Problem 1

Identifying a potential outlier
Seven successive measurements of the charge stored on a capacitor(μ C) are: $45.7, 53.2, 48.4,$
$45.1, 51.4, 62.1$ and 49.3. The sixth reading appears anomalously large. Apply Chauvenets
criterion to ascertain whether this data point should be rejected. Having decided whether to
keep six or seven data points, calculate the mean, standard deviation and standard error of
the charge.

Problem 2

Uniform distribution
A probability distribution function of interest in error analysis is the uniform distribution.
It is dened as,
$$P_U(x; \bar{x}, a) = \left\{ \begin{array}{ll} 1/a, & \text{if } \bar{x} - a/2 \le x \le \bar{x} + a/2 \\ 0, & \text{otherwise.} \end{array} \right\}$$
Here the parameter $\bar{x}$ is the mean of the distribution, and a is the interval in which the
probability distribution is uniform. Show that (i) the distribution $P_U(x; \bar{x}, a)$ is normalised;
(ii) the mean of the distribution is indeed $\bar{x}$; (iii) the standard deviation is given by $\sigma = a/\sqrt{12}$.

Problem 3

We define the standard deviation δ of a data-set x_i by,
$$s^2 = \mathbb{E}\big[(x_i - \bar{x})^2\big],$$
where $\mathbb{E}$ represents the expectation value and $\bar{x} = \mathbb{E}[x_i]$. Show that for the standard error of
the mean,
$$\sigma^2 = \mathbb{E}\big[(\bar{x} - X_T)^2\big],$$
the relationship between s and σ is
$$\sigma = \sqrt{\frac{N}{N-1}} \; s, \tag{1}$$
where X_T is the true value and N is the number of points in the data-set.

Problem 4

X is a normally distributed random variable. This is sometimes denoted as $X \sim N(\mu, \sigma^2)$,
where μ and σ^2 are the mean and variance respectively. If $Y = e^X$, calculate the probability

$P(Y \leq y)$. Express your answer in the form of an integral. Obtain a numerical value for some y, μ and σ. (Use Matlab or Mathematica or any other software.)

Problem 5

For a Poisson distribution

$$\mathbb{P}(n; \bar{n}) = \frac{1}{n!} e^{-\bar{n}} \bar{n}^n$$

show that

1. the sum of all the probabilities for $n = 0, 1, 2, 3, \cdots \infty$ is 1.

2. average n is indeed $\bar{n}$.

3. the variance of n is also $\bar{n}$.

4. In the limit of large $\bar{n}$, the Poison distribution approaches a Gaussian distribution. For this part you may need to use Stirling's approximation for large $\bar{n}$:

$$\log n! \approx n \log n - n \quad \text{and}$$
$$\frac{|n - \bar{n}|}{\bar{n}} \ll 1$$

Problem 6

Suppose X is an independent, identically distributed random variable with a uniform distribution in the interval $[0,1]$. Take the sum

$$S_N = X_1 + X_1 + \cdots + X_N.$$

Show numerically (on a computer) how the distribution of S_N approaches a normal distribution as N increases. For this question, I would like to see a narrative description on how you generate data, what are the plots and how do you interpret them. Generate a diagram similar to Figure 3.7 of Hughes's book. (See overleaf.)

Problem 7

Suppose that X has a probability distribution function:

$$p(x) = \frac{1}{2} \sin x \text{ for } x \in [0, \pi]$$

and is zero elsewhere. Calculate its mean, variance, skewness, and kurtosis. Also, compare it with the skewness and kurtosis of a normal distribution with the same mean and variance.

Total Marks: 15
Due Date: September 19, 2019,
Thursday 11 am.

Homework 1

PHY 300/500
September 13, 2019