Total Marks: 15
Due Date: October 1, 2019,
Tuesday 1 pm.

Homework 2

PHY 300/500
September 24, 2019

Problem 1

Gradient descent method

A student in our lab performs an experiment on the Hall effect. A sample is placed in a fixed magnetic field. She takes a series of measurements of the current I and the corresponding Hall voltage V. These measurements are given below,

$$I\ (mA) = \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0\}$$
$$V\ (V) = \{0.2, 0.4, 0.6, 0.9, 1.1, 1.2, 1.4, 1.7, 1.8, 2.1, 2.2, 2.5, 2.7, 2.9, 3.1, 3.4, 3.5, 3.9, 4.1, 4.3\}$$

The uncertainty in each reading of the Hall voltage is 0.1 V.

1. A linear fit is proposed to model the data. Write down an expression for the χ^2 statistic. Use a computer program to make a contour plot.
2. Analytically determine the gradient of the χ^2 variable . Write a computer program that uses gradient descent to minimize χ^2. What are the optimal values of the parameters describing the data?
Hint: Use the norm of the slope at each parameter value.
3. Show the evolution of the parameters, as the algorithm iterates, on the contour plot from part 1.
4. Furthermore, make plots of χ^2 vs. the iteration count.
5. Investigate the impact of varying the step size.
6. Using **PhysPlot**, load the dataset and perform least squares curve fitting. What are the values of the best-fit parameters?

Problem 2

Newton's technique

In Newton's method, the minimum in the parameter space $\{\mathbf{x}\}$ is found through the update equation

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \big(\mathbf{H}(\bar{\mathbf{x}}_k)\big)^{-1}\vec{\nabla}\chi^2(\bar{\mathbf{x}}_k),$$

where $\vec{\nabla}\chi^2(\mathbf{x}_k)$ is the normalized gradient in the parameter space, computed at the current values of the parameter $\bar{\mathbf{x}}_k$, $H(x_k)$ is the Hessian,

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$$H(\mathbf{x}_k) = \left.\begin{pmatrix} \frac{\partial^2 \chi^2}{\partial x_1^2} & \cdots & \frac{\partial^2 \chi^2}{\partial x_1 \ldots \partial x_m} \\ & \vdots & \\ \frac{\partial^2 \chi^2}{\partial x_1 \ldots \partial x_m} & \cdots & \frac{\partial^2 \chi^2}{\partial x_m^2} \end{pmatrix}\right|_{\mathbf{x_k}},$$

and the superscript (-1) denotes the matrix inverse. Solve problem 1 using Newton's technique.