

Question 7.7 from Hughes, "Measurements and their Uncertainties".

Problem 4

Question 7.8 from Hughes, "Measurements and their Uncertainties".

Problem 5

A student varies the orientation of a polarizer in steps of $10°$, starting from $0°$ and going up to $270°$. Polarized laser light passes through the polarizer and falls on a detector which measures the following intensities (in the same order and in arbitrary units).

$$\begin{aligned}\text{Intensity} = \{&0.0558, -0.0274, -0.1127, -0.2253, -0.2098, -0.2725, -0.3086, -0.2611, -0.2681, \\ &-0.2607, -0.1492, -0.0891, -0.0157, 0.0506, 0.1276, 0.1229, 0.2285, 0.2074, 0.1596, \\ &0.1071, 0.0166, -0.0053, -0.1177, -0.2067, -0.2031, -0.2898, -0.3173, -0.3006\}.\end{aligned}$$

I propose that a suitable fit for the data-set is

$$\text{Intensity} = A + B\cos^2(C\theta + D),$$

where θ is the polarizer angle in radians. Use simulated annealing to find the best estimate of $\{A, B, C, D\}$. Decrease the temperature, by say, 1% at each step of the iterative annealing process.