Electromagnetic induction and damping: Quantitative experiments using a PC interface

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A bar magnet, attached to an oscillating system, passes through a coil periodically, generating a series of electromotive force pulses. A novel method for the quantitative verification of Faraday's law is described which eliminates all errors associated with angular measurements, thereby revealing subtle features of the underlying mechanics. When electromagnetic damping is activated by short-circuiting the coil, a distinctly linear decay of the oscillation amplitude is observed. A quantitative analysis reveals an interesting interplay of the electromagnetic and mechanical time scales.
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I. INTRODUCTION

Laboratory experiments on Faraday's law of electromagnetic induction most often involve a bar magnet moving (falling) through a coil and a measurement of the induced electromotive force (emf) pulse.[1–4] Several parameters can be varied, such as the velocity of the magnet, the number of turns in the coil, and the strength of the bar magnet. The observed proportionality of the peak induced emf on the number of turns in the coil and the magnet velocity provides a quantitative verification of the Faraday's law.

It usually is convenient to attach the magnet to an oscillating system, so that it passes through the coil periodically, generating a series of emf pulses. This arrangement allows the peak emf to be easily determined by charging a capacitor with the rectified coil output. A simple, yet remarkably robust setup that utilizes this concept involves a rigid semicircular frame of aluminum of radius R pivoted at the center (O) of the circle (see Fig. 1). The whole frame can oscillate freely in its own plane about a horizontal axis passing through O. A rectangular bar magnet is mounted at the center of the arc and the arc passes through a coil C of suitable cross-sectional area.[5] The positions of the weights W_1 and W_2 can be adjusted to bring the mean position of the bar magnet vertically below the pivot O, and the position of the coil is adjusted so that its center coincides with this mean position ($\theta=0$) of the magnet. The angular amplitude can be read by means of a scale and pointer. The magnet velocity can be controlled by choosing different angular amplitudes, allowing the magnetic flux to be changed at different rates.

It is much more instructive to monitor the induced emf in the coil through a PC interface, which can be readily realized by a low-cost, convenient data-acquisition module. We have used a module based on the serial interface COBRA-3 and its accompanying software.[6] In this article we describe experiments designed to take advantage of the computer interface. These experiments allow for a quantitative and pedagogical study of the angular (position) dependence of the magnetic flux through the coil, verification of Faraday's law of induction, and electromagnetic damping, thereby revealing subtle features of the underlying mechanics.

II. INDUCED emf PULSE

The equation for the induced emf $\mathcal{E}(t)$ as a function of time t can be written as

$$\mathcal{E}(t)=\frac{d\Phi}{dt}=\frac{d\Phi}{d\theta}\frac{d\theta}{dt}, \tag{1}$$

expressing the combined dependence on the angular gradient $d\Phi/d\theta$ and the angular velocity $\omega(\theta)=d\theta/dt$. This combined dependence is reflected in the time dependence of $\mathcal{E}(t)$, and a typical emf pulse is shown in Fig. 2; the pulse shape is for one half cycle of oscillation, starting from the extreme position of the bar magnet ($\theta=\theta_0$). As the magnet approaches the coil, the induced emf initially rises, then turns over and starts falling as the magnet enters the coil and the magnetic flux begins to saturate, and finally changes sign as the magnet crosses the center of the coil ($\theta=0$) where the flux undergoes a maximum. Thus $\mathcal{E}$ vanishes at the point where the angular velocity of the magnet is maximum.

From Fig. 2 the magnitude of $\mathcal{E}$ is seen to be significant only in a very narrow time interval of about 200 ms, which is much smaller than the oscillation time period ($T\approx 2$ s). This large difference in the two time scales implies that the magnetic flux through the coil falls off very rapidly as the magnet moves away from its center, so that $d\Phi/d\theta$ is significant only in a very narrow angular range (typically about 5°) on either side of the mean position. Because $d\Phi/d\theta=0$ at $\theta=0$, it follows that $d\Phi/d\theta$ is strongly peaked very close to the mean position, which accounts for the emf pulse shape seen in Fig. 2. This separation of the electromagnetic and mechanical time scales has interesting consequences for the electromagnetic damping, as discussed in Sec. IV.

III. MAGNETIC FLUX THROUGH THE COIL

To quantitatively study the magnetic flux through the coil, the `integrate` feature of the software is especially convenient. From Eq. (1) the time integral of the induced emf directly yields the change in the magnetic flux, $\Delta\Phi$, corresponding to the limits of integration. If the lower limit of integration t_i corresponds to the extreme position of the magnet, $\theta(t_i)=\theta_0$,[7] where the magnetic flux through the coil is negligible (valid only for large θ_0), the magnetic flux $\Phi(\theta)$ for different angular positions $\theta(t)$ of the magnet is obtained as

$$\Phi(t)\approx\int_{t_i}^{t}\mathcal{E}(t')\,dt'. \tag{2}$$

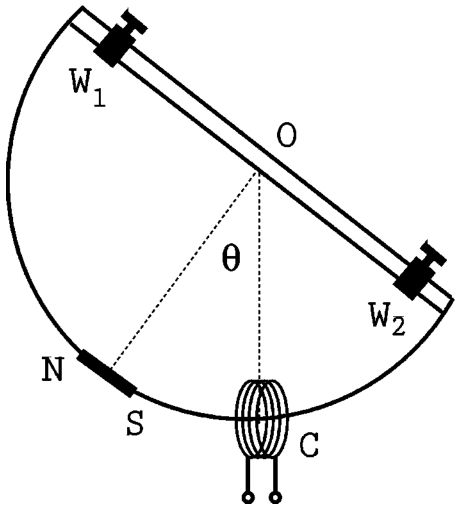

Fig. 1. A magnet (NS) attached to an oscillating system passes through a coil (C) periodically, generating a series of emf pulses.

Figure 3 shows a plot of $\Phi(t)$ vs t for a large angular amplitude ($\theta_0 \sim 30°$). The time interval during which $\Phi(t)$ is significant ($\sim$200 ms) is a very small fraction of the oscillation time period (about 2 s), confirming that the magnetic flux changes very rapidly as the magnet crosses the center of the coil. Because the angular velocity of the magnet is nearly constant in the central region, the time scale in Fig. 3 can be easily converted to the angular ($\theta = \omega t$) or linear ($x = R\theta$) scale. The points of inflection, where $d\Phi/dt$ (and therefore $d\Phi/d\theta$) are extremum, are at 4200 and 4300 ms, precisely where the peaks occur in the emf pulse (see Fig. 2).

IV. VERIFICATION OF FARADAY'S LAW

For $\theta_0 \geqslant 5°$, the angular velocity of the bar magnet is very nearly constant in the narrow angular range near the mean position, and hence the peak emf $\mathcal{E}_{max}$ is approximately given by

$$\mathcal{E}_{max} \approx \left(\frac{d\Phi}{d\theta}\right)_{max} \omega_{max}. \tag{3}$$

The maximum angular velocity ω_{max} depends on θ_0 through the simple relation (see the Appendix)

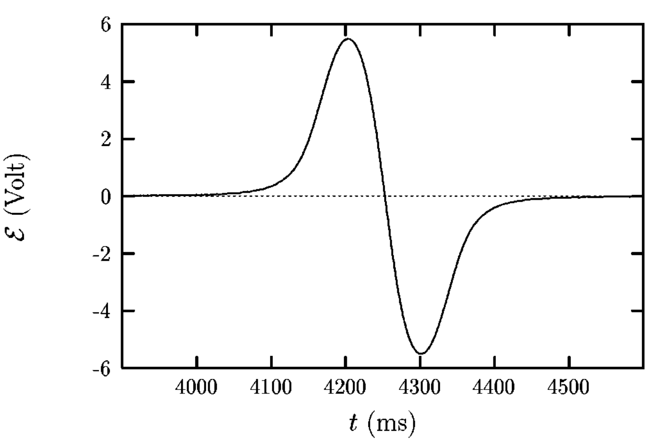

Fig. 2. A typical induced emf pulse.

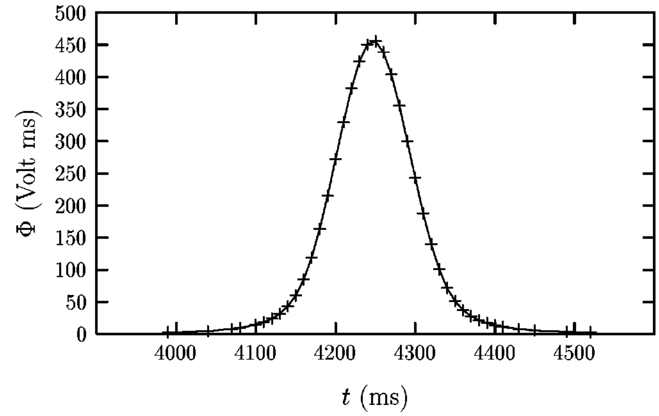

Fig. 3. Plot of the magnetic flux Φ through the coil with time t, showing the rapid change as the magnet crosses the center of the coil.

$$\omega_{max} = \frac{4\pi}{T} \sin(\theta_0/2), \tag{4}$$

where T is the period of (small) oscillations. Therefore if $\theta_0/2$ (in rad) is small compared to 1, then ω_{max} is nearly proportional to θ_0, and hence $\mathcal{E}_{max}$ approximately measures the angular amplitude θ_0.

Equation (3) provides a simple way for students to quantitatively verify Faraday's law. A plot of the peak emf $\mathcal{E}_{max}$ (conveniently obtained using the software feature `extrema`) versus ω_{max} [evaluated from Eq. (4)] for different angular amplitudes should show a linear dependence (for large θ_0). Although this behavior could be easily verified by students, the deviation from linearity expected for low angular amplitudes ($\theta_0 \sim 5°$), for which the θ dependence of the angular velocity $d\theta/dt$ is not negligible, turned out to be quite elusive. This small deviation was presumably washed out by the large percentage errors in θ_0 measurements, especially for small angles, precisely where this deviation is more pronounced.

An alternative approach, which eliminates the need for measuring the oscillation amplitude θ_0, is proposed below. If we take the time derivative of the induced emf and set $\theta = 0$, where the angular velocity is maximum, we obtain

$$\left(\frac{d\mathcal{E}}{dt}\right)_{\theta=0} = \left(\frac{d^2\Phi}{d\theta^2}\right)_{\theta=0} \omega_{max}^2, \tag{5}$$

which relates the slope of $\mathcal{E}$ at the mean position to ω_{max}^2. Because this relation holds for *all* amplitudes θ_0, it may be used to obtain ω_{max} for different angular amplitudes *without* the need for any angular measurement. The slope at the mean position (near zero crossing) is easily measured through linear interpolation (see Fig. 2). Thus, a plot of $\mathcal{E}_{max}$ vs $\sqrt{(d\mathcal{E}/dt)_{\theta=0}}$ should show both features of interest—the linear behavior for large angular amplitudes and the deviation for very low amplitudes. The key advantage of this plot lies in completely eliminating the errors associated with measurements of the oscillation amplitudes θ_0.

In our experiment the oscillations were started with a large initial angular amplitude, and during the gradual decay of oscillations, the peak voltages $\mathcal{E}_{max}$ (on both sides of the mean position) and the slope $(d\mathcal{E}/dt)_{\theta=0}$ were measured for a large number of pulses, so as to cover the full range from

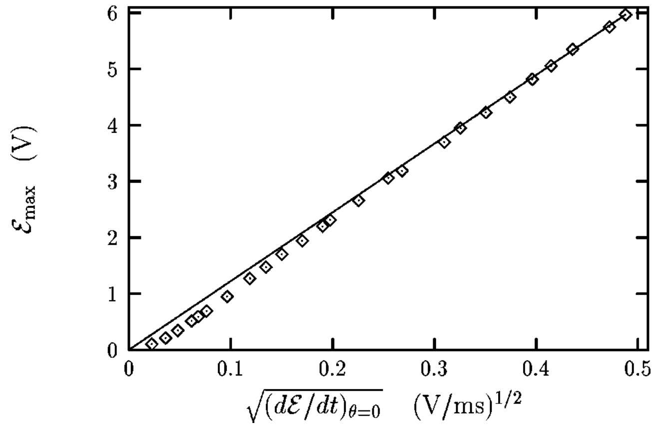

Fig. 4. Plot of $\mathcal{E}_{max}$ vs $(d\mathcal{E}/dt)^{1/2}_{\theta=0}$, showing the deviation from a straight line at low angular amplitudes.

large to very small angular amplitudes. Figure 4 shows the plot of the averaged $\mathcal{E}_{max}$ vs $\sqrt{(d\mathcal{E}/dt)_{\theta=0}}$, clearly showing the deviation from linearity for low angular amplitudes. To provide an idea of the angular amplitude scale, the peak voltage of ~ 6 V corresponds to an angular amplitude $\theta_0 \approx 35°$, so that the deviations become pronounced when $\theta_0 \approx 5°$.

V. ELECTROMAGNETIC DAMPING

To study the nature of the electromagnetic damping in the oscillating system, the coil was short-circuited by a low resistance (220 Ω). The oscillations were started with a large initial amplitude (with $\theta_0/2 \ll 1$), and the voltage $V(t)$ across the resistor was studied as a function of time. As the oscillations decayed, the peak voltages V_{max} for sample pulses were recorded at roughly equal time intervals. This voltage $V(t)$ is proportional to the current through the circuit, and hence to the induced emf $\mathcal{E}(t)$. Because the peak emf $\mathcal{E}_{max}$ is approximately proportional to the oscillation amplitude (except when the amplitude becomes too small), a plot of V_{max} vs t actually exhibits the decay of the oscillation amplitude with time.

Although an exponential decay of amplitude is more commonly encountered in damped systems, the plot of the normalized peak voltage $V_{max}(t)/V_{max}(0)$ vs t shows a distinctly linear decay (see Fig. 5). To distinguish the electromagnetic damping from other sources (for example, friction and air resistance), the same experiment was repeated in the open-circuit configuration, in which case electromagnetic damping is absent. In this case the amplitude decay is, as expected, much weaker, but significantly it is still approximately linear.

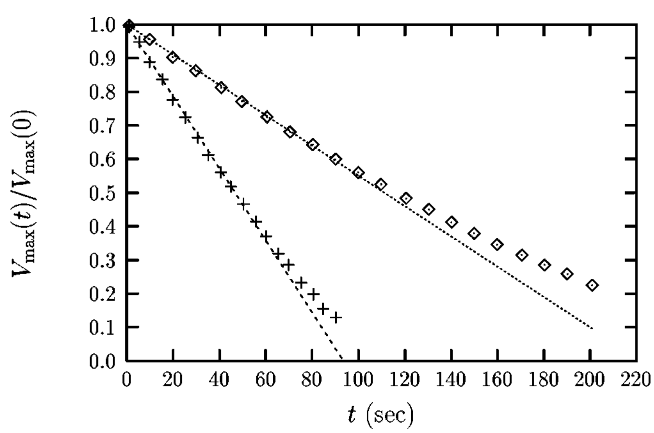

Fig. 5. The normalized peak voltage $V_{max}(t)/V_{max}(0)$ vs time t for the short-circuit (+) and open-circuit (◇) cases, showing nearly linear fall off.

A quantitative analysis of the energy loss provides an explanation for this nearly linear decay in both cases. We first consider the electromagnetic energy loss. Neglecting radiation losses, the main source of energy loss is Joule heating in the coil due to the induced current. If we integrate over one cycle, we have

$$\Delta E_{\text{one cycle}} = \int i^2 \mathcal{R}\, dt = \frac{1}{\mathcal{R}} \int \mathcal{E}^2 dt$$

$$= \frac{1}{\mathcal{R}} \int \left(\frac{d\Phi}{d\theta}\right)^2 \left(\frac{d\theta}{dt}\right)^2 dt, \tag{6}$$

where $\mathcal{R}$ is the coil resistance. Equation (6) may be further simplified because $d\Phi/d\theta$ is significant only in a narrow angular range near $\theta=0$ and rapidly vanishes outside. For amplitudes not too small, the angular velocity $d\theta/dt$ is nearly constant ($\approx \omega_{max}$) in this narrow angular range, and taking it outside the integral, we obtain

$$\Delta E_{\text{one cycle}} \approx \frac{\omega_{max}}{\mathcal{R}} \int \left(\frac{d\Phi}{d\theta}\right)^2 d\theta. \tag{7}$$

Because the angular integral is nearly independent of the initial amplitude θ_0, and therefore of ω_{max}, the energy loss per cycle is proportional to ω_{max}, and therefore to $\sqrt{E}$. On a long time scale ($t \gg T$), we therefore have

$$\frac{dE}{dt} = -k\sqrt{E}. \tag{8}$$

If we integrate Eq. (8) with the initial condition $E(0)=E_0$, we obtain

$$\sqrt{E_0} - \sqrt{E} \propto t, \tag{9a}$$

$$\Rightarrow \omega_{0,max} - \omega_{max} \propto t, \tag{9b}$$

$$\Rightarrow \mathcal{E}_{0,max} - \mathcal{E}_{max} \propto t, \tag{9c}$$

indicating linear decay of the peak emf, and therefore of the amplitude, with time.

We now consider the energy loss in the open-circuit case, where the damping is due to frictional losses. A frictional force proportional to velocity, as due to air resistance at low velocities, will result in an exponential decay of the oscillation amplitude. However, a function of the type $e^{-\alpha t}$ did not provide a good fit. On the other hand, assuming a constant frictional torque τ at the pivot, which is not unreasonable considering the contact forces at the pivot, we obtain for the energy loss in one cycle

$$\Delta E_{\text{one cycle}} = \int \tau\, d\theta = \tau\, 4\theta_0 \propto \omega_{max} \propto \sqrt{E}. \tag{10}$$

Equation (10) is similar to the earlier result of Eq. (8) for electromagnetic damping, yielding a linear decay of the oscillation amplitude with time and providing a much better fit to the observed data, as seen in Fig. 5. The deviation from linearity at large times is presumably due to a small air resistance term. In fact, if a damping term $-k_1 E$ due to air resistance is included in Eq. (8), the differential equation is easily solved, and the solution provides an excellent fit to the data. Finally, another term $-k_2E^{3/2}$ should be included in Eq. (8), arising from the reduction in the average centripetal force $Ml\omega^2\theta_0^2/2$ with the oscillation amplitude θ_0, which decreases the frictional force at the pivot due to the reduction in the normal reaction.

VI. SUMMARY

A pedagogically instructive study of electromagnetic induction and damping is made possible by attaching a PC interface to a conventional setup for studying Faraday's law. By eliminating all errors associated with angular measurements, the novel method applied to the verification of Faraday's law reveals subtle features associated with the underlying mechanics. A quantitative analysis of the distinctly linear decay of the oscillation amplitude due to electromagnetic damping reveals an interesting interplay of the electromagnetic and mechanical time scales.

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APPENDIX

If the system is released from rest with an angular displacement θ_0, then from energy conservation we have

$$\tfrac{1}{2}I\omega_{\max}^2 = Mgl(1-\cos\theta_0), \tag{A1}$$

where M is the mass of the system, I its moment of inertia about the pivot O, and l the distance from O to the center of gravity. For small oscillations, the equation of motion is $I\ddot{\theta} = -(Mgl)\theta$, so that the time period is given by

$$T = 2\pi\sqrt{\frac{I}{Mgl}}. \tag{A2}$$

If we eliminate I/Mgl from these two equations, we obtain

$$\omega_{\max} = \frac{4\pi}{T}\sin(\theta_0/2). \tag{A3}$$

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[3]J. A. Manzanares, J. Bisquert, G. Garcia-Belmonte, and M. Fernández-Alonso, "An experiment on magnetic induction pulses," Am. J. Phys. **62**, 702–706 (1994).
[4]P. Carpena, "Velocity measurements through magnetic induction," Am. J. Phys. **65**, 135–140 (1997).
[5]In our experimental setup the coil has a diameter of about 10 cm, about the same length, and consists of several thousand turns of insulated copper wire, and has a resistance of about 1000 Ω.
[6]PHYWE Systeme Gmbh, Göttingen, Germany.
[7]Identifying the time t_i for a given induced emf pulse $\mathcal{E}(t)$ is a good exercise for students.