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is 4.

Since the LOS rotation vector can be written as

$$\boldsymbol{\omega} = (\mathbf{v} \times \mathbf{r})/r^2,$$

then

$$\mathbf{a} = (K/r^2)\,(\mathbf{v} \times \mathbf{r}) \times \mathbf{v}. \qquad (1)$$

Clearly, **a** is perpendicular to **v**, making it purely a turning command, and its sense is to turn **v** into **r**.

In applying Eq. (1) to a computer simulation, one would probably work in an earth-fixed rectangular coordinate system. If $\mathbf{r}_m$, $\mathbf{r}_t$, $\mathbf{v}_m$, and $\mathbf{v}_t$ are the positions and velocities of missile and target, then, in Eq. (1), $\mathbf{r} = \mathbf{r}_t - \mathbf{r}_m$ and $\mathbf{v} = \mathbf{v}_m - \mathbf{v}_t$. One must expand Eq. (1) to obtain the x, y, and z components of **a** as functions of the components of $\mathbf{r}_m$, $\mathbf{r}_t$, $\mathbf{v}_m$, and $\mathbf{v}_t$.

Accelerations from Eq. (1) are commands to the missile: How will it respond to the commands? The most elementary procedure is to assume perfect response, letting the commands from Eq. (1) become *actual* missile acceleration. Two time integrations of **a** then yield missile position, while one integration of (constant) $\mathbf{v}_t$ yields target position. Note our tacit assumption that Eq. (1) specifies the total missile acceleration; forces of thrust, drag, gravity, etc., being neglected. In many missile designs, thrust for sustained flight is adjusted to be almost equal and opposite to drag, and the acceleration command includes a constant gravity bias term almost equal and opposite to the acceleration of gravity. Thus one can obtain a simple yet reasonably realistic guided missile simulation neglecting these factors.

For simulation on a computer, select a constant cycle time dt, and compute results for $t = 0$, $t = dt$, $t = 2dt$, etc., up until intercept occurs. Make dt small enough so that a hundred or more integration steps will be needed. In each compute cycle, first evaluate the components of **a**. Then assume **a** remains constant in that cycle, and use the kinematics of constant acceleration to advance the motion by one time increment. (Of course, there are elegant methods of numerical integration for use here, alternately.) Give your missile and target any reasonable initial positions and velocities, start the simulation, and watch your missile hit the target (almost) every time.

Reconstructed nineteenth-century experiment with physical pendula

Thomas B. Greenslade, Jr. and Aaron J. Owens[a)]
Kenyon College, Gambier, Ohio 43022
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We have used a reconstruction of a multiple physical pendulum apparatus which is now in the collection of the Smithsonian Institution as the basis for an independent investigation in our sophomore level Intermediate Mechanics course.

The multiple pendulum set is shown in Fig. 1. The original, which was donated to the Smithsonian by the Colgate University Physics department, has a bracket mounted at one end of the top bar which probably supported a simple pendulum. There is also a device for releasing the pendula simultaneously, suggesting that the device was intended for qualitative demonstrations of the relative rates of oscillation. The only reference to this apparatus which we have found is in Steele's *Fourteen Weeks in Natural Philosophy*,[1] which shows a device with the four shapes which we used, plus a simple pendulum of the same length. Steele's rather confusing qualitative discussion of the illustration emphasizes the different locations of the center of oscillation, the point at which all of the mass of the body would have to be concentrated to give same period as the physical pendulum.

The basic equation for the period of a physical pendulum is

$$T = 2\pi(I/mgl)^{1/2},$$

where I is the moment of inertia about the point of suspension, l is the distance from the point of suspension to the center of mass, and m is the mass of the pendulum. The students were required to find I and l for each of the four geometrical shapes and hence find an expression for T. They were then to make an experimental determination of the period and compare it with the predicted value.

The results which were obtained are shown in Table I. The overall length of a pendulum was denoted by h and the maximum radius by R. Within the tolerances possible on a woodworking lathe, the values of h and R were the same for all four bodies. The students were encouraged to make maximum use of symmetry and the perpendicular and parallel axis theorems in doing their calculations.

We did not expect good agreement between experiment and theory because the bases of the circular pyramids are chamfered, and the vertices are truncated. This will result in errors in both the moments of inertia and the centers of mass, and confirms our suspicion that the original demonstration was qualitative.

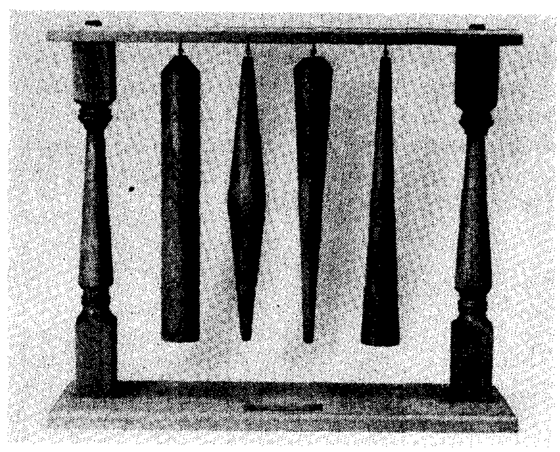

Fig. 1. Multiple physical pendulum set based on a similar piece of apparatus at the National Museum of History and Technology of the Smithsonian Institution (Catalog No. 318,740).

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Table I. Working equations and theoretical and experimental values of the periods of oscillation for the four geometrical shapes. The moment of inertia for a given shape is I, and the distance from the point of suspension to the center of mass is l. For the pendula shown in Fig. 1, $h = 0.377$ m and $R = 0.025$ m.

	Cone suspended from vertex	Cone suspended from base	Double cone	Cylinder
I	$(3/20)m(R^2+4h^2)$	$(3/20)m[R^2+(2/3)h^2]$	$(3/20)m[R^2+(11/6)h^2]$	$(m/12)(3R^2+4h^2)$
l	$3h/4$	$h/4$	$h/2$	$h/2$
T (theoretical) (sec)	1.103	0.782	0.915	1.008
T (experimental) (sec)	1.093 ± 0.008	0.848 ± 0.008	0.940 ± 0.011	1.003 ± 0.006

The apparatus was constructed by Gregory Sesler, of Kenyon College as a project for his Natural Philosophy course in the spring of 1978.

[a]Present address: Bartol Research Foundation of the Franklin Institute, University of Delaware, Newark, DE 19711.

[1]J. Dorman Steele, *Fourteen Weeks in Natural Philosophy* (Barnes, New York, 1869), p. 60.

Elliptical motion from a ball and spring

Mark Greiner
Xavier University, Cincinnati, Ohio 45238
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A spring and mass can behave quite erratically at times. A swinging or pendular motion can disrupt vertical oscillations under certain conditions. This phenomenum has been discussed by Olsson[1] and other authors.[2,3] The following note observes what further conditions are needed to cease any transfer of energy between the different modes of oscillation.

The energy transfer between the pendular and vertical motions was most pronounced when the angular frequency of the spring ω_s was twice the frequency of the pendulum ω_p and, in general, when

$$\omega_s/\omega_p = 2/n, \qquad n = 1, 2, 3, \ldots.$$

For $n = 1$, we set

$$(k/m)^{1/2} = 2(g/L)^{1/2}, \tag{1}$$

$$mg/k = L/4. \tag{2}$$

From Fig. 1, we see that the unstretched length of the spring is L_0, L is the length after stretching, and ΔL is the difference between L and L_0 which is due to the force exerted by the ball of mass m. Since $F = -k\Delta L$, then $\Delta L = mg/k$ for any mass on a spring. From Fig. 1 and Eq. (2), L can be solved in terms of L_0 which yields

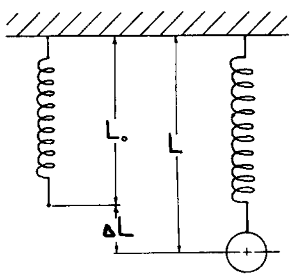

Fig. 1. Simple spring and mass oscillator. The force exerted by the spring is $-k\Delta L$, where k is the spring constant.

$$L = 4L_0/3. \tag{3}$$

To get $\omega_s = 2\omega_p$, add just enough mass to stretch the spring by one-third of its original length, neglecting the mass of the spring. This can be derived theoretically[1] rather than by this observational approach.

Where Olsson treated the cases for coupled oscillations, here we show that there exists a unique case where the equations of motion actually uncouple and yield independent vertical and pendular motions. This occurs when $\omega_s = \omega_p$. What is necessary to have the spring period equal to the period of the pendulum? If it is possible to meet these conditions, then the motion should be elliptical. The conditions are found in a manner similar to the above, which yields

$$L_0 = 0. \tag{4}$$

This is simply stating that the length of the pendulum must be attributed entirely to a stretching of the spring.

Olsson stated that "Since z_0 (equilibrium length of spring with mass, i.e., $L_0 + L$) is always greater than L_0, then subharmonic modes of the Mathieu equation corresponding to $n = 2, 3, 4, \ldots$, are never possible." [1] Ordinarily, this is impossible for the diagram shown in Fig. 1 because every spring has a nonzero equilibrium length. But by making a change in the apparatus, the condition for $\omega_s = \omega_p$ can be fulfilled. In Fig. 2, there is no mass present and the end of the string is at L_0. Now attach a mass to the end of the string so that the center of mass is where the end of the string was in Fig. 2 and let it hang freely. This is shown in Fig. 3. Now the entire length of the pendulum is due to a stretch of the spring and the frequencies of the two modes of oscillation are equal. It may also be mentioned that whatever mass is chosen, the entire length of the pendulum will still be attributed to a stretching of the spring and the

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